Gradient discretization of two-phase flows in fractured and deformable porous media

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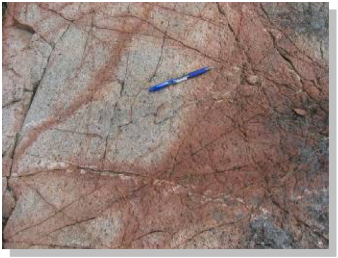
NEMESIS – NEw generation MEthods for numerical SImulationS

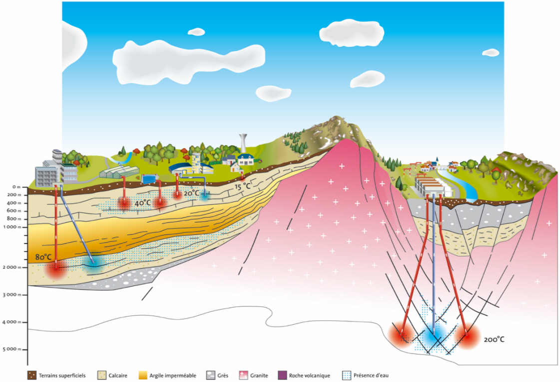
June 14-15, 2021

Fractured/faulted porous media: multiple scales (figures from J. R. de Dreuzy, Geosciences Rennes and Inria)

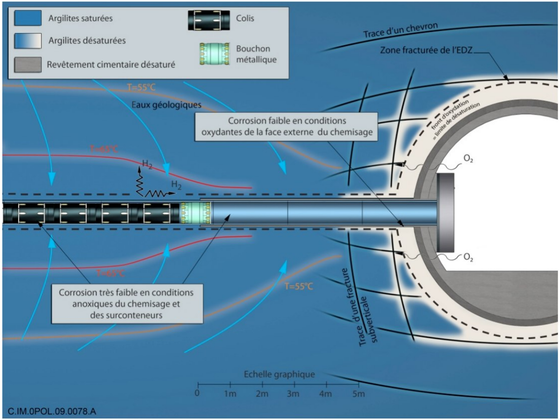


Fractured/faulted porous media: applications

- Oil and gas
- Hydrogeology
- Geothermal energy
- Geological storages
- Soil remediation
- ...



Motivation



- Representing the fracture network of the Excavation Damaged Zone (EDZ)
- Capturing the influence of gas pressure on fractures width

Outline

- Modelling concepts
- Discretization
  - Gradient discretization
  - Convergence analysis
- Numerical examples
  - Gas injection
  - Drying by suction
- Extensions
  - Discontinuous pressure model
  - Frictional contact

Two-phase Darcy flow: generalized Darcy law

$\alpha = \mathrm{w}$: wetting phase

$\alpha = \mathrm{nw}$: non wetting phase

p^α : phase pressure

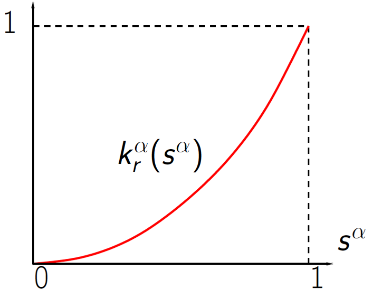
s^α : phase saturation

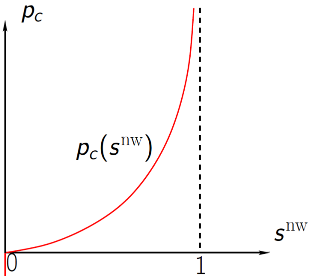
$$\mathbf{q}^\alpha = -\underbrace{\frac{k_r^\alpha(s^\alpha)}{\mu^\alpha}}_{\eta^\alpha(s^\alpha)} \mathbb{K}(\mathbf{x})(\nabla p^\alpha - \rho^\alpha \mathbf{g})$$

$k_r^\alpha(s^\alpha)$: phase relative permeability

$\eta^\alpha(s^\alpha)$: phase mobility

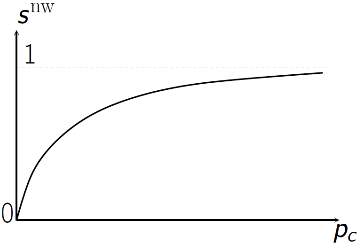
$p_c(s^{\mathrm{nw}}) = p^{\mathrm{nw}} - p^{\mathrm{w}}$: capillary pressure





Two-phase Darcy flow: incompressible flow in phase pressure formulation

$$s^{\mathrm{nw}} = S^{\mathrm{nw}}(p_c),$$
$$s^{\mathrm{w}} = S^{\mathrm{w}}(p_c) = 1 - S^{\mathrm{nw}}(p_c)$$



$$\left\{ \begin{array}{l} \partial_t(\phi S^\alpha(p_c)) + \mathrm{div}(\mathbf{q}^\alpha) = 0, \;\; \alpha = \mathrm{nw}, \mathrm{w}, \\ p_c = p^{\mathrm{nw}} - p^{\mathrm{w}} \end{array} \right.$$

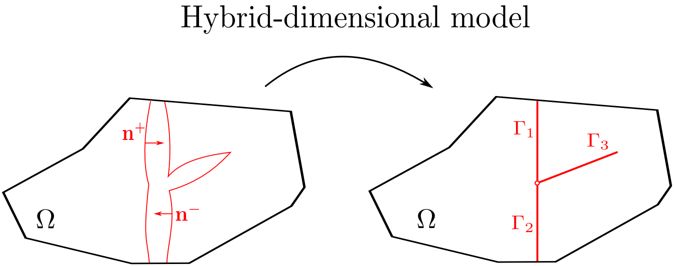
with

$$\mathbf{q}^\alpha = -\eta^\alpha(S^\alpha(p_c))\, \mathbb{K}(\mathbf{x})(\nabla p^\alpha - \rho^\alpha \mathbf{g}), \quad \alpha = \mathrm{nw}, \mathrm{w}.$$

Hybrid-dimensional model

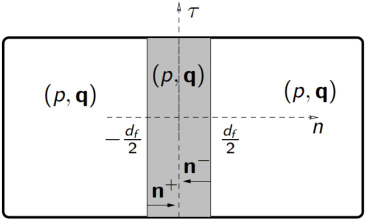
[Granet et al 2001], [Jaffré et al. 2002], [Bogdanov et al 2003], [Faille et al 2003], [Karimi Fard 2004], [Jaffré et al. 2005], [Angot et al. 2009], [Girault et al 2015], [Hanowski et al 2016]

- **Dimensional reduction**: averaging the model equations over the fracture width
- **Objectives**: facilitate the mesh generation and lower the number of degrees of freedom



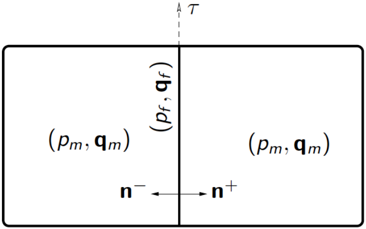
Hybrid-dimensional model (flow)

equi-dimensional model:



$\rightsquigarrow$

hybrid-dimensional model:



$$\begin{cases} \operatorname{div}(\mathbf{q}) = h(\mathbf{x}), \\ \mathbf{q} = -\mathbb{K}(\mathbf{x})\nabla p. \end{cases} \quad \rightsquigarrow \quad \begin{cases} \operatorname{div}(\mathbf{q}_m) = h_m, \\ \mathbf{q}_m = -\mathbb{K}_m \nabla p_m, \\ \operatorname{div}_\tau(\mathbf{q}_f) - [\![\mathbf{q}_m]\!] = h_f \\ \mathbf{q}_f = -d_f \mathbb{K}_f \nabla_\tau p_f. \end{cases}$$

with $[\![\mathbf{q}_m]\!] = \mathbf{q}_m \cdot \mathbf{n}^+ + \mathbf{q}_m \cdot \mathbf{n}^-$.

$+$ **transmission conditions** at matrix fracture interfaces.

Main modelling concepts (flow)

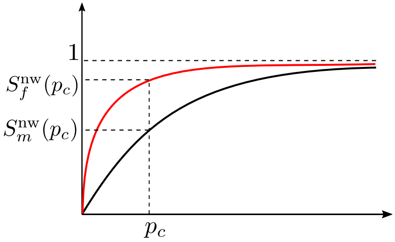
- **Poiseuille's law** for the *tangential velocity* in the fractures, extended to two-phase flow using generalized Darcy laws

$$\mathbf{q}_f^\alpha = -\eta_f^\alpha(s_f^\alpha)\left(\frac{1}{12}d_f^3\right)\nabla_\tau p_f^\alpha$$

- **Continuous phase pressures** at matrix fracture interfaces:

$$p_m^\alpha = p^\alpha \quad \text{and} \quad p_f^\alpha = \gamma p^\alpha$$

- **Discontinuous saturations** at matrix fracture interfaces due to different capillary pressure functions



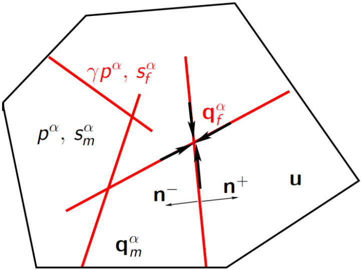
Two-phase hybrid-dimensional Darcy flow: $\alpha \in \{\mathrm{nw}, \mathrm{w}\}$

$$\partial_t \left(\phi_m s_m^\alpha\right) + \mathrm{div}\left(\mathbf{q}_m^\alpha\right) = h_m^\alpha,$$

$$\mathbf{q}_m^\alpha = -\eta_m^\alpha(s_m^\alpha)\mathbb{K}_m \nabla p^\alpha,$$

$$\partial_t \left(d_f s_f^\alpha\right) + \mathrm{div}_\tau(\mathbf{q}_f^\alpha) - [\![\mathbf{q}_m^\alpha]\!] = h_f^\alpha$$

$$\mathbf{q}_f^\alpha = -\eta_f^\alpha(s_f^\alpha)\left(\frac{1}{12}d_f^3\right)\nabla_\tau \gamma p^\alpha,$$



$$s_m^\alpha = S_m^\alpha(p_c),$$

$$s_f^\alpha = S_f^\alpha(\gamma p_c).$$

with $[\![\mathbf{q}_m^\alpha]\!] = \mathbf{q}_m^\alpha \cdot \mathbf{n}^+ + \mathbf{q}_m^\alpha \cdot \mathbf{n}^-$.

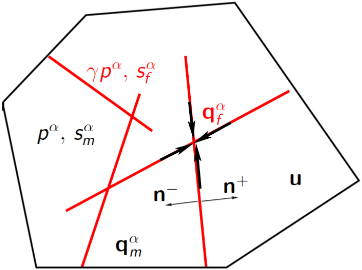
Linear poro-elastic mechanical model

$$-\text{div}\, \sigma^T(\mathbf{u}) = \mathbf{f},$$

with

$$\sigma^T(\mathbf{u}) = \sigma(\mathbf{u}) - b\, p_m^E \mathbb{I},$$

$$\sigma(\mathbf{u}) = 2\mu\, \epsilon(\mathbf{u}) + \lambda\, \text{div}(\mathbf{u})\, \mathbb{I}.$$

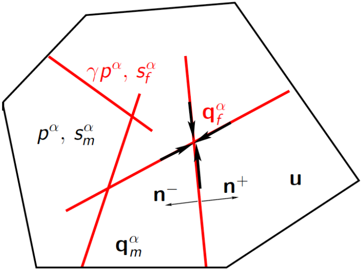


Fracture model with **no contact**: $\sigma^T(\mathbf{u})\mathbf{n}^\pm = -p_f^E \mathbf{n}^\pm$,

p_m^E, p_f^E: matrix and fracture **equivalent pressures**.

Closure laws

$$\begin{cases} \partial_t \phi_m = b \, \mathrm{div} \partial_t \mathbf{u} + \frac{1}{M} \partial_t p_m^E, \\ d_f = -[\![\mathbf{u}]\!]. \end{cases}$$



Following [Coussy], the *equivalent pressures* p_m^E and p_f^E are based on the **capillary energy density** $U_{\mathrm{rt}}(p_c) = \int_0^{p_c} q \left(S_{\mathrm{rt}}^{\mathrm{nw}}\right)'(q) dq$, $\mathrm{rt} = m, f$ and

$$p_m^E = \sum_{\alpha \in \{\mathrm{nw}, \mathrm{w}\}} p^\alpha \; s_m^\alpha - U_m(p_c),$$

$$p_f^E = \sum_{\alpha \in \{\mathrm{nw}, \mathrm{w}\}} \gamma p^\alpha \; s_f^\alpha - U_f(\gamma p_c).$$

Choice of the equivalent pressures

It is motivated by the energy estimate resulting formally from:

$$\sum_{\alpha\in\{\mathrm{nw},\mathrm{w}\}} p^\alpha \partial_t\Big(\phi_m S_m^\alpha(p_c)\Big) - b\, p_m^E \mathrm{div}(\partial_t \mathbf{u}) \\ = \partial_t\Big(\phi_m U_m(p_c) + \tfrac{1}{2M}(p_m^E)^2\Big),$$

$$\sum_{\alpha\in\{\mathrm{nw},\mathrm{w}\}} \gamma p^\alpha \partial_t\Big(d_f S_f^\alpha(\gamma p_c)\Big) + p_f^E [\![\partial_t \mathbf{u}]\!] \\ = \partial_t\Big(d_f U_f(\gamma p_c)\Big).$$

Weak solution: $\bar{p}^\alpha$ and $\bar{\mathbf{u}}$ such that for all smooth test functions $\bar{\varphi}^\alpha$ and $\bar{\mathbf{v}}$:

$$\int_0^T \int_\Omega \Big(-\bar{\phi}_m \bar{s}_m^\alpha \partial_t \bar{\varphi}^\alpha + \eta_m^\alpha(\bar{s}_m^\alpha) \mathbb{K}_m \nabla \bar{p}^\alpha \cdot \nabla \bar{\varphi}^\alpha \Big) \mathrm{d}\mathbf{x}\mathrm{d}t$$
$$+ \int_0^T \int_\Gamma \Big(-\bar{d}_f \bar{s}_f^\alpha \partial_t \gamma \bar{\varphi}^\alpha + \eta_f^\alpha(\bar{s}_f^\alpha) \frac{\bar{d}_f^3}{12} \nabla_\tau \gamma \bar{p}^\alpha \cdot \nabla_\tau \gamma \bar{\varphi}^\alpha \Big) \mathrm{d}\sigma(\mathbf{x})\mathrm{d}t$$
$$- \int_\Omega \bar{\phi}_m^0 \bar{s}_m^{\alpha,0} \bar{\varphi}^\alpha(0,\cdot) \mathrm{d}\mathbf{x} - \int_\Gamma \bar{d}_f^0 \bar{s}_f^{\alpha,0} \gamma \bar{\varphi}^\alpha(0,\cdot) \mathrm{d}\sigma(\mathbf{x})$$
$$= \int_0^T \int_\Omega h_m^\alpha \bar{\varphi}^\alpha \mathrm{d}\mathbf{x}\mathrm{d}t + \int_0^T \int_\Gamma h_f^\alpha \gamma \bar{\varphi}^\alpha \mathrm{d}\sigma(\mathbf{x})\mathrm{d}t, \quad \alpha \in \{\mathrm{w}, \mathrm{nw}\},$$

$$\int_0^T \int_\Omega \Big(\sigma(\bar{\mathbf{u}}) : \varepsilon(\bar{\mathbf{v}}) - b \bar{p}_m^E \mathrm{div}(\bar{\mathbf{v}}) \Big) \mathrm{d}\mathbf{x}\mathrm{d}t + \int_0^T \int_\Gamma \bar{p}_f^E \, [\![\bar{\mathbf{v}}]\!] \mathrm{d}\sigma(\mathbf{x})\mathrm{d}t$$
$$= \int_0^T \int_\Omega \mathbf{f} \cdot \bar{\mathbf{v}} \mathrm{d}\mathbf{x}\mathrm{d}t,$$

Gradient discretization: motivation

- Abstract discretization framework accounting for a large class of conforming and non conforming discretizations (FEM, FV, HMM, HHO, VEM, ...) and based on
  - Vector spaces of discrete unknowns
  - Reconstruction operators
  - Discrete variational formulation

- Allow a generic stability and convergence analysis under general properties such as
  - Coercivity (discrete Poincaré inequality)
  - Consistency
  - Limit Conformity (for non-conforming methods)
  - Compacity

Gradient discretization of the poro-mechanical model

Two-phase flow

$X^0_{\mathcal{D}_p}$ = space of discrete unknowns. Reconstruction operators:

- gradient operators on matrix and fracture network

$$\nabla^m_{\mathcal{D}_p} : X^0_{\mathcal{D}_p} \to L^\infty(\Omega)^d, \qquad \nabla^f_{\mathcal{D}_p} : X^0_{\mathcal{D}_p} \to L^\infty(\Gamma)^{d-1};$$

- **piecewise-constant** function operators on matrix and fracture network

$$\Pi^m_{\mathcal{D}_p} : X^0_{\mathcal{D}_p} \to L^\infty(\Omega), \qquad \Pi^f_{\mathcal{D}_p} : X^0_{\mathcal{D}_p} \to L^\infty(\Gamma).$$

Assume $\|v\|_{\mathcal{D}_p} := \|\nabla^m_{\mathcal{D}_p} v\|_{L^2(\Omega)^d} + \|d_0^{3/2} \nabla^f_{\mathcal{D}_p} v\|_{L^2(\Gamma)^{d-1}}$ to be a norm on $X^0_{\mathcal{D}_p}$.

Poromechanics

$X^0_{\mathcal{D}_\mathbf{u}}$ = space of discrete unknowns. Reconstruction operators:

- symmetric gradient operator $\mathbb{e}_{\mathcal{D}_\mathbf{u}} : X^0_{\mathcal{D}_\mathbf{u}} \to L^2(\Omega, \mathcal{S}_d(\mathbb{R}))$,
- displacement function operator $\Pi_{\mathcal{D}_\mathbf{u}} : X^0_{\mathcal{D}_\mathbf{u}} \to L^2(\Omega)^d$,
- normal jump function operator $[\![\cdot]\!]_{\mathcal{D}_\mathbf{u}} : X^0_{\mathcal{D}_\mathbf{u}} \to L^4(\Gamma)$.

Assume $\|\mathbf{v}\|_{\mathcal{D}_\mathbf{u}} := \|\mathbb{e}_{\mathcal{D}_\mathbf{u}}(\mathbf{v})\|_{L^2(\Omega)}$ to be a norm on $X^0_{\mathcal{D}_\mathbf{u}}$.

Gradient scheme

Find $p^\alpha \in (X^0_{\mathcal{D}_p})^{N+1}$ and $\mathbf{u} \in (X^0_{\mathcal{D}_\mathbf{u}})^{N+1}$ such that for all $\varphi^\alpha \in (X^0_{\mathcal{D}_p})^{N+1}$, $\mathbf{v} \in (X^0_{\mathcal{D}_\mathbf{u}})^{N+1}$:

$$\int_0^T \int_\Omega \Big(\delta_t \Big(\phi_{\mathcal{D}} \Pi^m_{\mathcal{D}_p} s^\alpha_m \Big) \Pi^m_{\mathcal{D}_p} \varphi^\alpha + \eta^\alpha_m (\Pi^m_{\mathcal{D}_p} s^\alpha_m) \mathbb{K}_m \nabla^m_{\mathcal{D}_p} p^\alpha \cdot \nabla^m_{\mathcal{D}_p} \varphi^\alpha \Big) \mathrm{d}\mathbf{x}\mathrm{d}t$$
$$+ \int_0^T \int_\Gamma \delta_t \Big(d_{f,\mathcal{D}_\mathbf{u}} \Pi^f_{\mathcal{D}_p} s^\alpha_f \Big) \Pi^f_{\mathcal{D}_p} \varphi^\alpha \mathrm{d}\sigma(\mathbf{x})$$
$$+ \int_0^T \int_\Gamma \eta^\alpha_f (\Pi^f_{\mathcal{D}_p} s^\alpha_f) \frac{d^3_{f,\mathcal{D}_\mathbf{u}}}{12} \nabla^f_{\mathcal{D}_p} p^\alpha \cdot \nabla^f_{\mathcal{D}_p} \varphi^\alpha \mathrm{d}\sigma(\mathbf{x})\mathrm{d}t$$
$$= \int_0^T \int_\Omega h^\alpha_m \Pi^m_{\mathcal{D}_p} \varphi^\alpha \mathrm{d}\mathbf{x}\mathrm{d}t + \int_0^T \int_\Gamma h^\alpha_f \Pi^f_{\mathcal{D}_p} \varphi^\alpha \mathrm{d}\sigma(\mathbf{x})\mathrm{d}t, \quad \alpha \in \{\mathrm{w}, \mathrm{nw}\},$$

$$\int_0^T \int_\Omega \Big(\sigma_{\mathcal{D}_u}(\mathbf{u}) : \epsilon_{\mathcal{D}_\mathbf{u}}(\mathbf{v}) - b(\Pi^m_{\mathcal{D}_p} p^E_m) \mathrm{div}_{\mathcal{D}_\mathbf{u}}(\mathbf{v}) \Big) \mathrm{d}\mathbf{x}\mathrm{d}t$$
$$+ \int_0^T \int_\Gamma (\Pi^f_{\mathcal{D}_p} p^E_f) [\![\mathbf{v}]\!]_{\mathcal{D}_\mathbf{u}} \mathrm{d}\sigma(\mathbf{x})\mathrm{d}t = \int_0^T \int_\Omega \mathbf{f} \cdot \Pi_{\mathcal{D}_\mathbf{u}} \mathbf{v} \mathrm{d}\mathbf{x}\mathrm{d}t,$$

Convergence result

There are $\bar{p}^\alpha \in L^2(\mathbb{T}; V^0)$ and $\bar{\mathbf{u}} \in L^\infty(\mathbb{T}; \mathbf{U}^0)$ satisfying the weak formulation s.t.

$\Pi^m_{\mathcal{D}^l_p} p^\alpha_l \rightharpoonup \bar{p}^\alpha$	weakly in $L^2(\mathbb{T}; L^2(\Omega))$,
$\Pi^f_{\mathcal{D}^l_p} p^\alpha_l \rightharpoonup \gamma\bar{p}^\alpha$	weakly in $L^2(\mathbb{T}; L^2(\Gamma))$,
$\Pi_{\mathcal{D}^l_{\mathbf{u}}} \mathbf{u}^l \rightharpoonup \bar{\mathbf{u}}$	weakly-$\star$ in $L^\infty(\mathbb{T}; L^2(\Omega)^d)$,
$\phi_{\mathcal{D}^l} \rightharpoonup \bar{\phi}_m$	weakly-$\star$ in $L^\infty(\mathbb{T}; L^2(\Omega))$,
$d_{f,\mathcal{D}^l_{\mathbf{u}}} \to \bar{d}_f$	in $L^\infty(\mathbb{T}; L^p(\Gamma))$ for $2 \leqslant p < 4$,
$\Pi^m_{D^l_p} S^\alpha_m(p^l_c) \to S^\alpha_m(\bar{p}_c)$	in $L^2(\mathbb{T}; L^2(\Omega))$,
$\Pi^f_{D^l_p} S^\alpha_f(p^l_c) \to S^\alpha_f(\gamma\bar{p}_c)$	in $L^2(\mathbb{T}; L^2(\Gamma))$,

where $\bar{\phi}_m = \bar{\phi}^0_m + b\ \mathrm{div}(\bar{\mathbf{u}} - \bar{\mathbf{u}}^0) + \dfrac{1}{M}(\bar{p}^E_m - \bar{p}^{E,0}_m)$ and $\bar{d}_f = -[\![\bar{\mathbf{u}}]\!]$.

Convergence analysis: main assumptions

- The sequences $(\mathcal{D}_p^l)_{l\in\mathbb{N}}$, $(\mathcal{D}_{\mathbf{u}}^l)_{l\in\mathbb{N}}$, $\{(t_n^l)_{n=0}^{N^l}\}_{l\in\mathbb{N}}$ of space time Gradient Discretizations satisfy **coercivity, consistency, limit-conformity and compactness** properties.

- There exist a solution $p_l^\alpha \in (X^0_{\mathcal{D}_p^l})^{N^l+1}$, $\mathbf{u}^l \in (X^0_{\mathcal{D}_{\mathbf{u}}^l})^{N^l+1}$ such that
  (i) $\phi_{\mathcal{D}^l}(t,\mathbf{x}) \geqslant \phi_{m,\min}$ for a.e. $(t,\mathbf{x}) \in \mathbb{T}\times\Omega$,
  (ii) $d_{f,\mathcal{D}_{\mathbf{u}}^l}(t,\mathbf{x}) \geqslant d_0(\mathbf{x})$ for a.e. $(t,\mathbf{x}) \in \mathbb{T}\times\Gamma$, where $d_0 \geqslant 0$ is continuous and vanishes only at the fracture tips.

- Mobility function $\eta_{\mathrm{rt}}^\alpha$ continuous, non-decreasing, such that

$$0 < \eta^\alpha_{\mathrm{rt},\min} \leqslant \eta^\alpha_{\mathrm{rt}}(s) \leqslant \eta^\alpha_{\mathrm{rt},\max} < +\infty \quad \forall s \in [0,1]$$

Main convergence result – Comments

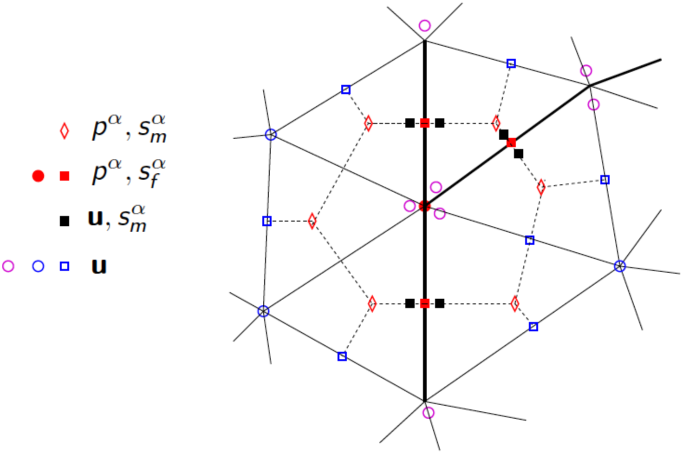
- **Existing analysis (Girault et al., '15): single-phase flow, linear case, d_f^3 freezed**

Main steps of the proof

- Energy estimates by suitable test functions
- Weak estimates on time derivatives
- Strong convergence of s_m^α
  - Separate matrix from fractures using cut-off functions
  - Time and space translates estimates
  - Recover compacity on the full domain from $s_m^\alpha \in [0,1]$
- Strong convergences of $d_f s_f^\alpha$, d_f^α and s_f^α
  - Uniform in time weak in space estimates + discrete Ascoli–Arzelà theorem
  - Isolate fracture tips for compacity in space of s_f
  - Recover compacity on the full domain from $s_f^\alpha \in [0,1]$
- Identification of the limit fields and weak solution

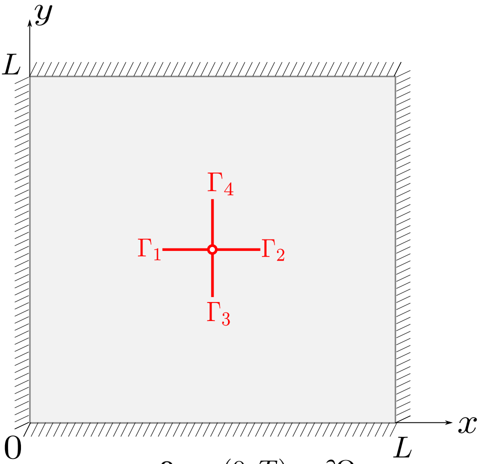
Numerical experiments: TPFA - $\mathbb{P}_2$ discretization

- Two-Point Flux Approximation (TPFA) scheme for Darcy
- $\mathbb{P}_2$ elements for mechanics



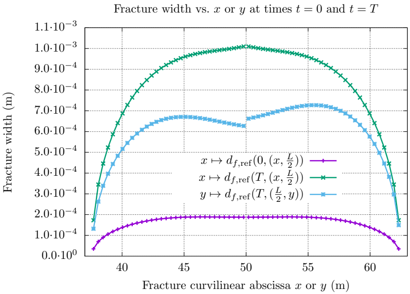
Numerical experiment: convergence test

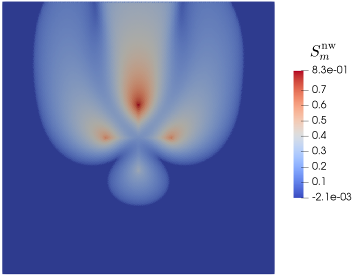
- Gas injection in the fractures
- $K_m = 3.\ 10^{-15}$ m^2
- $\phi_m^0 = 0.2$,
- $p_{c,m}(s) = -10^4 \log(1-s)$,
- $p_{c,f}(s) = -10 \log(1-s)$,
- $\lambda = 1.5$ GPa,
- $\mu = 2$ GPa,
- $M = 18.4$ GPa,
- $b = 0.81$,
- $T = 1000$ days

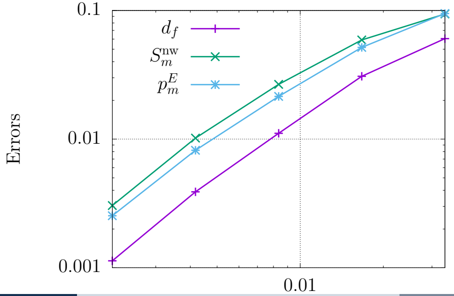


$\mathbf{u} = \mathbf{0}$ on $(0,T) \times \partial\Omega$

Numerical experiment: convergence test







Numerical experiment: fixed point algorithm [Kim et al 2011, Girault et al 2016]

We define the following "fixed stress" type fixed point function:

$$\mathbf{g}_{p,\mathbf{u}} \begin{pmatrix} \widetilde{\mathbf{u}} \\ \widetilde{p}_m^E \\ \widetilde{p}_f^E \end{pmatrix} = \begin{pmatrix} \mathbf{u} \\ p_m^E \\ p_f^E \end{pmatrix}, \text{ with } \begin{pmatrix} \widetilde{\mathbf{u}} \\ \widetilde{p}_m^E \\ \widetilde{p}_f^E \end{pmatrix} \underset{\substack{\text{Darcy} \\ \text{Solve}}}{\longrightarrow} \begin{pmatrix} p_m^E \\ p_f^E \end{pmatrix} \underset{\substack{\text{Mechanics} \\ \text{Solve}}}{\longrightarrow} \mathbf{u},$$

where the Darcy solve of the time step n uses

$$\phi_m = \phi_m^{n-1} + \mathrm{div}\Big(\widetilde{\mathbf{u}} - \mathbf{u}^{n-1}\Big) + \frac{1}{M}\Big(p_m^E - p_m^{E,n-1}\Big) + C_{r,m}\Big(p_m^E - \widetilde{p}_m^E\Big),$$

$$d_f = d_f^{n-1} - [\![\widetilde{\mathbf{u}} - \mathbf{u}^{n-1}]\!] + C_{r,f}\Big(p_f^E - \widetilde{p}_f^E\Big),$$

in the accumulation terms and $d_f = d_f^{n-1} - [\![\widetilde{\mathbf{u}} - \mathbf{u}^{n-1}]\!]$ in the fracture conductivity.

Fixed point vs non linear GMRes acceleration

We compare the fixed point algorithm to its non linear GMRes acceleration for different time stepping and either $C_{r,m} = C_{r,f} = 0$ (only for nlgmres) or

$$C_{r,m} = \frac{b^2}{2\mu + 2\lambda}, \quad C_{r,f} = \widetilde{d}_f C_{r,m}, \quad \widetilde{d}_f = 10^{-3}\,\mathrm{m},$$

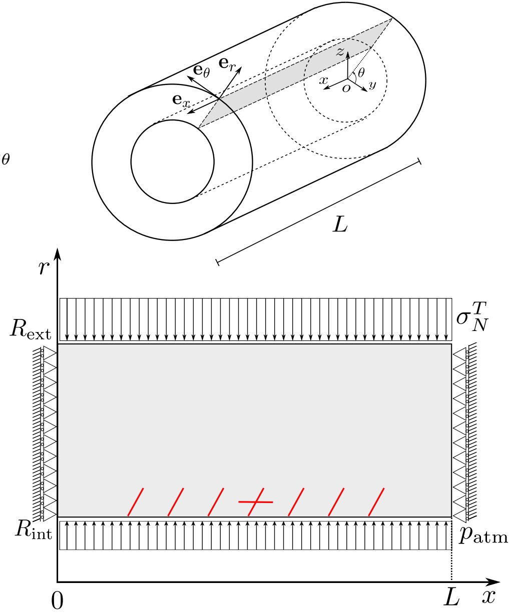
δt^{init}	Algorithm	$N_{\Delta t}$	N_{Newton}	$N_{\text{FixedPoint}}$	CPU (s)
0.1 d	Fixed-point	139	2621	2060	238
0.1 d	nlgmres	139	1088	705	118
0.1 d	nlgmres $C_{r,\mathrm{rt}} = 0$	139	1300	900	140
0.025 d	Fixed-point	153	8009	7351	675
0.025 d	nlgmres	153	1588	1118	159
0.025 d	nlgmres $C_{r,\mathrm{rt}} = 0$	153	1349	903	139
10^{-3} d	Fixed-point	x	x	x	x
10^{-3} d	nlgmres	187	2370	1601	208
10^{-3} d	nlgmres $C_{r,\mathrm{rt}} = 0$	187	2197	1453	192

Numerical experiment: desaturation by suction

- Porous medium initially water-saturated and prestressed:

  $$\sigma^0 = \sigma_x^0 \mathbf{e}_x \otimes \mathbf{e}_x + \sigma_r^0 \mathbf{e}_r \otimes \mathbf{e}_r + \sigma_\theta^0 \mathbf{e}_\theta \otimes \mathbf{e}_\theta$$

- Atmospheric gas pressure and fixed gas saturation at the bottom boundary
- Axisymmetric model with a 2D triangular mesh of 28945 cells of the xr-domain $(0, 10\,m) \times (5\,m, 35\,m)$
- Fracture network in the EDZ: 7 oblique fractures and 1 horizontal fracture
- Data set provided by Andra

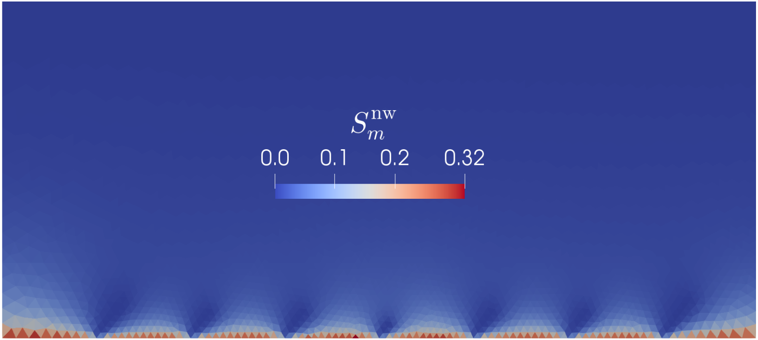


Data set

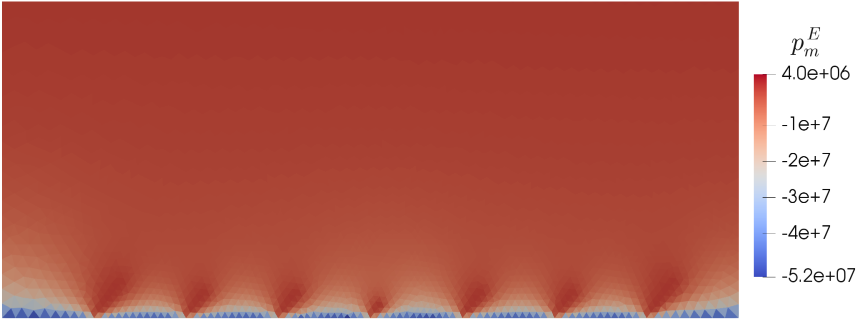
- **Lamé coefficients**: $\lambda = 1.5\,\mathrm{GPa}$, $\mu = 2\,\mathrm{GPa}$
- **Biot coefficient**: $b = 1$
- **Biot Modulus**: $M = 1\,\mathrm{GPa}$
- **Pre-stresses**: $\sigma_x^0 = 16\,\mathrm{MPa}$, $\sigma_r^0 = \sigma_\theta^0 = 12\,\mathrm{MPa}$
- **Initial aperture**: $d_f = 1$ cm,
- **Matrix rock type**: Callovo Oxfordian argilite
  - $K_m = 5 \cdot 10^{-20}\,\mathrm{m}^2$, $\phi_m^0 = 0.15$
  - $p_{c,m}(s) = -2.\ 10^8 \log(1-s)$,
- **Fracture rock type**:
  - $K_f = d_f^2/12$,
  - $p_{c,f}(s) = -10^2 \log(1-s)$,
- **Simulation time**: 200 years

Numerical experiment: desaturation by suction

Gas matrix saturation at final time



Numerical experiment: desaturation by suction

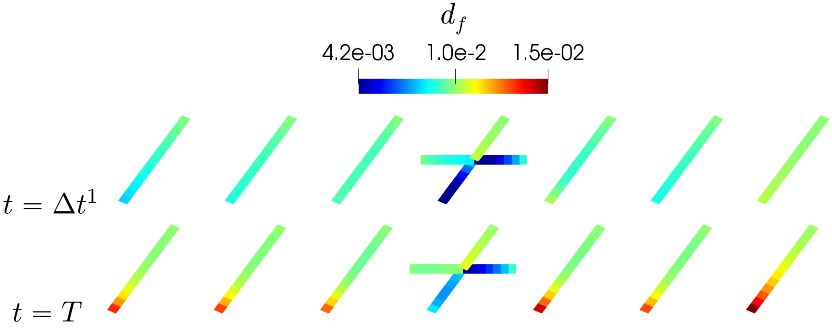


Numerical experiment: desaturation by suction

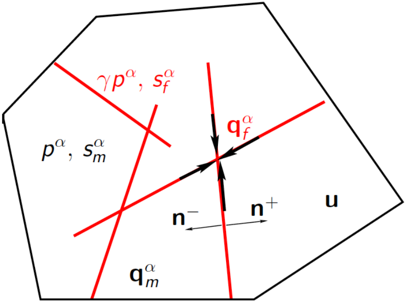
Porosity at final time

Numerical experiment: desaturation by suction

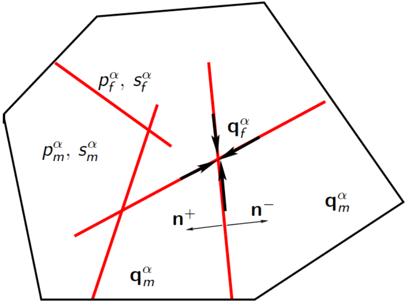
Fracture aperture at first time step and at final time



Continuous pressure model

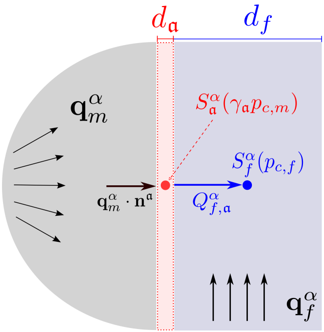


discontinuous pressure model



Continuous vs. discontinuous pressure models

- **Discontinuous pressure model** ($\mathfrak{a} = \pm$, damaged rock) [Droniou et al 2019]:



$$Q_{f,\mathfrak{a}}^\alpha \approx \eta_{\mathfrak{a}}^\alpha(S_{\mathfrak{a}}^\alpha(\gamma_{\mathfrak{a}} p_{c,m}))K_{f,\mathbf{n}}\left(\frac{\gamma_{\mathfrak{a}} p_m^\alpha - p_f^\alpha}{\frac{d_f}{2}} - \rho^\alpha \mathbf{g}\cdot\mathbf{n}^{\mathfrak{a}}\right)^+ - \eta_f^\alpha(S_f^\alpha(p_{c,f}))K_{f,\mathbf{n}}\left(\frac{\gamma_{\mathfrak{a}} p_m^\alpha - p_f^\alpha}{\frac{d_f}{2}} - \rho^\alpha \mathbf{g}\cdot\mathbf{n}^{\mathfrak{a}}\right)^-$$

$$\mathbf{q}_m^\alpha \cdot \mathbf{n}^{\mathfrak{a}} - Q_{f,\mathfrak{a}}^\alpha = d_{\mathfrak{a}}\phi_{\mathfrak{a}}\partial_t S_{\mathfrak{a}}^\alpha(\gamma_{\mathfrak{a}} p_{c,m})$$

- **Continuous pressure model** $\left(\frac{K_{f,\mathbf{n}}}{d_f} \gg \frac{K_{m,\mathbf{n}}}{L}\right)$:

$$\gamma_+ p_m^\alpha = \gamma_- p_m^\alpha = p_f^\alpha$$

Gradient scheme for the discontinuous pressure model

Find $(p_m^\alpha, p_f^\alpha) \in (X^0_{\mathcal{D}_p})^{N+1}$ and $\mathbf{u} \in (X^0_{\mathcal{D}_\mathbf{u}})^{N+1}$ such that for all $(\varphi_m^\alpha, \varphi_f^\alpha) \in (X^0_{\mathcal{D}_p})^{N+1}$, $\mathbf{v} \in (X^0_{\mathcal{D}_\mathbf{u}})^{N+1}$:

$$\begin{aligned}
&\int_0^T \int_\Omega \Big(\delta_t\Big(\phi_\mathcal{D}\Pi^m_{\mathcal{D}_p} s_m^\alpha\Big)\Pi^m_{\mathcal{D}_p}\varphi_m^\alpha + \eta_m^\alpha(\Pi^m_{\mathcal{D}_p} s_m^\alpha)\mathbb{K}_m \nabla^m_{\mathcal{D}_p} p_m^\alpha \cdot \nabla^m_{\mathcal{D}_p}\varphi_m^\alpha\Big)\mathrm{d}\mathbf{x}\mathrm{d}t \\
&+ \int_0^T \int_\Gamma \delta_t\Big(d_{f,\mathcal{D}_\mathbf{u}}\Pi^f_{\mathcal{D}_p} s_f^\alpha\Big)\Pi^f_{\mathcal{D}_p}\varphi_f^\alpha \mathrm{d}\sigma(\mathbf{x}) \\
&+ \int_0^T \int_\Gamma \eta_f^\alpha(\Pi^f_{\mathcal{D}_p} s_f^\alpha)\frac{d^3_{f,\mathcal{D}_\mathbf{u}}}{12}\nabla^f_{\mathcal{D}_p} p_f^\alpha \cdot \nabla^f_{\mathcal{D}_p}\varphi_f^\alpha \mathrm{d}\sigma(\mathbf{x})\mathrm{d}t \\
&+ \sum_{\mathfrak{a}=\pm}\int_0^T \int_\Gamma \Big(Q^\alpha_{f,\mathfrak{a}}[\![\varphi^\alpha]\!]^\mathfrak{a}_{\mathcal{D}_p} + d_\mathfrak{a}\phi_\mathfrak{a}\delta_t\Big(\mathbb{T}^\mathfrak{a}_{\mathcal{D}_p} s^\alpha_\mathfrak{a}\Big)\mathbb{T}^\mathfrak{a}_{\mathcal{D}_p}\varphi^\alpha_m\Big)\mathrm{d}\sigma(\mathbf{x})\mathrm{d}t \\
&= \int_0^T \int_\Omega h_m^\alpha \Pi^m_{\mathcal{D}_p}\varphi_m^\alpha \mathrm{d}\mathbf{x}\mathrm{d}t + \int_0^T \int_\Gamma h_f^\alpha \Pi^f_{\mathcal{D}_p}\varphi_f^\alpha \mathrm{d}\sigma(\mathbf{x})\mathrm{d}t, \quad \alpha \in \{\mathrm{w}, \mathrm{nw}\},
\end{aligned}$$

$$\begin{aligned}
&\int_0^T \int_\Omega \Big(\sigma_{\mathcal{D}_u}(\mathbf{u}) : \epsilon_{\mathcal{D}_\mathbf{u}}(\mathbf{v}) - b(\Pi^m_{\mathcal{D}_p} p_m^E)\mathrm{div}_{\mathcal{D}_\mathbf{u}}(\mathbf{v})\Big)\mathrm{d}\mathbf{x}\mathrm{d}t \\
&\quad + \int_0^T \int_\Gamma (\Pi^f_{\mathcal{D}_p} p_f^E)[\![\mathbf{v}]\!]_{\mathcal{D}_\mathbf{u}}\mathrm{d}\sigma(\mathbf{x})\mathrm{d}t = \int_0^T \int_\Omega \mathbf{f}\cdot \Pi_{\mathcal{D}_\mathbf{u}}\mathbf{v}\mathrm{d}\mathbf{x}\mathrm{d}t
\end{aligned}$$

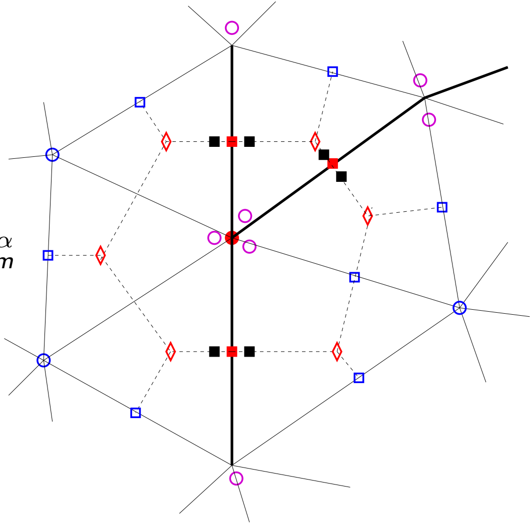
Main convergence result

There are $(\bar{p}_m^\alpha, \bar{p}_f^\alpha) \in L^2(\mathbb{T}; V_m^0 \times V_f^0)$ and $\bar{\mathbf{u}} \in L^\infty(\mathbb{T}; \mathbf{U}^0)$ satisfying the weak formulation s.t.

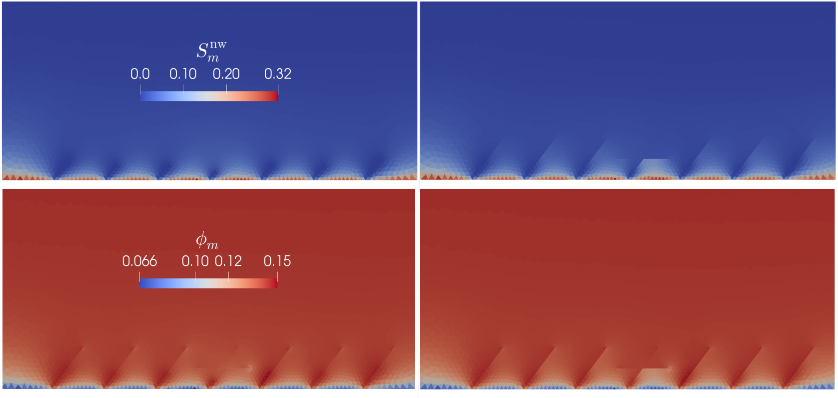
$\Pi^m_{\mathcal{D}_p^l} p^\alpha_{m,l} \rightharpoonup \bar{p}^\alpha_m$	weakly in $L^2(\mathbb{T}; L^2(\Omega))$,
$\Pi^f_{\mathcal{D}_p^l} p^\alpha_{f,l} \rightharpoonup \bar{p}^\alpha_f$	weakly in $L^2(\mathbb{T}; L^2(\Gamma))$,
$\Pi_{\mathcal{D}_{\mathbf{u}}^l} \mathbf{u}^l \rightharpoonup \bar{\mathbf{u}}$	weakly-$\star$ in $L^\infty(\mathbb{T}; L^2(\Omega)^d)$,
$\phi_{\mathcal{D}^l} \rightharpoonup \bar{\phi}_m$	weakly-$\star$ in $L^\infty(\mathbb{T}; L^2(\Omega))$,
$d_{f,\mathcal{D}_{\mathbf{u}}^l} \to \bar{d}_f$	in $L^\infty(\mathbb{T}; L^p(\Gamma))$ for $2 \leqslant p < 4$,
$\Pi^m_{D_p^l} S^\alpha_m(p^l_{c,m}) \to S^\alpha_m(\bar{p}_{c,m})$	in $L^2(\mathbb{T}; L^2(\Omega))$,
$\Pi^f_{D_p^l} S^\alpha_f(p^l_{c,f}) \to S^\alpha_f(\bar{p}_{c,f})$	in $L^2(\mathbb{T}; L^2(\Gamma))$,
$Q^\alpha_{f,\mathfrak{a}} \rightharpoonup \bar{Q}^\alpha_{f,\mathfrak{a}}$	weakly in $L^2(\mathbb{T}; L^2(\Gamma))$.

TPFA - $\mathbb{P}_2$ discretization

- ◊ p_m^α, s_m^α
- ● ■ p_f^α, s_f^α
- ■ $\mathbf{u}, p_m^\alpha, s_{\mathfrak{a}}^\alpha, s_m^\alpha$
- ○ ○ □ $\mathbf{u}$

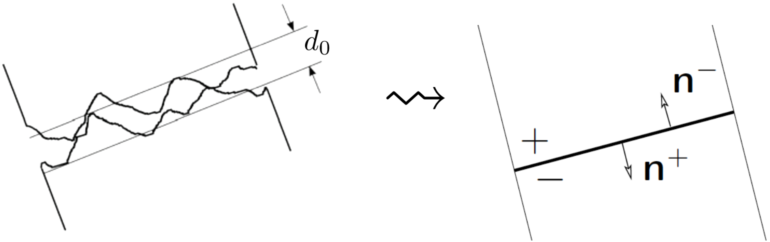


Numerical experiment: desaturation by suction



Gas saturation (top) and porosity (bottom) given by the continuous (left) and discontinuous (right) models

Extension to Coulomb frictional contact



- Jump of the displacement field: $[\![\mathbf{u}]\!] = \mathbf{u}^+ - \mathbf{u}^-$
- Normal and tangential jumps:
  $[\![\mathbf{u}]\!]_n = [\![\mathbf{u}]\!] \cdot \mathbf{n}^+, \quad [\![\mathbf{u}]\!]_\tau = [\![\mathbf{u}]\!] - [\![\mathbf{u}]\!]_n \mathbf{n}^+$
- Stress vectors :

$$\mathbf{T}^\pm(\mathbf{u}) = \sigma^T(\mathbf{u})\mathbf{n}^\pm + p_f^E \mathbf{n}^\pm$$

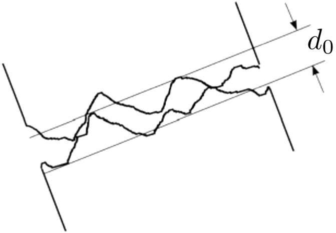
- Normal and and tangential stresses :

$$T_n(\mathbf{u}) = \mathbf{T}^+(\mathbf{u}) \cdot \mathbf{n}^+, \quad \mathbf{T}_\tau(\mathbf{u}) = \mathbf{T}^+(\mathbf{u}) - (\mathbf{T}^+(\mathbf{u}) \cdot \mathbf{n}^+)\mathbf{n}^+$$

Extension to Coulomb frictional contact [Garipov et al 2016, Berge et al 2019]

Contact conditions

$$\begin{cases} \mathbf{T}^+(\mathbf{u}) + \mathbf{T}^-(\mathbf{u}) = \mathbf{0}, \\ T_n(\mathbf{u}) \leqslant 0,\ [\![\mathbf{u}]\!]_n \leqslant 0,\ [\![\mathbf{u}]\!]_n\ T_n(\mathbf{u}) = 0 \end{cases}$$



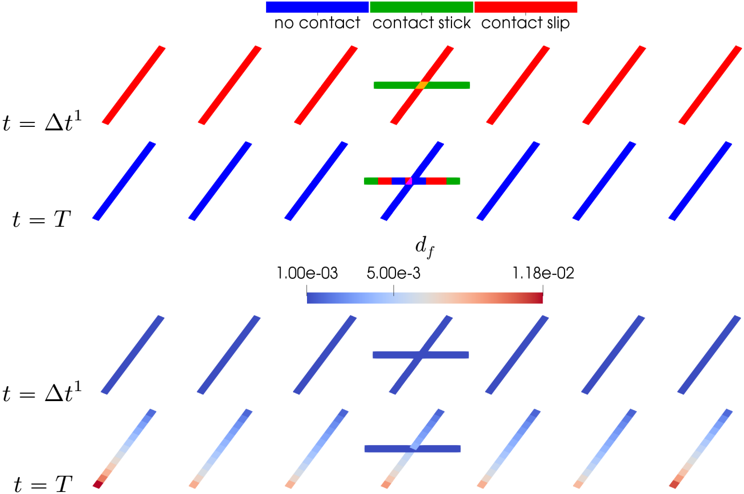
Aperture : $d_f = d_0 - [\![\mathbf{u}]\!]_n$

Slip-stick conditions
($F \geqslant 0$ friction coeff.) :

$$\begin{cases} |\mathbf{T}_\tau(\mathbf{u})| \leqslant -F\ T_n(\mathbf{u}) \\ \mathbf{T}_\tau(\mathbf{u}) \cdot [\![\dot{\mathbf{u}}]\!]_\tau - F\ T_n(\mathbf{u}) |[\![\dot{\mathbf{u}}]\!]_\tau| = 0 \end{cases}$$

Desaturation by suction with contact

Fracture contact state and aperture at first time step and at final time



Desaturation by suction with contact

Gas matrix saturation at final time

Desaturation by suction with contact

Porosity at final time

Conclusions & perspectives

Conclusions

- Numerical analysis of the poro-mechanical coupling for two-phase flows in deformable and fractured porous media
  - open fractures, no contact
  - full nonlinear coupling
  - Continuous and discontinuous pressure models
  - gradient discretization framework

Perspectives

- Contact, slip, friction between fracture surfaces (ongoing)
- More advanced discretizations in 3D

Thanks for your attention