

# Curriculum vitæ

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## 1 Personal information

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### 1.1 Education

6/12/2010 **Habilitation** (*Habilitation à Diriger des Recherches*), [Université Paris-Est](#), *Nonconforming methods for PDEs with diffusion*

28/3/2006 **Ph.D. Thesis**, [Università di Bergamo](#), *Discontinuous Galerkin methods for the incompressible Navier–Stokes equations*, supervisor **F. Bassi**. Part of my Ph.D. thesis was carried out at EPFL (see below)

11/7/2002 **Master in Engineering**, [Università di Bergamo](#), 110/110 with honors

### 1.2 Current positions

1/9/2012–pres. **Full professor** (PR1C) at [Institut Montpelliérain Alexander Grothendieck \(IMAG\)](#), [Université de Montpellier \(UM\)](#)

1/10/2014–pres. Head of the [ACSIOM research team](#) (25 permanent members)

### 1.3 Previous positions

1/4/2007–31/8/2012 **Senior researcher** at the Department of Applied Mathematics of [IFP Energies Nouvelles \(IFPEN\)](#)

1/2/2006–31/3/2007 **Post-doctoral researcher** at the [Centre d’Enseignement et de Recherche en Mathématiques et Calcul Scientifique \(CERMICS\)](#), [École des Ponts ParisTech \(ENPC\)](#)

1/1/2005–30/6/2006 **Visiting Ph.D. assistant**, [CMCS](#), [École Polytechnique Fédérale Lausanne \(EPFL\)](#)

### 1.4 Fellowships, awards, and distinctions

4–5/2018 STaRs invited professor (“Supporting Talented Researchers”) at [Università di Bergamo](#)

1/9/2016 CNRS professor appointment (1 year) at [Institut Henri Poincaré \(Paris\)](#)

2016 ITALY (*Italian TALented Young researchers*) fellowship, [Università di Bergamo](#), Italy

2012–pres. French national award for Doctoral Supervision and Research

### 1.5 Memberships

2015–pres. Member of [SMAI](#) (French Society of Industrial and Applied Mathematics)

2015–pres. Member of the French Research Network [MaNu](#) (*Mathématiques pour le Nucléaire*)

## 2 Supervision of doctoral and post-doctoral fellows

### 2.1 Supervision of Ph.D. students

- 2018–pres. **Ilaria Fontana**, contact problems in dam modelling
- 2018–pres. **André Harnist**, Hybrid High-Order methods for non-Newtonian fluids
- 2018–pres. **Pierre Matalon**, Algebraic multi-grid solvers for Hybrid High-Order methods
- Def. 2018 **Michele Botti**, Advanced polyhedral discretization methods for poromechanical modelling, in collaboration with BRGM. TEL manuscript [tel-01871074](#)
- Def. 2018 **Florent Chave**, Hybrid High-Order methods for interface problems. TEL manuscript [tel-01881007](#)
- Def. 2017 **Rita Riedlbeck**, A posteriori-based adaptive algorithms for poro-mechanics. TEL manuscript [tel-01676709](#). R. Riedlbeck is now research manager at **TWT**
- Def. 2016 **Joubine Aghili**, Numerical resolution of partial differential equations with variable coefficients. TEL manuscript [tel-01616910](#) J. Aghili is now post-doctoral researcher at Université de Nice
- Def. 2013 **Jean-Marc Gratien**, Development of a domain-specific embedded language for lowest-order methods on general meshes. TEL manuscript [tel-00926232](#). Co-director with C. Prud'homme (professor, Univ. Strasbourg). J.-M. Gratien is now research engineer at IFPEN
- Def. 2013 **Simon Lemaire**, Hybrid finite volume methods for poro-mechanics. TEL manuscript [tel-00957292](#). Co-supervisor with R. Eymard (professor, Univ. Paris-Est). S. Lemaire is now researcher (*Chargé de Recherche*) at INRIA
- Def. 2013 **Soleiman Yousef**, A posteriori error estimates and adaptivity for the SAGD proceeding, co-supervisor with M. Vohralík (senior researcher, INRIA) and V. Girault (professor, UPMC — Univ. Pierre et Marie Curie). S. Yousef is now research engineer at IFPEN

I also supervised the Ph.D. students **Lorenzo Botti** and **Sissel Mundal** during their 6 months stay at IFPEN.

### 2.2 Supervision of post-doctoral fellows

- 2018–present **Saghar Heidari**, Advanced aspects of Hybrid High-Order methods for applications in computational physics
- 2017–present **Daniel Castanon Quiroz**, Advanced implementation of Hybrid High-Order methods
- 2017–2018 **Alice Raeli**, Hybrid High-Order methods on octree meshes
- 2016–2017 **Francesco Bonaldi**, Advanced discretization methods for plate problems. F. Bonaldi is now post-doctoral researcher at MOX, Politecnico di Milano
- 2016–2017 **Roberta Tittarelli**, A posteriori error estimators for incompressible problems. R. Tittarelli is now Associate Professor (*Maître de Conférences*) at Université de Besançon
- 2008–2009 **Ivan Kapyrin**, Multi-points finite volume methods for porous media flows. I. Kapyrin is now Senior Researcher at the Institute of Numerical Mathematics of the Russian Academy of Sciences

### 2.3 Participation in Ph.D. theses committees

- 2018 C. Marcati\* (Université Pierre et Marie Curie)
- 2017 A. Raeli\* (Université de Bordeaux), A. Della Rocca\* (Politecnico di Milano, Italy), S. Zonca (Politecnico di Milano, Italy)
- 2016 R. Porcù\* (Politecnico di Milano, Italy), K. Haddaoui\* (Université Pierre et Marie Curie Paris 6)
- 2015 V. Baron\* (Univ. Nantes, France), K. Mallem\* (Aix-Marseille Univ., France)
- 2014 J. Bonelle (EDF-Univ. Paris-Est), A. Duran (UM)
- 2013 S. Gérald\* (ONERA-UPMC, referee), M. Cathala (UM), A. Baldit (UM)
- 2012 J. Richard (UM), T. Hai Ong\* (Univ. Paris-Est, France).

\* Referee

### 3 Teaching activities

#### 3.1 Post-graduate courses (Ph.D. level)

- 2018 *An introduction to the convergence analysis of discretisation methods for PDEs with application to Hybrid High-Order methods* (4h), Univ. Bergamo (Italy)
- 2016 *Hybrid High-Order methods* (6h), Institut Henri Poincaré (Paris), cf. <http://imag.edu.umontpellier.fr/event/ihp-nmpdes>
- 2016 *An introduction to Hybrid High-Order methods* (3h), Università di Bergamo (Italy)
- 2015 *An introduction to Hybrid High-Order methods* (3h), CEA-EDF-Inria School (Paris), cf. <https://www.inria.fr/centre/paris/agenda/ecole-cea-edf-inria>
- 2015 *Hybrid High-Order methods and applications* (18h), doctoral school I2S, Univ. Montpellier
- 2015 *Discontinuous Galerkin methods and applications* (4h), École de Mécanique des Fluides Numériques (Porquerolles, France), cf. <https://ecolemf.n.limsi.fr>
- 2016 *An introduction to Hybrid High-Order methods* (3h), Università di Bergamo (Italy)
- 2013 *Discontinuous Galerkin methods and applications* (6h), École de Mécanique des Fluides Numériques (Porquerolles, France), cf. <https://ecolemf.n.limsi.fr/doku.php?id=2013:start>
- 2012 *Discontinuous Galerkin methods and applications* (20h), doctoral school I2S, Univ. Montpellier

#### 3.2 Undergraduate courses

**Legend:** CM = *Cours Magistral* (Masterclass), TD = *Travaux Dirigés* (Exercices), TP = *Travaux Pratiques* (Practical exercises). In France 1h CM = 1.5h TD, 1h TD = 1.5h TP; LX = Xth year of Licence, MX = Xth year of Master

##### 3.2.1 As professor at University of Montpellier

- 2017–2018 **Analyse Numérique 3** (M2, 33 CM), **Modélisation Numérique** (M2, 7 CM), **Analyse Numérique Matricielle** (L2, 21 CM + 12 TD)
- 2016–2017 **Analyse Numérique des EDP 3** (M2, 33 CM), **Analyse Numérique Matricielle** (L2, 21 CM + 12 TD)
- 2015–2016 **Analyse Numérique des EDP 3** (M2, 33 CM), **Analyse Numérique Matricielle** (L2, 21 CM + 12 TD), **Algèbre Linéaire et Analyse 1** (2 x 48 TD)
- 2014–2015 **Calcul scientifique et Applications** (M2, 28 CM), **Algèbre Linéaire Analyse 1** (48 TD), **Optimisation numérique** (M1, 24 CM + 15 TD + 12 TP), **Biomaths** (L1, 36 TD)
- 2013–2014 **Calcul scientifique et Applications** (M2, 30 CM), **Algèbre Linéaire Analyse 1** (78 TD + 6 CM)
- 2012–2013 **Calcul scientifique et Applications** (M2, 30 CM), **Algèbre Linéaire Analyse 1** (78 TD + 6 CM), **Analyse Numérique Matricielle** (21 CM + 12 TD)

##### 3.2.2 Other undergraduate courses in France

- 2011–2012 **Discontinuous Galerkin Methods and Applications** (M2, Univ. Paris 5, 24h CM), **Calcul Scientifique** (L3, Ecole des Ponts ParisTech, 27h CM)
- 2010–2011 **Discontinuous Galerkin Methods and Applications** (M2, Univ. Paris 5, 24h CM), **Calcul Scientifique** (L3, Ecole des Ponts ParisTech, 27h CM)
- 2009–2010 **Discontinuous Galerkin Methods and Applications** (M2, Univ. Paris 5, 24h CM), **Calcul Scientifique** (L3, Ecole des Ponts ParisTech, 27h CM)
- 2008–2009 **Discontinuous Galerkin Methods and Applications** (M2, Univ. Paris 5, 10 CM), **Calcul Scientifique** (L3, Ecole des Ponts ParisTech, 27h CM)
- 2007–2008 **Calcul Scientifique** (L3, Ecole des Ponts ParisTech, 27h CM)

### 3.2.3 Supervision of M.D. theses

- 2018 **Alessandra Guglielmana**, A low-order method for linear elasticity on general meshes
- 2018 **André Harnist**, Applications of Hybrid High-Order methods to computational mechanics
- 2016 **Bastien Hamlat**, Discontinuous Galerkin methods for free-surface flows
- 2015 **Michele Botti**, Nonconforming discretization methods for poro-mechanics
- 2015 **Florent Chave**, Hybrid High-Order methods for the Cahn–Hilliard problem, in collaboration with Saint–Gobain
- 2013 **Rita Riedlbeck**, Spectral methods for the incompressible Navier–Stokes equations
- 2009 **Soleiman Yousef**, Finite volume methods for petroleum reservoir modelling
- 2005 **Nicoletta Franchina**, Discontinuous Galerkin methods for problems in fluid mechanics
- 2004 **Pietro Gabbiadini**, Development of a Matlab code for brake modelling, in collaboration with Freni Brembo

## 4 Scientific outreach

### 4.1 Editorial activity

I have acted as a referee for all the major international journals in Numerical Analysis and Scientific Computing. I have acted as referee for several national research agencies (ANR France, CONACYT Chile, FWF Austrian Science Found, PRIN Italy).

- 2016 Editor for the volume *Numerical methods for PDES: State of the Art Techniques* of the SEMA-SIMAI Springer series

### 4.2 Organization of scientific meetings

- 2019 Co-organizer of the POEMS 2019 conference at **CIRM** (29 Apr.– 3 May 2019). See <https://conferences.cirm-math.fr/1954.html>
- 2019 Co-organizer of the mini-symposium *Theoretical and computational advances in polygonal and polyhedral methods*, **MAFELAP 2019** (Brunel University, London)
- 2017 Co-organizer of the mini-symposia *Polyhedral Methods and Applications* and *Recent advances on polyhedral discretizations*, **ENUMATH 2017** international conference (Bergen, Norway)
- 2016 Coordinator of the **IHP thematic quarter *Numerical Methods for PDEs***. The quarter included one summer school and three international conferences:
  - Introductory school (IESC, Corse, 5–9 Sept. 2016)
  - *Advanced numerical methods: recent developments, analysis, and applications* (IHP, 3–7 Oct. 2016)
  - *Recent developments in numerical methods for model reduction* (IHP, 7–10 Nov. 2016)
  - *Industry and mathematics* (IHP, 21–23 Nov. 2016)
Detailed information at <http://imag.edu.umontpellier.fr/event/ihp-nmpdes> An IHP thematic quarter requires two years of preparation after the project is selected. Two special issues and a book resulted from this thematic quarter.
- 2007 Co-organizer of the international workshop *Discontinuous Galerkin Methods: From theoretical developments to industrial applications* (Bergamo, Italy)

### 4.3 Selection of recent invited presentations

For some of the following presentations, slides are available on my web page <http://imag.umontpellier.fr/~di-pietro>.

### 4.3.1 Outside France

- Mar. 2019 Invited seminar at [SISSA](#) (Italy)
- Oct. 2018 Invited seminar at [Univ. Udine](#) (Italy)
- May 2018 STaRs (“Supporting Talented Researchers”) invited seminar (4h) at [Univ. Bergamo](#) (Italy)
- Dec. 2017 Invited seminar at [Univ. Bergamo](#) (Italy)
- July 2017 Plenary speaker at the [POEMS 2017](#) international workshop (Univ. Milano Bicocca)
- July 2017 Invited doctoral mini-course at [Univ. Bergamo](#)
- July 2017 Invited speaker at the international workshop *Recent Advances and Challenges in Discontinuous Galerkin Methods and Related Approaches* (IMA, Univ. of Minnesota, USA). Declined owing to scheduling conflict
- Dec. 2016 Invited seminar at [MOX](#), Politecnico di Milano (Italy)
- June 2016 Invited speaker at the [ECCOMAS 2016](#) conference, minisymposium *High-Order methods for polygonal and polyhedral meshes* (declined because the date conflicted with another commitment)
- June 2016 Invited speaker at the [MAFELAP 2016](#) conference (Brunel University, UK), minisymposia *PDE discretization methods on polygonal and polyhedral meshes* and *Hybridizable discontinuous Galerkin methods*
- May 2016 Invited speaker at the [ZHACM Colloquium](#) Univ. Zürich-ETHZ
- Mar. 2016 Invited speaker at the international workshop *Variational Multiscale and Stabilized Finite Elements*, Magdeburg (declined because the date conflicted with another commitment)
- Feb. 2016 Invited seminar at Univ. di Pavia-IMATI
- Sept. 2015 Invited speaker at the *eXtended Discretization Methods 2015* conference, minisymposium *Polygonal and polyhedral methods* (Ferrara, Italy)
- July 2015 Invited lecturer for the Ph.D. course *An introduction to Hybrid High-Order methods*, Univ. di Bergamo
- June 2015 Invited lecturer at the CEA-EDF-INRIA school *New Trends in Compatible Discretizations* (Paris)
- June 2015 Invited speaker at the international workshop *Discontinuous Galerkin Methods and Applications* (Paris)
- Feb. 2015 Invited seminar at Univ. Milano Bicocca (Italy)
- July 2014 Invited speaker at the *World Congress on Computational Mechanics XI*, minisymposium *Structure-preserving and polyhedral discretizations* (Barcelona, Spain)
- Feb. 2013 Invited seminar at [MOX](#) (Politecnico di Milano)
- Dec. 2011 Invited seminar at Univ. Bergamo
- June 2011 Invited plenary speaker at the *Finite Volumes for Complex Applications VI* conference (Prague, Czech Republic)
- May 2011 Invited seminar at the Department of Mathematics, Univ. of Sussex (UK)

### 4.3.2 In France

- May 2018 Invited speaker at the minisymposium on *Polyhedral methods and applications, 44e Congrès National d’Analyse Numérique*, Cap d’Agde
- Nov. 2017 Invited plenary speaker at the *Journées Multiphasiques et Incertitudes* Nantes
- Apr. 2017 Invited seminar at UMPA, Lyon
- Mar. 2017 Invited seminar at Institut de Mathématiques de Bordeaux
- Jan. 2017 Invited lecturer at the *29ème séminaire de mécanique des fluides numérique*, IHP Paris
- Sept. 2016 Invited seminar at *EDF research lab Chatou*, Paris
- Sept. 2016 Invited seminar at the *Laboratoire de Mécanique et Génie Civil*, Univ. de Montpellier
- May 2016 Invited lecturer at the *Journées Numériques*, Laboratoire Jean Dieudonné, Univ. de Nice
- June 2015 Invited lecturer at the CEA-EDF-INRIA school *New Trends in Compatible Discretizations* (Paris)
- June 2015 Invited lecturer at the international workshop *Méthode de Galerkin discontinue et ses applications*, CNAM, Paris

- June 2015 Invited lecturer at the *École de Mécanique des Fluides Numérique 2015* (Porquerolles, France)
- Mar. 2015 Invited seminar at Département de Mathématiques d'Orsay, Univ. Paris 11
- Jan. 2015 Invited seminar at Institut Camille Jordan, Lyon
- Oct. 2014 Invited seminar at Saint-Gobain-CNRS research unit *Surface du Verre et Interfaces*, Paris Aubervilliers
- Jan. 2014 Invited seminar at EDF research lab Clamart, Paris
- Oct. 2013 Invited seminar at I2M, Aix-Marseille Univ.
- June 2013 Invited lecturer at the *École de Mécanique des Fluides Numérique 2013* (Porquerolles, France)
- Jan. 2013 Invited seminar at LAMSID, EDF, Paris Clamart
- Dec. 2012 Invited seminar at Laboratoire J. A. Dieudonné, Nice
- Oct. 2012 Invited speaker at the workshop *Innovative schemes and highly performing methods for the numerical simulation of fluid flows*, Marseille
- Apr. 2012 Invited speaker at the *Workshop on complex grids and fluid flows*, Lyon
- Dec. 2011 Invited seminar at Laboratoire de Mathématiques de Besançon
- Nov. 2011 Invited seminar at Institut de Mathématiques de Bordeaux
- May 2011 Invited seminar at LAGA, Univ. Paris 13

#### 4.4 Press

MaddMaths [interview](#) by M. Briani (in Italian): *Daniele Di Pietro: l'analisi numerical come antidoto contro noia e frustrazione*, rubrica *Giovani matematici crescono*

## 5 Institutional responsibilities

### 5.1 Main responsibilities

- Oct. 2018 **Director elect** of IMAG, taking office in 2020
- Oct. 2014–pres. Head of the **ACSIOM research team** <http://imag.edu.umontpellier.fr/acsiom> (25 permanent researchers)
- Oct. 2014–pres. Member of the board of directors of **IMAG**
- Sept. 2015–pres. In charge of the second year of the Master *Modeling and Numerical Analysis*, cf. <http://tinyurl.com/UM-M2-MANU>
- Sept. 2013–pres. Member of the board of the **Department of Mathematics** of the University of Montpellier
- June 2017–pres. Member of the *Commission de Section 26* (local expert committee for Applied Mathematics)
- 2012–2015 In charge of the first year of the Master *Mathématiques, Statistique et Applications*

### 5.2 Participation in selection committees

- 2016 President of the selection committee for a post of **Associate Professor** (ref. 26MCF99, Université de Montpellier)
- 2015 President of the selection committee for a post of **Full Professor** (ref. 2526PR4118, Université de Nîmes, France)
- 2014 Member of the selection committee for a post of **Associate Professor** (MAT/08, ref. 2010/MAT3, Politecnico di Milano, Italy)
- 2014 President of the selection committee for a post of **Full Professor** (ref. 26PR4171, Université de Montpellier)

## 6 Research funding track-record

### 6.1 Academic research projects

#### 6.1.1 As Principal Investigator (PI)

Reference	Timeframe	Funding	Description
ANR-10-LABX-0002-01	2017–2018	47 700€	Co-funding for the project <i>Development of a HHO method for the direct simulation of turbulent flows in Code_Saturne</i>
ANR HHOMM	2015–2019	172 224€	<i>Hybrid High-Order Methods on polyhedral Meshes</i> . Only JCJC project* in Numerical Analysis funded in the 2015 call. Details at <a href="http://imag.umontpellier.fr/~di-pietro/HHOMM.html">http://imag.umontpellier.fr/~di-pietro/HHOMM.html</a>
NUMEV 2014-2-006	2015–2018	50 000€	Co-funding for the Ph.D. thesis of M. Botti
UFI Vinci	2015–2018	90 000€	Ph.D. thesis of F. Chave
ERT IFPEN-LJLL	2008–2013	220 000€	<i>Enhanced oil recovery and geological sequestration of CO<sub>2</sub>: mesh adaptivity, a posteriori error control, and other advanced techniques</i> , co-PI with M. Vohralík

\* JCJC projects are the only form of individual projects funded by ANR, and are reserved to researchers whose Ph.D. thesis has been defended up to 10 years prior to the call

#### 6.1.2 As co-investigator

Reference	Timeframe	Funding	Description
ANR fast4hho*	2017–2021	1 067 770€	<i>Fast Solvers for robust discretisations in CFD</i> (PI: F. Hülse-mann)
ANR HAMM	2010–2014	1 060 721€	<i>Hybrid Architectures and Multiscale Methods</i> (PI: C. Prud'homme)
ANR VFSitCom	2009–2012	180 000€	<i>Volumes Finis pour Situations Complexes</i> (PI: J. Droniou)

\* In charge of the local team at IMAG. ANR contribution: 465 686€

### 6.2 Industrial collaborations as PI

Reference	Timeframe	Funding	Description
EDF	2017–2020	36 000€	Co-funding for the project <i>Development of a HHO method for the direct simulation of turbulent flows in Code_Saturne</i>
EDF	2016–2019	10 000€	Scientific collaboration
BRGM	2014–2018	60 000€	Co-funding for the Ph.D. thesis of M. Botti
Saint-Gobain	2015–2016	15 000€	<i>Hybrid High-Order methods for the Cahn–Hilliard equation</i> , fundamental research program Phi-Zero
EDF	2014–2017	135 000€	Funding for the Ph.D. thesis of R. Riedlbeck + scientific collaboration



## 7 Research

My main research topics include:

- Advanced numerical methods for PDEs
- A priori and a posteriori error analysis
- Fluid- and solid-mechanics
- Porous media flows
- Modern implementation techniques

### 7.1 Summary of five selected publications

Title: **Mathematical Aspects of Discontinuous Galerkin Methods**

Reference: D. A. Di Pietro and A. Ern, *Mathématiques & Applications*, 69. Springer, Heidelberg

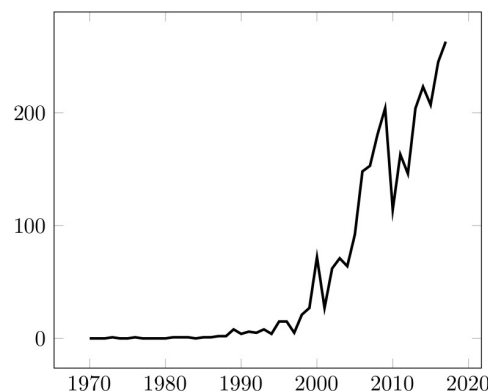


Figure 1: Publications per year in MathSciNet whose title contains the string “discontinuous Galerkin”.

Discontinuous Galerkin (dG) methods have gained an immense popularity over the last 20 years, with a steadily increasing number of scientific publications on this subject; see Figure 1. **This monograph, which treats in a mathematically rigorous fashion both basic and advanced topics, has rapidly become an established reference on dG methods.**

A **wide range of steady and unsteady, linear and nonlinear model problems** are covered, elaborating from advection-reaction and diffusion problems up to the Navier–Stokes equations and Friedrichs’ systems.

For each linear problem, we show how to derive optimally convergent dG methods by seeking the three key properties identified in the abstract analysis framework and corresponding to Lax’s principle: stability, consistency, and continuity. We then carry out a complete convergence analysis leading to quasi-optimal estimates in various norms. An original point of the analysis is that the dependence on the problem coefficients is explicitly tracked, leading to methods and estimates that are **robust with respect to the underlying physics**.

For nonlinear problems, we prove additional **compact embeddings** which enable us to carry out a convergence analysis towards minimal regularity solutions (in this case, of course, without estimating of the convergence rate). As a paradigmatic example for the use of these advanced techniques we consider the Navier–Stokes equations, for which dG has nowadays become a reference numerical method.

This monograph additionally collects a number of valuable **fundamental numerical analysis results** whose interest goes beyond the specific method and applications. We mention here, in particular: (i) an **abstract analysis framework** adapted to fully non-conforming methods, which serves as a basis for the development and analysis of dG methods for linear problems; (ii) a set of novel **functional analysis results for broken polynomial and Sobolev spaces on general meshes**. The treatment of general polyhedral meshes is an extremely original point, and it proved a farsighted choice which contributed to the very recent developments of **Polyhedral Element Methods** (a family to which HHO belongs).

**Both Finite Element (FE) and Finite Volume (FV) viewpoints are utilized and bridged** to convey the main

ideas underlying the design of the approximation. The FV viewpoint is the most natural choice for hyperbolic problems, where dG methods can be regarded as high-order schemes relying on a piecewise polynomial approximation. The FE viewpoint, on the other hand, appears naturally when dealing with elliptic and parabolic problems, with dG methods being regarded as a means to incorporate the conformity constraints by penalisation.

Previous publications that have contributed to this book include [40, 42–45, 49–51, 53–57]. This monograph has, in turn, given rise or contributed to a large set of publications, impossible to cover here in detail.

**Title: A hybrid high-order locking-free method for linear elasticity on general meshes**

Reference: D. A. Di Pietro, A. Ern, *Computer Methods in Applied Mechanics and Engineering*, 2015, 283:1–21

**This is the paper where Hybrid High-Order (HHO) methods were originally introduced and, despite being relatively recent, it already ranks second among my most cited publications according to the MathSciNet database.** HHO methods and the underlying ideas have breakthrough features which set them as a reference for future simulators. Several academic and industrial projects have been funded to develop HHO methods. We recall here, in particular, the **ANR HHOMM** and **ANR fast4hho** projects selected in the 2015 and 2017 calls, respectively; see Section 6.1. Several **industrial collaborations** are also active on HHO methods; see again Section 6.2. In particular, HHO methods have already been implemented and used in **industrial simulators** in fluid- and solid-mechanics including, in particular, Code\_Saturne <https://www.code-saturne.org> and Code\_Aster <https://www.code-aster.org>.

This paper focuses on the development of a **locking-free HHO method for quasi-incompressible linear elasticity** on general meshes including polyhedral elements and non-matching interfaces. The proposed method relies on a pure-displacement (primal) formulation, and leads to a symmetric, positive definite system matrix with compact stencil. The only globally coupled discrete unknowns are vector-valued polynomials of degree  $k \geq 1$  on the mesh faces, so that the lowest order version of the scheme only requires 4 (resp. 9) DOFs per face in 2 (resp. 3) space dimensions. The key idea is to reconstruct discrete counterparts of the symmetric gradient and divergence operators inside each mesh element in terms of the local discrete unknowns by solving inexpensive, trivially parallel local problems. The global problem is then assembled element-wise using these operators conjointly with a high-order stabilization bilinear form.

**Locking-free error estimates are derived** for the energy norm and for the  $L^2$ -norm of the displacement, with optimal convergence rates on general meshes of order  $(k + 1)$  and  $(k + 2)$ , respectively. These results are confirmed numerically, and the accuracy vs. CPU cost is evaluated on both standard and polygonal meshes.

It is worth mentioning that **this paper has also provided a positive answer to the long-standing open question: “Can superconvergence of the potential unknowns be achieved on general meshes for Hybridizable Discontinuous Galerkin methods?”**; cf. [25] for the details.

Earlier works that have contributed to the ideas in this paper include [1] (analysis tools on general meshes), [34] (functional interpretation of discrete operators) and [44] (early introduction of the analysis techniques). Subsequent developments include, among others, [3–8, 62, 10–13, 16–20, 65, 23, 25–28, 30, 32, 33, 36].

**Title: A Hybrid high-Order method for Leray–Lions elliptic equations on general meshes**

Reference: D. A. Di Pietro and J. Droniou, *Mathematics of Computation*, 2017, 86(307):2159–2191.

This work, along with its companion paper [19], deals with the application of Hybrid High-Order (HHO) methods to the discretisation of **nonlinear Leray–Lions problems**. In terms of novel developments, this is an extremely rich paper, as it contains: (i) the first application of HHO methods to **nonlinear elliptic operators** in a non-Hilbertian setting; (ii) the first **convergence analysis for HHO methods relying on compactness techniques**; (iii) a plethora of **discrete functional analysis tools** for HHO spaces; (iv) key approximation results for **projectors on polynomial spaces over polytopal elements**.

As for the linear case considered in [36], the HHO method proposed here hinges on discrete unknowns that are broken polynomials of degree up to  $k \geq 0$  over the mesh elements and faces. Based on these discrete unknowns, **a reconstruction of the gradient is performed inside each element using a richer space than in [36]**. This reconstruction is conceived so that, composed with the local interpolator applied to a smooth

function, it coincides with the  $L^2$ -orthogonal projector on the space of polynomials of total degree  $\leq k$  of the function gradient. As observed in [13, Section 4.1], this richer reconstruction is required in the nonlinear (and non-Hilbertian) setting, where the flux of a polynomial potential is not necessarily a polynomial itself. The discretisation is then completed by a local stabilisation term which generalises the classical HHO penalisation strategy to the non-Hilbertian setting. The resulting method has several assets, including the support for arbitrary approximation orders and general polytopal meshes.

The convergence analysis is carried out using a compactness technique. Extending this technique to HHO methods has prompted us to develop a set of **discrete functional analysis tools** including, in particular: (i) (direct and) reverse **continuous Lebesgue and Sobolev embeddings for local polynomial spaces** over polytopal elements; (ii) **continuous and compact Sobolev embeddings for global HHO spaces**. The interest of these results **goes beyond the specific method and application considered here**. As an example, local continuous Lebesgue and Sobolev embeddings have played a key role in the analysis of the HHO method for the Cahn–Hilliard problem considered in [24]. The compact global Sobolev embeddings, on the other hand, are lie at the heart of the convergence analysis of the HHO method for the Navier–Stokes equations proposed in [14].

Another very important set of results of general interest contained in this paper concerns the **boundedness and approximation properties of projectors on local polynomial spaces**. More specifically, this paper contains new proofs of general  $L^p$ -stability and  $W^{s,p}$ -approximation properties for the  $L^2$ -projector on polytopal elements, which have been later generalised in [19] to cover general  $W$ -bounded projectors; see Theorems 1 and 2 therein.

While very recent, this paper has already given rise or contributed to a string of extremely relevant publications, including, in particular, [13, 14, 60, 16, 17, 63, 64, 18, 24].

**Title: Adaptive regularization, linearization, discretization, and a posteriori error control for the two-phase Stefan problem**

Reference: D. A. Di Pietro, M. Vohralík, S. Yousef, *Mathematics of Computation*, 2015, 84(291):153–186

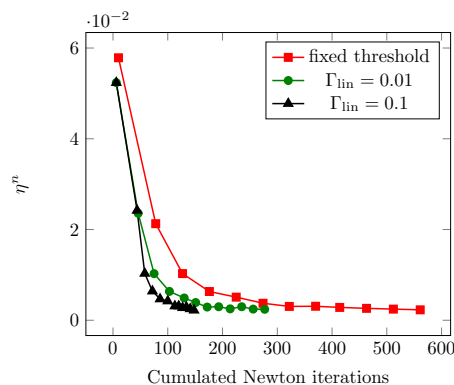


Figure 2: Cumulated Newton iteration: adaptive algorithm vs. fixed threshold.

The subject of this work is the time-dependent two-phase Stefan problem, for which a posteriori error estimates and adaptive numerical solution strategies are devised. The Stefan problem is encountered in phase change modeling, and it allows one to predict the advancement of the interface between phases. Its salient feature is that the enthalpy-temperature law accounts for the latent heat, and is therefore **neither strictly monotone nor of class  $C^1$** . From a numerical point of view, this lack of regularity induces a **highly nonlinear behavior** which can make convergence problematic. Additionally, as customary for interface problems, local adaptation is required to ensure an efficient use of the computational resources.

In this work, we develop rigorous a posteriori error estimators and utilize them to conceive a **fully adaptive algorithm**, that is, one where all the numerical parameters and the stopping criteria are adaptively selected. The key point, in our case, is to **separately estimate the space, time, regularization, and linearization error components** in the analysis. This is done here by translating all of these error components into quantities that

are homogeneous to a flux norm using **equilibrated reconstructions** performed at a local level.

Based on this decomposition of the error, a fully adaptive algorithm is devised including the online choice of the regularization parameter, an adaptive stopping criterion for the Newton solver, and the local adaptation of the space- and time-meshes. Roughly speaking, the innovative idea that this algorithm implements in a rigorous manner is the following: there is no need to further reduce the regularization parameter when the corresponding regularization error is negligible with respect to the dominant error components (space and time); similarly, there is no need to iterate further in the Newton algorithm if the corresponding linearization error is only a small fraction of the dominant error components (space, time, and regularization). Figure 2 shows that significant gains in terms of cumulated Newton iterations can be achieved using adaptive criteria.

Additionally, the proposed estimators are conceived so as to yield a **guaranteed and fully computable upper bound** for the dual norm of the residual, as well as for the  $L^2(L^2)$ -error on the temperature and the  $L^2(H^1)$ -error on the enthalpy.

It is worth emphasizing that **the ideas developed in this work are general**, and could be applied to (potentially) any problem where a conservative flux can be identified. In [37], e.g., these ideas are applied to the modelling of **multi-phase multi-component porous media flows**, and it is shown that the computational cost can be reduced by a factor 10 with respect to a standard industrial simulator. Further developments of the ideas in this paper can be found in [66, 22, 38, 39].

An **industrial collaboration with EDF** (CIFRE Ph.D. thesis of R. Riedlbeck, cf. Section 2.1) focusing on a posteriori-based adaptive algorithms for **poro-mechanics** with application to the numerical modelling of nuclear waste storage ended in 2017. A second CIFRE Ph.D. thesis with the same research group at EDF will start in November 2018.

**Title: An extension of the Crouzeix–Raviart space to general meshes with application to quasi-incompressible linear elasticity and Stokes flow**

Reference: D. A. Di Pietro, S. Lemaire, *Mathematics of Computation*, 2015, 84(291):1–31

The idea underlying Galerkin-type discretization methods consists in replacing continuous functional spaces by (possibly nonconforming) finite-dimensional approximations. One classical example is the Crouzeix–Raviart space, which can be constructed starting from a mesh of the domain composed of simplices. In several applications, however, handling elements of more general shapes is required.

In this work, **we introduce a novel discrete functional space on general polytopal meshes which mimics two important properties of the standard Crouzeix–Raviart space**, namely the continuity of mean values at interfaces and the existence of a Fortin interpolator. These properties are key to the design of stable and inexpensive methods in fluid- and solid-mechanics.

The discrete functional space is defined from element and face unknowns by working on a simplicial submesh. Specifically, the key ingredients are a reconstruction of the potential obtained mimicking a local integration by parts formula, and a stabilising contribution obtained by penalising high-order differences. A carefully selected penalty parameter yields the continuity of the mean value across interfaces.

Two applications are considered in which the properties of the discrete space play an important role:

- (i) the design of a **lowest-order locking-free primal (as opposed to mixed) method for quasi-incompressible linear elasticity** on general meshes. The key point is here the construction of a discrete divergence operator that satisfies a suitable commuting property combined with a least-square penalty of interface jumps;
- (ii) the design of an **inf-sup stable method for the Stokes equations** on general polygonal or polyhedral meshes. In this context, we also propose a general modification, applicable to any suitable discretization, which guarantees that **the velocity approximation is unaffected by the presence of large irrotational body forces** provided that a Helmholtz decomposition of the right-hand side is available.

This work contains several inspirational ideas that have later led to the development of HHO methods; cf., in particular, [31, 36]. In addition, it is one of very few works where the ideas of embedding local reconstructions into the definition of the scheme and the weak enforcement of inter-element continuity by least-square penalty

meet. Contributions to the ideas developed in this work are found in [44] (use of reconstructions to formulate a variational method) and in [42] (nonconforming methods for linear elasticity).

## 7.2 On MathSciNet

According to the MathSciNet database, my works have been **cited 931 times by 730 authors**. My ten most cited publications:

- 227 cit. D. A. Di Pietro and A. Ern. *Mathematical aspects of discontinuous Galerkin methods*, volume 69 of *Mathématiques & Applications*. Springer-Verlag, Berlin, 2012.
- 67 cit. D. A. Di Pietro and A. Ern. A hybrid high-order locking-free method for linear elasticity on general meshes. *Comput. Meth. Appl. Mech. Engrg.*, 283:1–21, 2015.
- 57 cit. F. Bassi, A. Crivellini, D. A. Di Pietro, and S. Rebay. An artificial compressibility flux for the discontinuous Galerkin solution of the incompressible Navier-Stokes equations. *J. Comput. Phys.*, 218(2):794–815, 2006.
- 49 cit. D. A. Di Pietro and A. Ern. Discrete functional analysis tools for discontinuous Galerkin methods with application to the incompressible Navier–Stokes equations. *Math. Comp.*, 79:1303–1330, 2010.
- 48 cit. D. A. Di Pietro, A. Ern, and S. Lemaire. An arbitrary-order and compact-stencil discretization of diffusion on general meshes based on local reconstruction operators. *Comput. Meth. Appl. Math.*, 14(4):461–472, 2014
- 45 cit. F. Bassi, L. Botti, A. Colombo, D. A. Di Pietro, and P. Tesini. On the flexibility of agglomeration based physical space discontinuous Galerkin discretizations. *J. Comput. Phys.*, 231(1):45–65, 2012.
- 37 cit. F. Bassi, A. Crivellini, D. A. Di Pietro, and S. Rebay. An implicit high-order discontinuous Galerkin method for steady and unsteady incompressible flows. *Comp. & Fl.*, 36(10):1529–1546, 2007.
- 36 cit. B. Cockburn, D. A. Di Pietro, A. Ern. Bridging the hybrid high-order and hybridizable discontinuous Galerkin methods. *ESAIM: Math. Model Numer. Anal. (M2AN)*, 50(3):635–650, 2016.
- 31 cit. D. A. Di Pietro, A. Ern, and J.-L. Guermond. Discontinuous Galerkin methods for anisotropic semi-definite diffusion with advection. *SIAM J. Numer. Anal.*, 46(2):805–831, 2008.
- 30 cit. D. A. Di Pietro, A. Ern. Hybrid high-order methods for variable-diffusion problems on general meshes, method. *C. R. Math. Acad. Sci. Paris*, 353:31–34, 2015.

## 8 Publications

### 8.1 Research monographs

- [1] D. A. Di Pietro and A. Ern. *Mathematical aspects of discontinuous Galerkin methods*. Vol. 69. *Mathématiques & Applications (Berlin) [Mathematics & Applications]*. Springer, Heidelberg, 2012, pp. xviii+384. ISBN: 978-3-642-22979-4 (Softcover) 978-3-642-22980-0 (eBook). DOI: [10.1007/978-3-642-22980-0](https://doi.org/10.1007/978-3-642-22980-0).

### 8.2 Edited books

- [2] D. A. Di Pietro, A. Ern, and L. Formaggia, eds. *Numerical Methods for PDEs. State of the Art Techniques*. Vol. 15. SEMA-SIMAI. Springer International Publishing, 2018. ISBN: 978-3-319-94675-7 (Hardcover) 978-3-319-94676-4 (eBook). DOI: [10.1007/978-3-319-94676-4](https://doi.org/10.1007/978-3-319-94676-4).

### 8.3 Papers in international peer-reviewed journals

- [3] L. Botti, D. A. Di Pietro, and J. Droniou. “A Hybrid High-Order method for the incompressible Navier–Stokes equations based on Temam’s device”. In: *J. Comput. Phys.* 376 (2019), pp. 786–816. DOI: [10.1016/j.jcp.2018.10.014](https://doi.org/10.1016/j.jcp.2018.10.014).

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