A Hybrid High-Order method for the incompressible Navier–Stokes problem robust for large irrotational body forces

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References

- HHO methods [DP and Ern, 2015]
- MHO for Stokes [Aghili, Boyaval, DP, 2015]
- Skew-symmetric for Navier–Stokes [DP and Krell, 2018]
- Temam's device [Botti, DP, Droniou, 2018]
- Pressure-robust in the sense of [Linke, 2014, Linke and Merdon, 2016]
  - Stokes, standard meshes [DP, Ern, Linke, Schieweck, 2016]
  - Navier–Stokes, standard meshes [Castañón Quiroz and DP, 2020]
  - Navier–Stokes, polyhedral meshes [Castañón Quiroz and DP, 2022]
- DDR/VEM for Stokes, polyhedral meshes [Beirão da Veiga, Dassi, DP, Droniou, 2022] $\rightarrow$ **see J. Droniou's talk!**

The incompressible Navier–Stokes equations

- Let $\nu > 0$, $\boldsymbol{f} \in \boldsymbol{L}^2(\Omega;\mathbb{R}^d)$, $\boldsymbol{U} \coloneqq \boldsymbol{H}_0^1(\Omega;\mathbb{R}^d)$, and $P \coloneqq L_0^2(\Omega)$
- The INS problem reads: Find $(\boldsymbol{u}, p) \in \boldsymbol{U} \times P$ s.t.

$$
\begin{aligned}
\nu a(\boldsymbol{u},\boldsymbol{v}) + t(\boldsymbol{u},\boldsymbol{u},\boldsymbol{v}) + b(\boldsymbol{v},p) &= \ell(\boldsymbol{f},\boldsymbol{v}) && \forall \boldsymbol{v} \in \boldsymbol{U},\\
-b(\boldsymbol{u},q) &= 0 && \forall q \in L^2(\Omega),
\end{aligned}
$$

with viscous and pressure-velocity coupling bilinear forms

$$
a(\boldsymbol{w},\boldsymbol{v}) \coloneqq \int_\Omega \boldsymbol{\nabla}\boldsymbol{w} : \boldsymbol{\nabla}\boldsymbol{v}, \quad b(\boldsymbol{v},q) \coloneqq -\int_\Omega q(\boldsymbol{\nabla}\cdot\boldsymbol{v})
$$

and convective trilinear form and forcing term

$$
t(\boldsymbol{w},\boldsymbol{v},\boldsymbol{z}) \coloneqq \int_\Omega \boldsymbol{\nabla}\boldsymbol{w}\boldsymbol{v}\cdot\boldsymbol{z} - \int_\Omega \boldsymbol{\nabla}\boldsymbol{w}\boldsymbol{z}\cdot\boldsymbol{v}, \quad \ell(\boldsymbol{f},\boldsymbol{v}) \coloneqq \int_\Omega \boldsymbol{f}\cdot\boldsymbol{v}
$$

The incompressible Navier–Stokes equations

- Let $\nu > 0$, $\boldsymbol{f} \in \boldsymbol{L}^2(\Omega; \mathbb{R}^d)$, $\boldsymbol{U} := \boldsymbol{H}_0^1(\Omega; \mathbb{R}^d)$, and $P := L_0^2(\Omega)$
- The INS problem reads: Find $(\boldsymbol{u}, p) \in \boldsymbol{U} \times P$ s.t.

$$\begin{aligned} \nu a(\boldsymbol{u}, \boldsymbol{v}) + t(\boldsymbol{u}, \boldsymbol{u}, \boldsymbol{v}) + b(\boldsymbol{v}, p) &= \ell(\boldsymbol{f}, \boldsymbol{v}) \qquad && \forall \boldsymbol{v} \in \boldsymbol{U}, \\ -b(\boldsymbol{u}, q) &= 0 && \forall q \in L^2(\Omega), \end{aligned}$$

  with viscous and pressure-velocity coupling bilinear forms

$$a(\boldsymbol{w}, \boldsymbol{v}) := \int_\Omega \boldsymbol{\nabla} \boldsymbol{w} : \boldsymbol{\nabla} \boldsymbol{v}, \quad b(\boldsymbol{v}, q) := -\int_\Omega q(\boldsymbol{\nabla} \cdot \boldsymbol{v})$$

  and convective trilinear form and forcing term

$$t(\boldsymbol{w}, \boldsymbol{v}, \boldsymbol{z}) := \int_\Omega \boldsymbol{\nabla} \boldsymbol{w} \boldsymbol{v} \cdot \boldsymbol{z} - \int_\Omega \boldsymbol{\nabla} \boldsymbol{w} \boldsymbol{z} \cdot \boldsymbol{v}, \quad \ell(\boldsymbol{f}, \boldsymbol{v}) := \int_\Omega \boldsymbol{f} \cdot \boldsymbol{v}$$

- With this formulation, p is the so-called Bernoulli pressure

$$p = p_{\mathsf{kin}} + \frac{1}{2}|\boldsymbol{u}|^2$$

A key remark

- Assume $b_1 = 0$. The following Helmholtz decomposition is classical:

$$\boldsymbol{f} = \boldsymbol{g} + \lambda \boldsymbol{\nabla} \psi,$$

with

$$\boldsymbol{g} \in \boldsymbol{\nabla} \times \boldsymbol{H}_0(\mathbf{curl}; \Omega) \text{ and } \psi \in H^1(\Omega) \text{ s.t. } \|\boldsymbol{\nabla} \psi\|_{\boldsymbol{L}^2(\Omega;\mathbb{R}^d)} = 1$$

- We have the following crucial property: For all $\boldsymbol{v} \in \boldsymbol{U}$,

$$\begin{aligned} \ell(\boldsymbol{g} + \lambda \boldsymbol{\nabla} \psi, \boldsymbol{v}) &= \ell(\boldsymbol{g}, \boldsymbol{v}) - \int_\Omega \lambda \psi \, (\boldsymbol{\nabla} \cdot \boldsymbol{v}) + \cancel{\int_{\partial\Omega} \lambda \psi \, (\boldsymbol{v} \cdot \boldsymbol{n}_\Omega)} \\ &= \ell(\boldsymbol{g}, \boldsymbol{v}) + b(\boldsymbol{v}, \lambda \psi) \end{aligned}$$

- Hence, with $(\boldsymbol{u}_{\boldsymbol{g}}, p_{\boldsymbol{g}})$ solution corresponding to the forcing term $\boldsymbol{g}$,

$$\boldsymbol{g} \leftarrow \boldsymbol{g} + \lambda \boldsymbol{\nabla} \psi \implies (\boldsymbol{u}_{\boldsymbol{g}}, p_{\boldsymbol{g}}) \leftarrow (\boldsymbol{u}_{\boldsymbol{g}}, p_{\boldsymbol{g}} + \lambda \psi)$$

Discrete spaces I

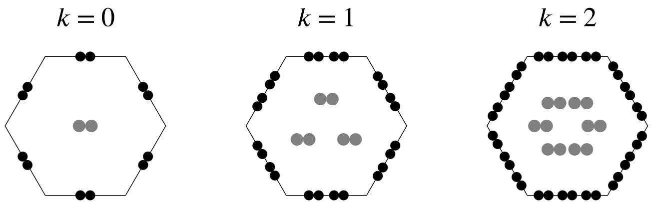


Figure: Local velocity space $\underline{\boldsymbol{U}}_T^k$ for $d = 2$ and $k \in \{0, 1, 2\}$

- Denote by $\mathcal{T}_h$ a polygonal/polyhedral mesh of Ω
- For $k \geq 0$, we define the global space of discrete velocity unknowns

$$\underline{\boldsymbol{U}}_h^k := \Big\{\underline{\boldsymbol{v}}_h = ((\boldsymbol{v}_T)_{T \in \mathcal{T}_h}, (\boldsymbol{v}_F)_{F \in \mathcal{F}_h}) \ : \\ \boldsymbol{v}_T \in \mathbb{P}^k(T)^d \quad \forall T \in \mathcal{T}_h \text{ and } \boldsymbol{v}_F \in \mathbb{P}^k(F)^d \quad \forall F \in \mathcal{F}_h\Big\}$$

- The restrictions to $T \in \mathcal{T}_h$ are $\underline{\boldsymbol{U}}_T^k$ and $\underline{\boldsymbol{v}}_T := (\boldsymbol{v}_T, (\boldsymbol{v}_F)_{F \in \mathcal{F}_T})$

Discrete spaces II

- The global interpolator $\underline{\boldsymbol{I}}_h^k : \boldsymbol{H}^1(\Omega; \mathbb{R}^d) \to \underline{\boldsymbol{U}}_h^k$ is s.t.

$$\underline{\boldsymbol{I}}_h^k \boldsymbol{v} := \left((\boldsymbol{\pi}_T^k \boldsymbol{v})_{T \in \mathcal{T}_h}, (\boldsymbol{\pi}_F^k \boldsymbol{v})_{F \in \mathcal{F}_h} \right)$$

- The velocity space strongly accounting for boundary conditions is

$$\underline{\boldsymbol{U}}_{h,0}^k := \left\{ \underline{\boldsymbol{v}}_h \in \underline{\boldsymbol{U}}_h^k \ : \ \boldsymbol{v}_F = \boldsymbol{0} \quad \forall F \in \mathcal{F}_h^{\mathrm{b}} \right\}$$

- The discrete pressure space is defined setting

$$P_h^k := \mathbb{P}^k(\mathcal{T}_h) \cap P$$

Viscous and pressure-velocity coupling terms I

- Let an element $T \in \mathcal{T}_h$ be fixed
- For all $l \geq 0$, the discrete gradient $\boldsymbol{G}_T^l : \underline{\boldsymbol{U}}_T^k \to \mathbb{P}^k(T;\mathbb{R}^{d\times d})$ is s.t.

$$\int_T \boldsymbol{G}_T^l \underline{\boldsymbol{v}}_T : \boldsymbol{\tau} = -\int_T \boldsymbol{v}_T \cdot (\boldsymbol{\nabla} \cdot \boldsymbol{\tau}) + \sum_{F \in \mathcal{F}_T} \int_F \boldsymbol{v}_F \cdot \boldsymbol{\tau} \boldsymbol{n}_{TF} \quad \forall \boldsymbol{\tau} \in \mathbb{P}^k(T;\mathbb{R}^{d\times d})$$

- For $l = k$ we have the following commutation property:

$$\boldsymbol{G}_T^k \underline{\boldsymbol{I}}_T^k \boldsymbol{v} = \boldsymbol{\pi}_T^k \boldsymbol{\nabla} \boldsymbol{v} \qquad \forall \boldsymbol{v} \in \boldsymbol{H}^1(T;\mathbb{R}^d),$$

as can be checked writing, for $\boldsymbol{\tau} \in \mathbb{P}^k(T;\mathbb{R}^{d\times d})$,

$$\int_T \boldsymbol{G}_T^l \underline{\boldsymbol{I}}_T^k \boldsymbol{v} : \boldsymbol{\tau} = -\int_T \cancel{\boldsymbol{\pi}_T^k} \boldsymbol{v} \cdot (\boldsymbol{\nabla} \cdot \boldsymbol{\tau}) + \sum_{F \in \mathcal{F}_T} \int_F \cancel{\boldsymbol{\pi}_F^k} \boldsymbol{v} \cdot \boldsymbol{\tau} \boldsymbol{n}_{TF} = \int_T \boldsymbol{\nabla} \boldsymbol{v} : \boldsymbol{\tau}$$

Viscous and pressure-velocity coupling terms II

- The viscous term is discretised by $a_h : \underline{\boldsymbol{U}}_h^k \times \underline{\boldsymbol{U}}_h^k \to \mathbb{R}$ s.t.

$$a_h(\underline{\boldsymbol{w}}_h, \underline{\boldsymbol{v}}_h) \coloneqq \sum_{T \in \mathcal{T}_h} a_T(\underline{\boldsymbol{w}}_T, \underline{\boldsymbol{v}}_T)$$

  with

$$a_T(\underline{\boldsymbol{w}}_T, \underline{\boldsymbol{v}}_T) \coloneqq \int_T \boldsymbol{G}_T^k \underline{\boldsymbol{w}}_T : \boldsymbol{G}_T^k \underline{\boldsymbol{v}}_T + s_T(\underline{\boldsymbol{w}}_T, \underline{\boldsymbol{v}}_T)$$

- Above, s_T is a polynomially consistent local stabilisation
- Pressure-velocity coupling is realised by $b_h : \underline{\boldsymbol{U}}_h^k \times \mathbb{P}^k(\mathcal{T}_h) \to \mathbb{R}$ s.t.

$$b_h(\underline{\boldsymbol{v}}_h, p_h) \coloneqq \sum_{T \in \mathcal{T}_h} \int_T D_T^k \underline{\boldsymbol{v}}_T \; p_T \quad \text{with} \quad D_T^k = \operatorname{tr} \boldsymbol{G}_T^k$$

Convective and forcing terms I

- Assume, for the moment being, $\mathcal{T}_h$ matching simplicial
- The div-conforming velocity reconstruction $\boldsymbol{R}_T^k : \underline{\boldsymbol{U}}_T^k \to \mathbb{RTN}^k(T)$ is s.t.

$$\boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T \cdot \boldsymbol{n}_{TF} = \boldsymbol{v}_F \cdot \boldsymbol{n}_{TF} \qquad \forall F \in \mathcal{F}_T,$$
$$\int_T \boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T \cdot \boldsymbol{w} = \int_T \boldsymbol{v}_T \cdot \boldsymbol{w} \qquad \forall \boldsymbol{w} \in \mathbb{P}^{k-1}(T; \mathbb{R}^d),$$

- The global counterpart $\boldsymbol{R}_h^k : \underline{\boldsymbol{U}}_h^k \to \mathbb{RTN}^k(\mathcal{T}_h)$ is defined setting

$$(\boldsymbol{R}_h^k \underline{\boldsymbol{v}}_h)_{|T} \coloneqq \boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T \qquad \forall T \in \mathcal{T}_h$$

- Crucially, it holds, setting $(D_h^k)_{|T} \coloneqq D_T^k$ for all $T \in \mathcal{T}_h$,

$$\boldsymbol{R}_h^k \underline{\boldsymbol{v}}_h \in \boldsymbol{H}(\mathrm{div}; \Omega) \quad \text{and} \quad \boldsymbol{\nabla} \cdot \boldsymbol{R}_h^k \underline{\boldsymbol{v}}_h = D_h^k \underline{\boldsymbol{v}}_h \quad \text{for all } \underline{\boldsymbol{v}}_h \in \underline{\boldsymbol{U}}_h^k$$

Convective and forcing terms II

- The convective term is approximated by $t_h : [\underline{\boldsymbol{U}}_h^k]^3 \to \mathbb{R}$ s.t.

$$t_h(\underline{\boldsymbol{w}}_h, \underline{\boldsymbol{v}}_h, \underline{\boldsymbol{z}}_h) \coloneqq \sum_{T \in \mathcal{T}_h} t_T(\underline{\boldsymbol{w}}_T, \underline{\boldsymbol{v}}_T, \underline{\boldsymbol{z}}_T)$$

  where, for all $T \in \mathcal{T}_h$,

$$t_T(\underline{\boldsymbol{w}}_T, \underline{\boldsymbol{v}}_T, \underline{\boldsymbol{z}}_T) \coloneqq \int_T \boldsymbol{G}_T^{2(k+1)} \underline{\boldsymbol{w}}_T \boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T \cdot \boldsymbol{R}_T^k \underline{\boldsymbol{z}}_T - \int_T \boldsymbol{G}_T^{2(k+1)} \underline{\boldsymbol{w}}_T \boldsymbol{R}_T^k \underline{\boldsymbol{z}}_T \cdot \boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T$$

- We have the following crucial non-dissipativity property:

$$t_h(\underline{\boldsymbol{w}}_h, \underline{\boldsymbol{v}}_h, \underline{\boldsymbol{v}}_h) = 0 \qquad \forall (\underline{\boldsymbol{w}}_h, \underline{\boldsymbol{v}}_h) \in \underline{\boldsymbol{U}}_h^k \times \underline{\boldsymbol{U}}_h^k$$

Convective and forcing terms III

Remark (Implementation of t_h)

The implementation of t_T does not require the actual computation of $\boldsymbol{G}_T^{2(k+1)}$. Instead, using its definition, we use the equivalent formulation:

$$
\begin{aligned}
t_T(\underline{\boldsymbol{w}}_T, \underline{\boldsymbol{v}}_T, \underline{\boldsymbol{z}}_T) = {} & \int_T \boldsymbol{\nabla} \boldsymbol{w}_T \boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T \cdot \boldsymbol{R}_T^k \underline{\boldsymbol{z}}_T - \int_T \boldsymbol{\nabla} \boldsymbol{w}_T \boldsymbol{R}_T^k \underline{\boldsymbol{z}}_T \cdot \boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T \\
& + \sum_{F \in \mathcal{F}_T} \int_F (\boldsymbol{w}_F - \boldsymbol{w}_T) \cdot \boldsymbol{R}_T^k \underline{\boldsymbol{z}}_T (\boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T \cdot \boldsymbol{n}_{TF}) \\
& - \sum_{F \in \mathcal{F}_T} \int_F (\boldsymbol{w}_F - \boldsymbol{w}_T) \cdot \boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T (\boldsymbol{R}_T^k \underline{\boldsymbol{z}}_T \cdot \boldsymbol{n}_{TF}).
\end{aligned}
$$

Convective and forcing terms IV

- The forcing term $\ell_T : \boldsymbol{L}^2(\Omega;\mathbb{R}^d) \times \underline{\boldsymbol{U}}_h^k \to \mathbb{R}$ is s.t.

$$\ell_h(\boldsymbol{\phi}, \underline{\boldsymbol{v}}_h) \coloneqq \int_\Omega \boldsymbol{\phi} \cdot \boldsymbol{R}_h^k \underline{\boldsymbol{v}}_h$$

- Recalling that $\boldsymbol{f} = \boldsymbol{g} + \lambda \boldsymbol{\nabla}\psi$, velocity invariance holds:

$$\ell_h(\boldsymbol{g} + \lambda \boldsymbol{\nabla}\psi, \underline{\boldsymbol{v}}_h) = \ell_h(\boldsymbol{g}, \underline{\boldsymbol{v}}_h) + b_h(\underline{\boldsymbol{v}}_h, \lambda \pi_h^k \psi) \qquad \forall \underline{\boldsymbol{v}}_h \in \underline{\boldsymbol{U}}_{h,0}^k$$

Discrete problem and main results I

Find $(\underline{\boldsymbol{u}}_h, p_h) \in \underline{\boldsymbol{U}}_{h,0}^k \times P_h^k$ s.t.

$$\begin{aligned} \nu a_h(\underline{\boldsymbol{u}}_h, \underline{\boldsymbol{v}}_h) + t_h(\underline{\boldsymbol{u}}_h, \underline{\boldsymbol{u}}_h, \underline{\boldsymbol{v}}_h) + b_h(\underline{\boldsymbol{v}}_h, p_h) &= \ell_h(\boldsymbol{f}, \underline{\boldsymbol{v}}_h) \quad && \forall \underline{\boldsymbol{v}}_h \in \underline{\boldsymbol{U}}_{h,0}^k, \\ -b_h(\underline{\boldsymbol{u}}_h, q_h) &= 0 && \forall q_h \in \mathbb{P}^k(\mathcal{T}_h) \end{aligned}$$

Theorem (λ-uniform a priori bound on the discrete velocity)

Recalling the decomposition $\boldsymbol{f} = \boldsymbol{g} + \lambda \boldsymbol{\nabla} \psi$,

$$\|\underline{\boldsymbol{u}}_h\|_{1,h} \lesssim \nu^{-1} \|\boldsymbol{g}\|_{\boldsymbol{L}^2(\Omega;\mathbb{R}^d)},$$

with $\|\cdot\|_{1,h}$ H^1-like norm on $\underline{\boldsymbol{U}}_{h,0}^k$.

Discrete problem and main results II

Theorem (λ-robust error estimate)

Assume, with $\alpha \in (0,1)$ and $C_{\Omega,\varrho} > 0$ only depending on Ω and on the mesh regularity, that the following data smallness condition holds:

$$\|\boldsymbol{g}\|_{\boldsymbol{L}^2(\Omega;\mathbb{R}^d)} \leq \alpha C_{\Omega,\varrho} \nu^2.$$

Then, under the additional regularity $\boldsymbol{u} \in \boldsymbol{H}^{k+2}(\mathcal{T}_h;\mathbb{R}^d)$ and $p \in H^1(\Omega)$,

$$\|\underline{\boldsymbol{u}}_h - \underline{\boldsymbol{I}}_h^k \boldsymbol{u}\|_{1,h} + \nu^{-1}\|p_h - \pi_h^k p\|_{L^2(\Omega)} \lesssim h^{k+1},$$

with hidden constant independent of h, ν, λ, and p, but possibly depending on Ω, ϱ, α, and bounded norms of $\boldsymbol{u}$.

Extension to general meshes

- Take now $\mathcal{T}_h$ general polytopal mesh with convex elements
- Let $T \in \mathcal{T}_h$, $\mathfrak{T}_T$ a matching simplicial submesh of T, and

$$\mathbb{RTN}^k(\mathfrak{T}_T) \coloneqq \left\{ \mathfrak{w} \in \boldsymbol{H}(\mathrm{div};T) \;:\; \mathfrak{w}_{|\tau} \in \mathbb{RTN}^k(\tau) \quad \forall \tau \in \mathfrak{T}_T \right\},$$
$$\mathbb{RTN}_0^k \coloneqq \mathbb{RTN}^k \cap \boldsymbol{H}_0(\mathrm{div};\Omega)$$

- The div-conforming velocity reconstruction solves:

Find $(\boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T, \psi) \in \mathbb{RTN}^k(\mathfrak{T}_T) \times \mathbb{P}^k(\mathfrak{T}_T)$ s.t.

$$(\boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T)_{|\sigma} = (\boldsymbol{v}_F \cdot \boldsymbol{n}_{TF})_{|\sigma} \quad \forall \sigma \in \mathfrak{F}_F, \forall F \in \mathcal{F}_T,$$
$$\int_T \boldsymbol{\nabla} \cdot \boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T \; \phi = \int_T D_T^k \underline{\boldsymbol{v}}_T \; \phi \quad \forall \phi \in \mathbb{P}^k(\mathfrak{T}_T),$$
$$\int_T \boldsymbol{R}_T^k \underline{\boldsymbol{v}}_T \cdot \mathfrak{w} + \int_T \boldsymbol{\nabla} \cdot \mathfrak{w} \; \psi = \int_T \boldsymbol{v}_T \cdot \mathfrak{w} \quad \forall \mathfrak{w} \in \mathbb{RTN}_0^k(\mathfrak{T}_T)$$

- Optimal λ-robust error estimates are obtained for $k \in \{0, 1\}$

Lid-driven cavity with modified forcing term I

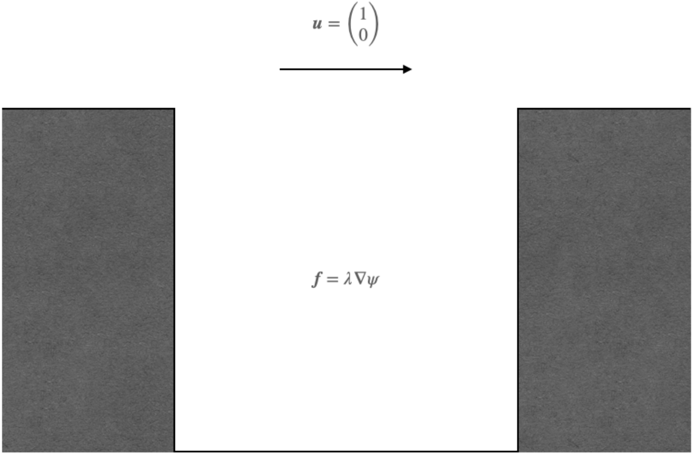


Figure: Problem description. We take $\nu = 10^{-3}$, corresponding to a Reynolds number of 1000

Lid-driven cavity with modified forcing term II

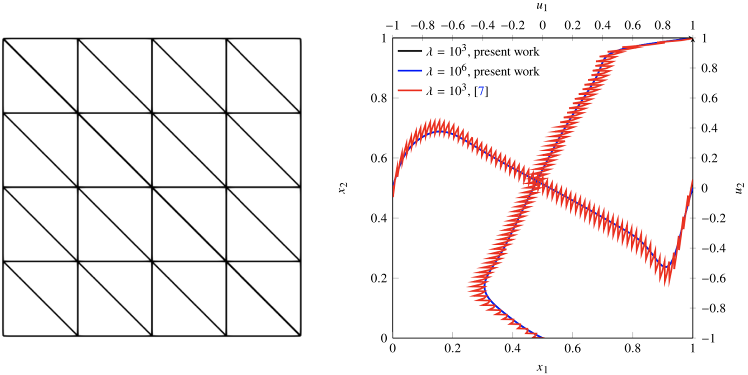
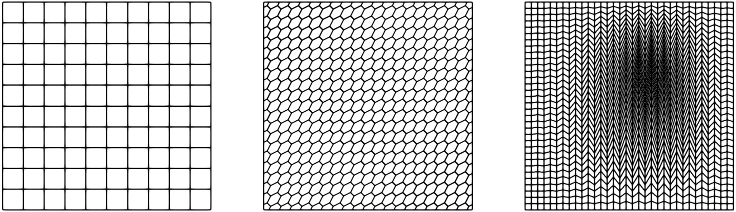


Figure: Mesh pattern for the simplicial version of the scheme and numerical results, including a comparison with the standard (non-λ-robust) HHO method of [Botti et al., 2019]

Lid-driven cavity with modified forcing term III



(a) Cartesian. (b) Hexagonal. (c) Kershaw.

Figure: Mesh types used for the polygonal/polyhedral version of the scheme

Lid-driven cavity with modified forcing term IV

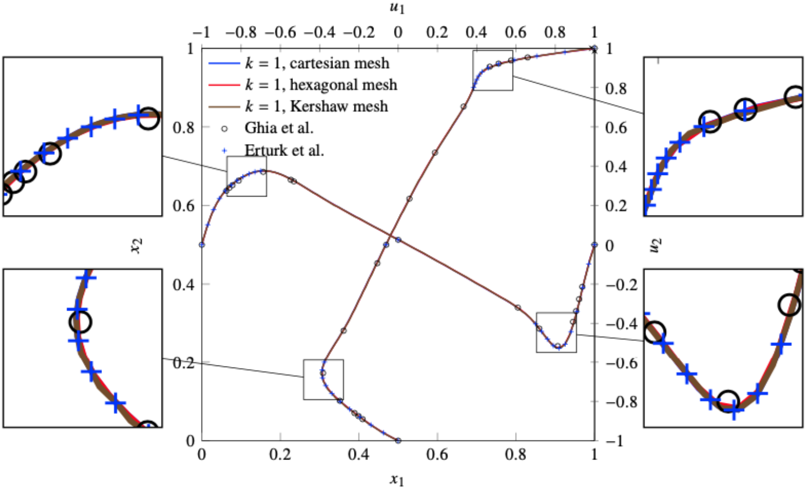


Figure: $\lambda = 0$, Re $= 1000$ and comparison with the literature

Lid-driven cavity with modified forcing term V

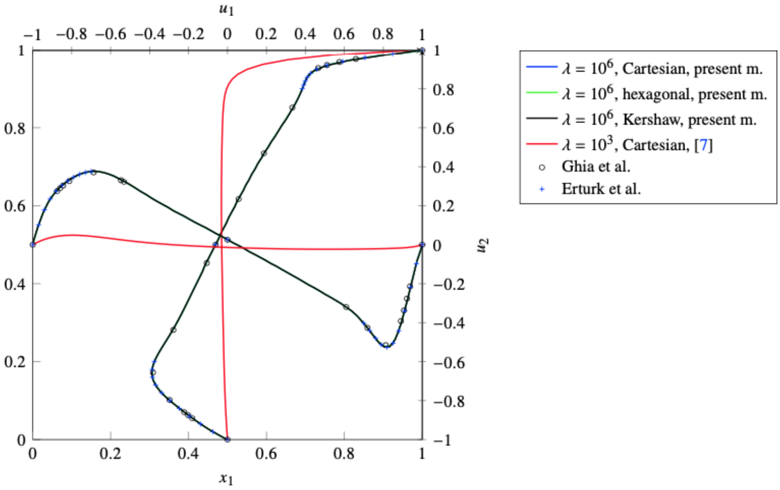


Figure: $\lambda = 10^6$, Re $= 1000$ and comparison with the literature (standard HHO method for $\lambda = 10^3$ and reference results for $\lambda = 0$)

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