A nonconforming Virtual Element Method for a biharmonic problem on polygonal meshes

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Outline

Introduction

- Many different methods to solve PDEs on polygonal meshes: Virtual Elements, Hybrid High-Order and Discontinuous Galerkin methods, Mimetic Finite Differences, Mixed/Hybrid Finite Volumes, ...
- Here we focus on the Virtual Element Method (VEM)

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FV: [Eymard,Gallouet, Herbin, Linke, 2012], HHO: [Chave, Di Pietro, Marche, Pigeonneau, 2016]

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Goal: Design a “weaker” nonconforming Virtual Element Method (VEM) for the discretization of the biharmonic problem on polygonal meshes.

The continuous problem

Let $\Omega \subset \mathbb{R}^2$ convex polygonal domain and $f \in L^2(\Omega)$

$$\begin{aligned} D\Delta^2 u &= f \qquad \text{in } \Omega \\ u = \partial_n u &= 0 \qquad \text{on } \Gamma = \partial\Omega \end{aligned}$$

$D = \frac{Et^3}{12(1-\nu^2)}$ bending rigidity, t thickness, E Young modulus, v Poisson's ratio.

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Weak formulation: Find $u \in V = \{v \in H^2(\Omega) : v = \partial_n u = 0 \text{ on } \Gamma\}$ s.t.

$$a(u,v) = F(v) \qquad \forall v \in V$$

where

$$a(u,v) = D\int_\Omega \nu \Delta u \Delta v + (1-\nu)(u_{,ij} v_{,ij}) dx \qquad F(v) = \int_\Omega f v dx.$$

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Setting $\|\cdot\|_V := |\cdot|_{2,\Omega}$ there exist $\alpha, M > 0$ such that

$$\begin{aligned} a(v, v) &\geq \alpha \|v\|_V^2 \qquad \forall v \in V \\ |a(u, v)| &\leq M \|u\|_V \|v\|_V \qquad \forall u, v \in V. \end{aligned}$$

Hence, there exists a unique solution $u \in V$.

Let $\sigma_{ij}(u) = \lambda(u_{,11} + u_{,22})\delta_{ij} + \mu u_{,ij}$ with Lamé parameters $\lambda = D\nu$, $\mu = D(1-\nu)$. We set:

$$\begin{aligned} M_{nn}(u) &= \sum_{i,j} \sigma_{ij} n_i n_j && \text{(normal bending moment)} \\ M_{nt}(u) &= \sum_{ij} \sigma_{ij} n_i t_j && \text{(twisting moment)} \\ T(u) &= \sigma_{ij,j} n_i + M_{nt,t} && \text{(normal shear force).} \end{aligned}$$

Let $K \subset \mathbb{R}^2$ be a polygonal domain and set

$$a^K(u, v) := D \int_K \nu \Delta u \Delta v + (1 - \nu)(u_{,ij} v_{,ij}) dx.$$

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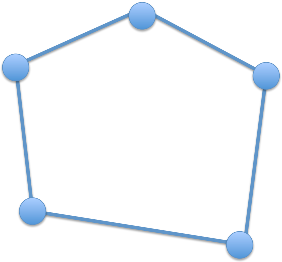
Integrations by parts yield the following useful equalities

$$\begin{aligned} a^K(u,v) &= D\left\{\int_K \Delta^2 uvdx + \int_{\partial K}(\Delta u - (1-\nu)u_{,tt})v_{,n}ds \right. \\ &\qquad \left. - \int_{\partial K}\left(\partial_n(\Delta u)v - (1-\nu)u_{,nt}v_{,t}\right)ds\right\} \\ &= D\left\{\int_K \Delta^2 uvdx + \int_{\partial K} M_{nn}(u)\partial_n v - \int_{\partial K} T(u)vds \right. \\ &\qquad \left. - \sum_{e\in\partial K}(M_{nt}(u), vn_{\partial e})_{\partial e}\right\}. \end{aligned}$$

where ∂e is the boundary of the edge e, $n_{\partial e}$ is the outwards normal to ∂e.

Towards nonconforming VEM discretization

- $\mathcal{T}_h$ decomposition of Ω into polygons K



- h_K is the diameter of K, i.e., the maximum distance between any two vertices of K.
- On $\mathcal{T}_h$ we make the following assumptions (for analysis only):

  there exists a fixed number $\rho_0 > 0$ independent of $\mathcal{T}_h$, such that for every element K (with diameter h_k) it holds

  1. K is star-shaped with respect to all the points of a ball of radius $\rho_0 h_K$
  2. every edge $e \in \mathcal{E}_h$ has length $|e| \geq \rho_0 h_K$

Low-order nonconforming VEM

Local nonconforming VEM space:

$$V_{h,2}^K = \{v_h \in H^2(K) : \Delta^2 v_h = 0, M_{nn}(v_h)_{|e} \in \mathbb{P}^0(e), T(v_h)_{|e} = 0 \quad \forall e \in \partial K\}$$

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The DOFs of $V_{h,2}^{K}$ are chosen as follows:

(d1) $v_h(\nu_i)$ for any vertex ν_i of K;

(d2) $\int_e \partial_n v_h ds$ for any edge e of ∂K.

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Remarks:

- Dofs (d1)-(d2) are unisolvent for $V_{h,2}^K$
- $\mathbb{P}^2(K) \subset V_{h,2}^K$.
- In case of a triangular mesh, we recover Morley's DOFs.
- The solution of the biharmonic problem appearing in the definition of $V_{h,2}^K$ is uniquely defined up to a linear function (which is however filtered by fixing, e.g., the values of the function in three non-aligned vertexes).

We now prove that dofs (d1)-(d2) are unisolvent for V_h^K.

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$$a^K(v_h, v_h) = \int_K \Delta^2 v_h v_h dx - \sum_{e \in \partial K} (M_{nt}(v_h), v_h n_{\partial e})_{\partial e} + \int_{\partial K} M_{nn}(v_h) \partial_n v_h ds$$

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Observing that $\Delta^2 v_h = 0$ and $M_{nn}(v_h)$ is constant on each edge, by setting to zero the degrees of freedom, we get $a^K(v_h, v_h) = 0$.

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Coercivity gives $v_h = 0$ (note that setting to zero the nodal values of v_h filters the linear polynomials)

The global non-conforming low-order VEM space is defined as

$$V_{h,2} := \Big\{ v_h : v_h|_K \in V_{h,2}^K, v_h \text{ continuous at internal vertexes}, \\ \int_e [\partial_n v_h] ds = 0 \ \forall e \in \mathcal{E}_h^i, \\ \int_e \partial_n v_h ds = 0 \ \forall e \in \mathcal{E}_h^\Gamma, v_h(\nu_i) = 0 \ \forall \nu_i \in \mathcal{V}_h^\Gamma \Big\},$$

where $[\cdot]$ is the usual jump operator.

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Note that functions in V_h are not continuous.

High-order nonconforming VEM

For $\ell > 2$, the local VEM space is defined as follows:

$$V_{h,\ell}^K = \{v_h \in H^2(K) : \Delta^2 v_h \in \mathbb{P}^{\ell-4}(K), M_{nn}(v_h)_{|e} \in \mathbb{P}^{\ell-2}(e), T(v_h) \in \mathbb{P}^{\ell-3}(e)\ \forall e \in \partial K\}$$

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The local DOFs are chosen as follows:

(D1) $v_h(\nu_i)$ for any vertex ν_i of K;

(D2) $\frac{1}{|K|}\int_K v_h p ds$ for any $p \in \mathbb{P}^{\ell-4}(K)$;

(D3) $\int_e \partial_n v_h q ds$ for any $q \in \mathbb{P}^{\ell-2}(e)$ and any edge e of ∂K;

(D4) $\frac{1}{|e|}\int_e v_h q ds$ for any $q \in \mathbb{P}^{\ell-3}(e)$ and any edge e of ∂K.

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Remarks:

- Dofs (D1)-(D4) are unisolvent for $V_{h,\ell}^K$;
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The global non-conforming VEM space is defined as

$$V_{h,\ell} := \Big\{ v_h : v_h|_K \in V_h^K, v_h \text{ continuous at internal vertexes,}$$
$$v_h(\nu_i) = 0 \ \forall \nu_i \in \mathcal{V}_h^\Gamma$$
$$\int_e [\partial_n v_h] q ds = 0 \ \forall q \in \mathbb{P}^{\ell-2}(e), \int_e [v_h] p ds = 0 \ \forall p \in \mathbb{P}^{\ell-3}(e) \ \forall e \in \mathcal{E}_h^i;$$
$$\int_e \partial_n v_h q ds = 0 \ \forall q \in \mathbb{P}^{\ell-2}(e), \int_e v_h p ds = 0 \ \forall p \in \mathbb{P}^{\ell-3}(e) \ \forall e \in \mathcal{E}_h^\Gamma \Big\}.$$

Local discrete bilinear form

We build a discrete local bilinear form $a_h^K(\cdot,\cdot) : V_{h,\ell}^K \times V_{h,\ell}^K \to \mathbb{R}$ such that

- (**ℓ-consistency**) $\forall p \in \mathbb{P}^\ell(K)$ $\forall v_h \in V_{h,\ell}^K$

$$a_h^K(p, v_h) = a^K(p, v_h)$$

- (**stability**) There exist positive constants α_*, α^* independent of h and K such that

$$\alpha_* a^K(v_h, v_h) \leq a_h^K(v_h, v_h) \leq \alpha^* a^K(v_h, v_h) \qquad \forall v_h \in V_{h,\ell}^K.$$

- (**computability**) $\forall u_h, v_h \in V_{h,\ell}^K$ the local bilinear form

$$a_h^K(u_h, v_h)$$

  is computable using only the DOFs of $V_{h,\ell}^K$.

Discrete VEM problem

Assemble in the usual way the global discrete bilinear form $a_h(\cdot,\cdot)$, i.e.

$$a_h(\cdot,\cdot) = \sum_{K\in\mathcal{T}_h} a_h^K(\cdot,\cdot).$$

The VEM discrete problem reads as follows: find $u_h \in V_{h,\ell}$ such that

$$a_h(u_h, v_h) = \langle f_h, v_h \rangle$$

for any $v_h \in V_{h,\ell}$, where $\langle f_h, v_h \rangle \simeq F(v_h)$.

Theorem (Antonietti, Manzini, V. 2016)

Under regularity assumptions on the mesh, there exists a unique solution $u_h \in V_{h,\ell}$. *Moreover, it holds*

$$|u - u_h|_{2,h} \lesssim h^{\ell-1} + \sup_{v_h \in V_{h,\ell}} \frac{\langle f - f_h, v_h \rangle}{|v_h|_{2,h}} + \sup_{v_h \in V_{h,\ell}} \frac{\mathcal{N}(u, v_h)}{|v_h|_{2,h}}$$

where

$$\begin{aligned} \mathcal{N}(u, v_h) \quad := \quad & D \sum_{K \in \mathcal{T}_h} \left\{ \int_{\partial K} (\Delta u - (1-\nu) u_{,tt}) v_{h,n} ds \right. \\ & \left. - \int_{\partial K} \left(\partial_n (\Delta u) v_h - (1-\nu) u_{,nt} v_{h,t} \right) ds \right\}. \end{aligned}$$

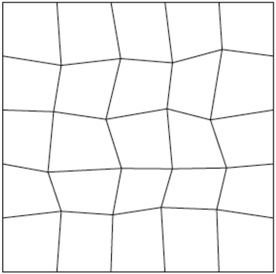
Corollary

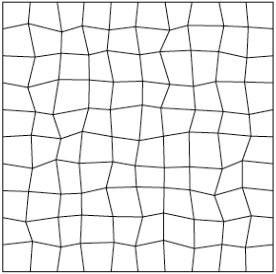
Under regularity assumption on the mesh, the unique solution $u_h \in V_{h,\ell}$ *satisfies the following error estimate, for* $\ell \geq 2$

$$|u - u_h|_{2,h} \lesssim h^{\ell-1}.$$

Numerical results

- Exact solution: $u(x, y) = x^2(1 - x)^2 y^2(1 - y)^2$
- Examples of employed polygonal meshes





Case $\ell = 2$

8 DOFs for each polygon

n	#dofs	Resid	$\mathcal{E}_{abs}(u_h)$	$\mathcal{E}_{rel}(u_h)$	Rate
0	$9.600\,10^{1}$	$3.823\,10^{-18}$	$6.671\,10^{-2}$	$9.364\,10^{-1}$	$--$
1	$3.410\,10^{2}$	$4.663\,10^{-18}$	$3.897\,10^{-2}$	$4.969\,10^{-1}$	0.999
2	$1.281\,10^{3}$	$7.007\,10^{-18}$	$2.053\,10^{-2}$	$2.559\,10^{-1}$	1.002
3	$4.961\,10^{3}$	$1.636\,10^{-17}$	$1.038\,10^{-2}$	$1.287\,10^{-1}$	1.015
4	$1.952\,10^{4}$	$3.694\,10^{-17}$	$5.224\,10^{-3}$	$6.467\,10^{-2}$	1.004
5	$7.744\,10^{4}$	$7.052\,10^{-17}$	$2.616\,10^{-3}$	$3.237\,10^{-2}$	1.004
6	$3.085\,10^{5}$	$1.771\,10^{-16}$	$1.309\,10^{-3}$	$1.620\,10^{-2}$	1

Case $\ell = 3$

16 DOFs for each polygon

n	#dofs	Resid	$\mathcal{E}_{abs}(u_h)$	$\mathcal{E}_{rel}(u_h)$	Rate
0	$2.160\,10^{2}$	$5.022\,10^{-18}$	$2.253\,10^{-2}$	$2.807\,10^{-1}$	$--$
1	$7.810\,10^{2}$	$9.850\,10^{-18}$	$7.419\,10^{-3}$	$9.185\,10^{-2}$	1.738
2	$2.961\,10^{3}$	$2.350\,10^{-17}$	$2.008\,10^{-3}$	$2.485\,10^{-2}$	1.961
3	$1.152\,10^{4}$	$4.610\,10^{-17}$	$5.214\,10^{-4}$	$6.452\,10^{-3}$	1.985
4	$4.544\,10^{4}$	$1.028\,10^{-16}$	$1.322\,10^{-4}$	$1.636\,10^{-3}$	2
5	$1.805\,10^{5}$	$2.122\,10^{-16}$	$3.330\,10^{-5}$	$4.121\,10^{-4}$	1.998
6	$7.194\,10^{5}$	$5.766\,10^{-16}$	$8.341\,10^{-6}$	$1.032\,10^{-4}$	2.002

Case $\ell = 4$

25 DOFs for each polygon

n	#dofs	Resid	$\mathcal{E}_{abs}(u_h)$	$\mathcal{E}_{rel}(u_h)$	Rate
0	$3.610\,10^{2}$	$5.301\,10^{-18}$	$3.589\,10^{-3}$	$4.441\,10^{-2}$	−−
1	$1.321\,10^{3}$	$5.054\,10^{-17}$	$6.776\,10^{-4}$	$8.385\,10^{-3}$	2.570
2	$5.041\,10^{3}$	$2.445\,10^{-17}$	$1.125\,10^{-4}$	$1.392\,10^{-3}$	2.681
3	$1.116\,10^{4}$	$5.485\,10^{-17}$	$3.693\,10^{-5}$	$4.569\,10^{-4}$	2.802
4	$1.968\,10^{4}$	$5.240\,10^{-17}$	$1.638\,10^{-5}$	$2.027\,10^{-4}$	2.865
5	$3.060\,10^{4}$	$8.506\,10^{-17}$	$8.633\,10^{-6}$	$1.068\,10^{-4}$	2.902
6	$4.392\,10^{4}$	$7.660\,10^{-17}$	$5.089\,10^{-6}$	$6.297\,10^{-5}$	2.925
7	$5.964\,10^{4}$	$1.076\,10^{-16}$	$3.245\,10^{-6}$	$4.016\,10^{-5}$	2.940

Case $\ell = 5$

35 DOFs for each polygon

n	#dofs	Resid	$\mathcal{E}_{abs}(u_h)$	$\mathcal{E}_{rel}(u_h)$	Rate
0	$5.310\,10^{2}$	$2.115\,10^{-18}$	$1.605\,10^{-3}$	$1.985\,10^{-2}$	$--$
1	$1.961\,10^{3}$	$5.428\,10^{-17}$	$1.372\,10^{-4}$	$1.698\,10^{-3}$	3.764
2	$7.521\,10^{3}$	$1.077\,10^{-17}$	$8.904\,10^{-6}$	$1.102\,10^{-4}$	4.069
3	$1.668\,10^{4}$	$8.361\,10^{-17}$	$1.765\,10^{-6}$	$2.184\,10^{-5}$	4.063
4	$2.944\,10^{4}$	$2.374\,10^{-17}$	$5.595\,10^{-7}$	$6.924\,10^{-6}$	4.044
5	$4.580\,10^{4}$	$1.146\,10^{-16}$	$2.296\,10^{-7}$	$2.842\,10^{-6}$	4.030
6	$6.576\,10^{4}$	$3.491\,10^{-17}$	$1.109\,10^{-7}$	$1.373\,10^{-6}$	4.023
7	$8.932\,10^{4}$	$1.648\,10^{-16}$	$6.142\,10^{-8}$	$7.600\,10^{-7}$	3.860

Conclusions

We presented:

- Nonconforming VEM based discretization for a biharmonic problem on polygonal meshes
- Energy norm a priori error estimates
- Numerical results

Future work:

- Build an efficient preconditioner

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