The RODIN project: an example of research collaboration with industry in the context of shape and topology optimization of structures

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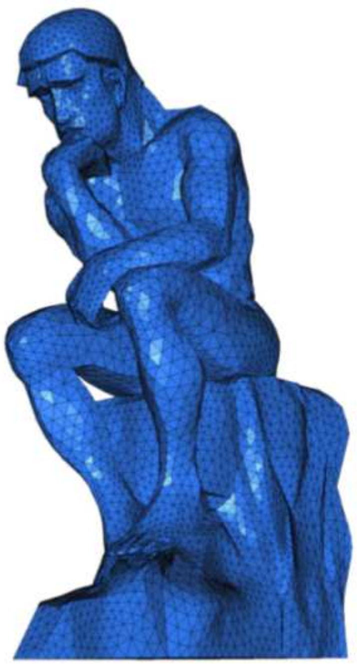
CONTENTS

RODIN project



Ecole Polytechnique,

UPMC, INRIA,

Renault, Airbus,

Safran, ESI group, etc.

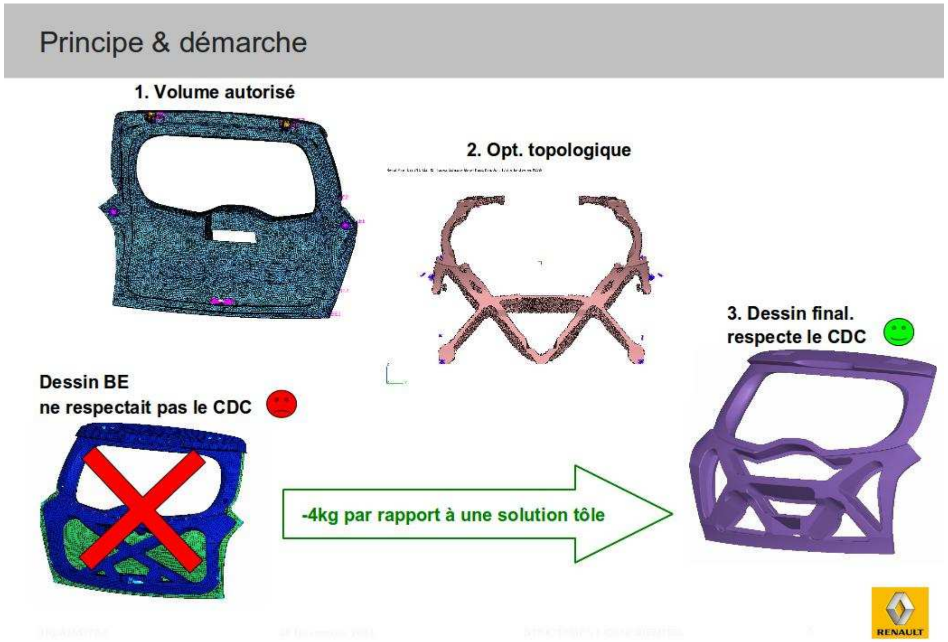
1. Review of the level set method for shape and topology optimization.

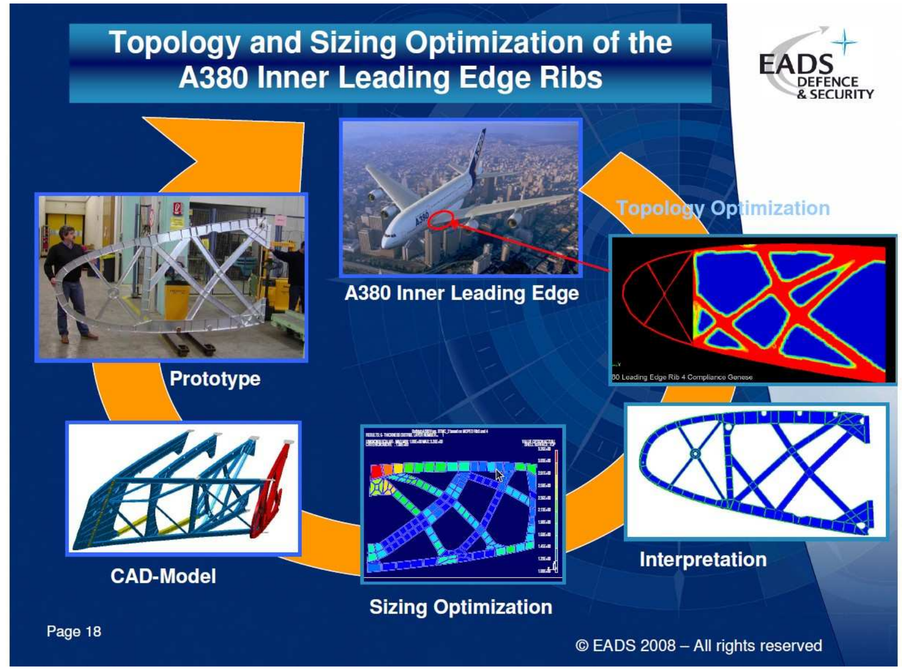
2. Thickness constraints.

3. Uncertainties and linearized worst-case design.

4. A level set based mesh evolution method.

-I- INTRODUCTION AND REVIEW





- ☞ Tremendous progresses were achieved on academic research about shape and topology optimization.
- ☞ There are already many commercial softwares which are heavily used by industry.
- ☞ Pending issues: manufacturability, robustness, geometric precision.

Definition of structural optimization

Shape optimization : minimize an objective function over a set of admissibles shapes Ω (including possible constraints)

$$\inf_{\Omega \in \mathcal{U}_{ad}} J(\Omega)$$

The objective function is evaluated through a partial differential equation (state equation)

$$J(\Omega) = \int_\Omega j(u_\Omega)\, dx$$

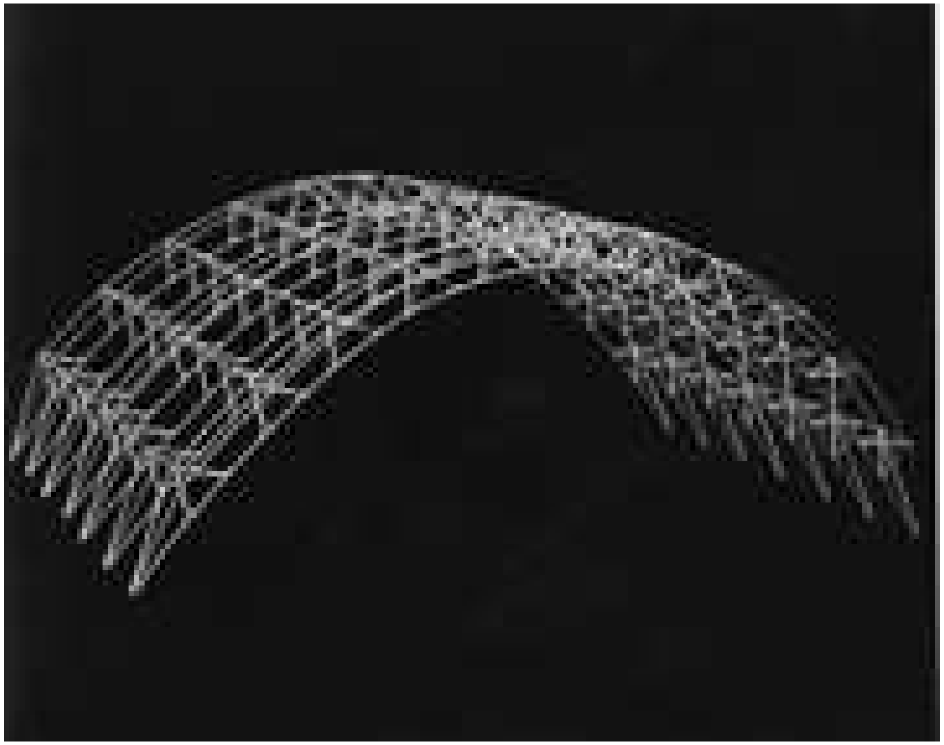
where u_Ω is the solution of

$$PDE(u_\Omega) = 0 \quad \text{in} \quad \Omega$$

Topology optimization : the optimal topology is unknown.

The art of structure is where to put the holes.

Robert Le Ricolais, architect and engineer, 1894-1977



The model of linear elasticity

Shape $\Omega \subset \mathbb{R}^d$ with free boundary Γ and fixed boundaries Γ_D, Γ_N.

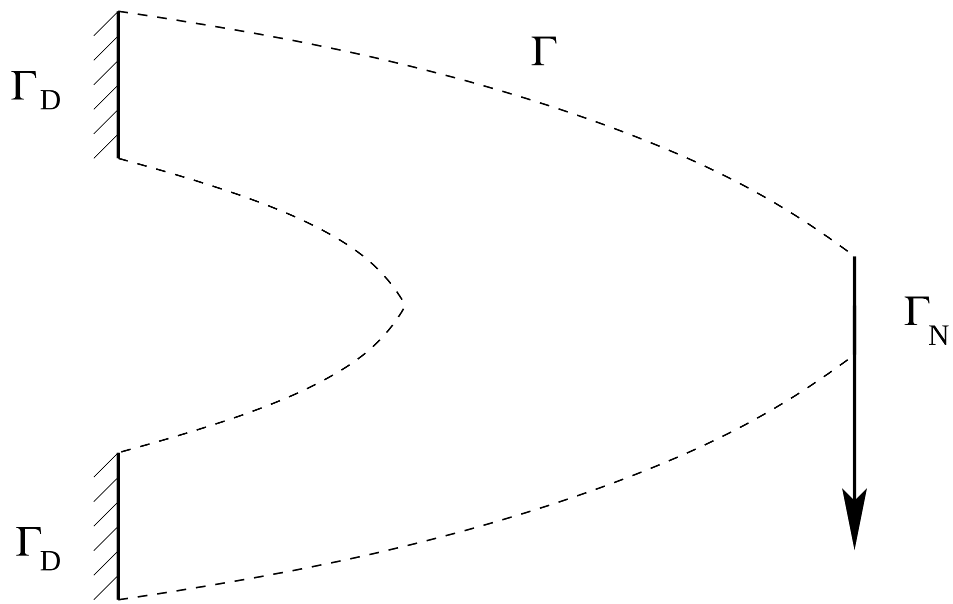
$$\begin{cases} -\operatorname{div}\left(A\, e(u)\right) = 0 & \text{in } \Omega \\ u = 0 & \text{on } \Gamma_D \\ \big(A\, e(u)\big)n = g & \text{on } \Gamma_N \\ \big(A\, e(u)\big)n = 0 & \text{on } \Gamma \end{cases}$$

- ☞ Applied load $g : \Gamma_N \to \mathbb{R}^d$
- ☞ Displacement $u : \Omega \to \mathbb{R}^d$
- ☞ Strain tensor $e(u) = \frac{1}{2}\left(\nabla u + \nabla^t u\right)$
- ☞ Stress tensor $\sigma = Ae(u)$, with A homog. isotropic elasticity tensor

Typical objective function: compliance

$$J(\Omega) = \int_{\Gamma_N} g \cdot u \, dx,$$

Admissible shapes



The **shape optimization** problem is $\inf_{\Omega \in \mathcal{U}_{ad}} J(\Omega)$,

where the set of admissible shapes is typically

$$\mathcal{U}_{ad} = \left\{ \Omega \subset D \text{ open set such that } \Gamma_D \bigcup \Gamma_N \subset \partial\Omega \text{ and } \int_\Omega dx = V_0 \right\},$$

with $D \subset \mathbb{R}^d$, a given "working domain" and V_0 a prescribed volume.

LEVEL SET METHOD

Main idea: coupling a front propagation algorithm with shape sensitivities

- Front propagation: level set algorithm of Osher and Sethian (JCP 1988).
- Shape capturing algorithm.
- Hadamard method for computing shape derivatives.
- Early references: Sethian and Wiegmann (JCP 2000), Osher and Santosa (JCP 2001), Allaire, Jouve and Toader (CRAS 2002, JCP 2004, CMAME 2005), Wang, Wang and Guo (CMAME 2003).

FRONT PROPAGATION BY LEVEL SET

Shape capturing method on a fixed mesh of the "working domain" D.

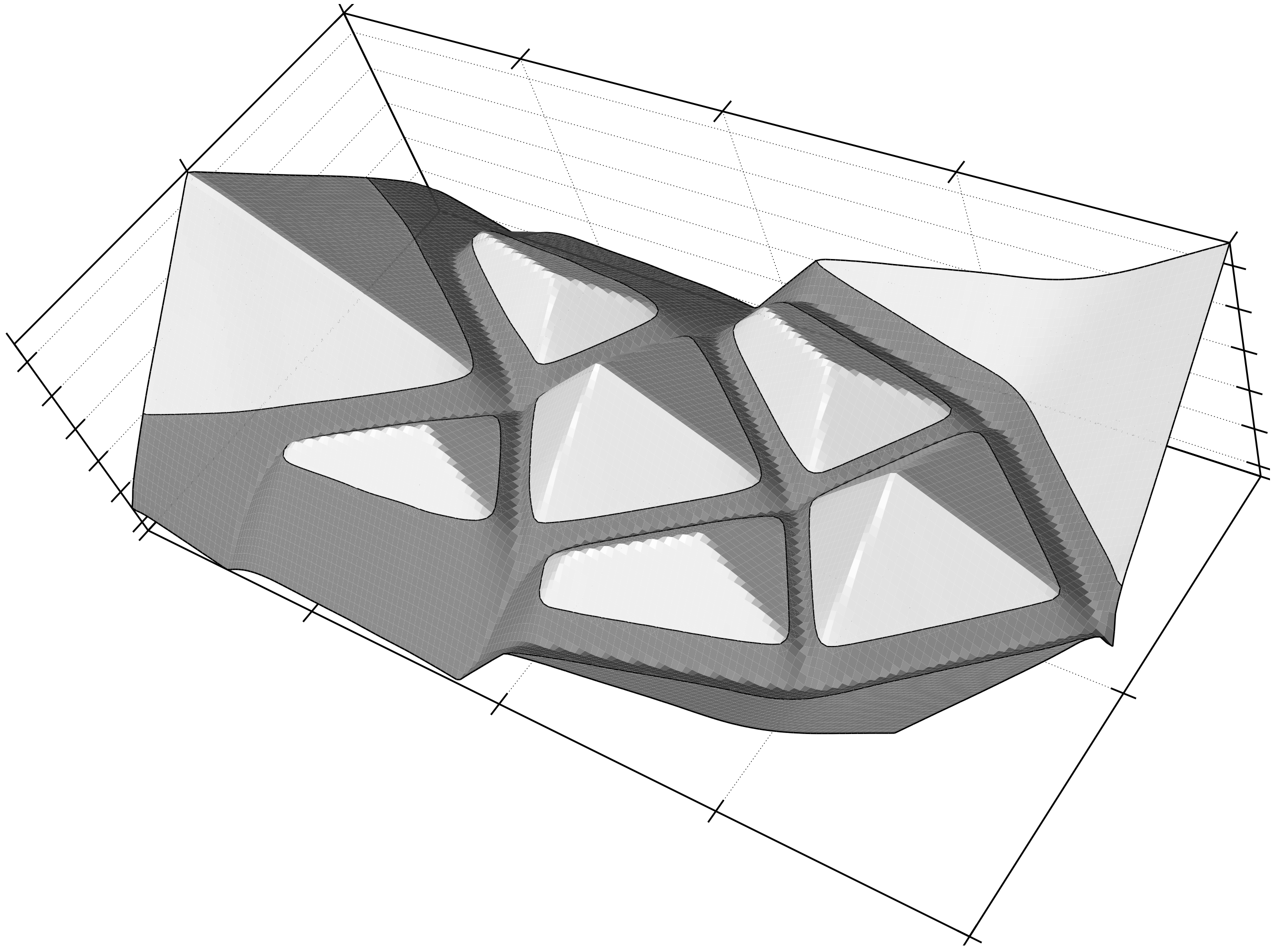
A shape Ω is parametrized by a **level set** function

$$\begin{cases} \psi(x) = 0 & \Leftrightarrow x \in \partial\Omega \cap D \\ \psi(x) < 0 & \Leftrightarrow x \in \Omega \\ \psi(x) > 0 & \Leftrightarrow x \in (D \setminus \Omega) \end{cases}$$

Assume that the shape $\Omega(t)$ evolves in time t with a normal velocity $V(t,x)$. Then its motion is governed by the following Hamilton Jacobi equation

$$\frac{\partial \psi}{\partial t} + V |\nabla_x \psi| = 0 \quad \text{in } D.$$

Example of a level set function



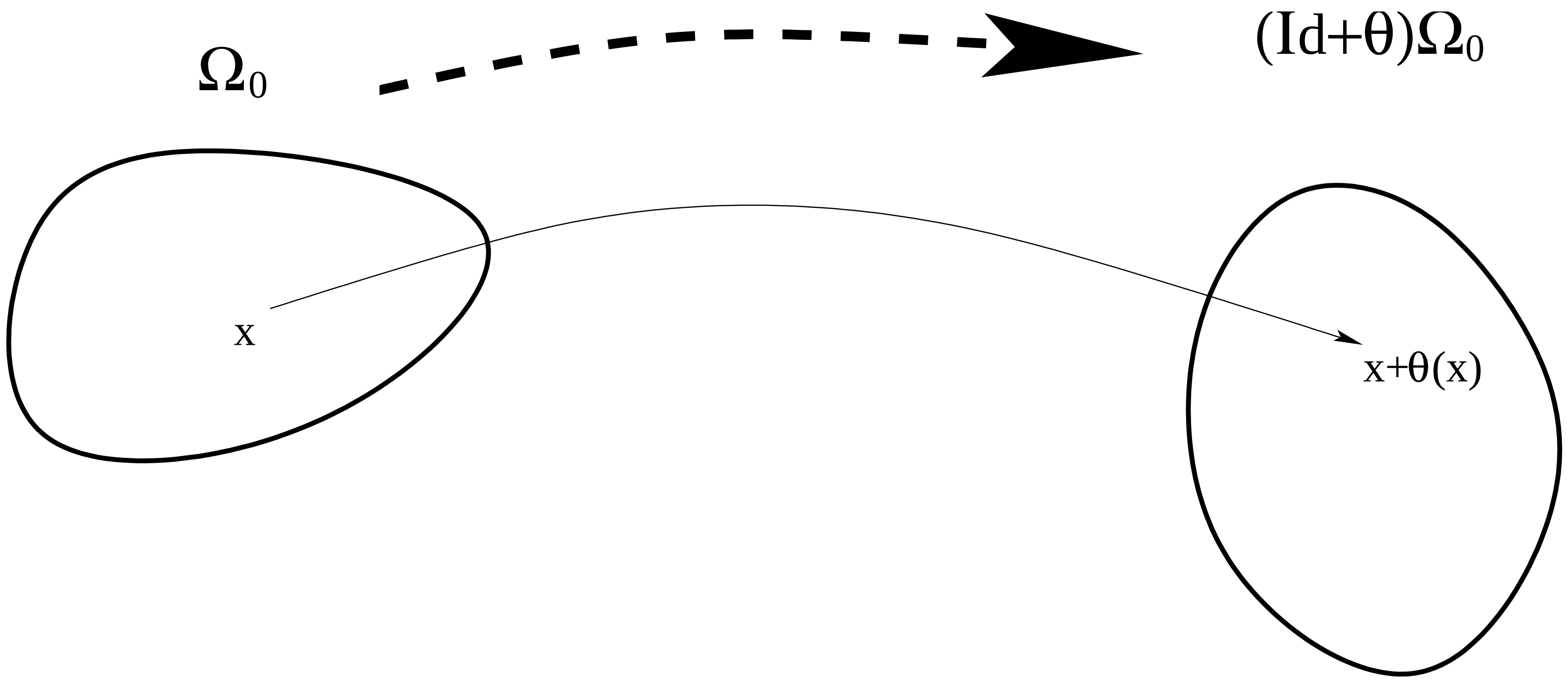
Advection velocity = shape gradient

The velocity V is deduced from the shape gradient of the objective function.

To compute this shape gradient we recall the well-known Hadamard's method.

Let Ω_0 be a reference domain. Shapes are parametrized by a vector field θ

$$\Omega = (\,\mathrm{Id} + \theta)\Omega_0 \quad \text{with} \quad \theta \in C^1(\mathbb{R}^d;\mathbb{R}^d).$$



Shape derivative

Definition: the shape derivative of $J(\Omega)$ at Ω_0 is the Fréchet differential of $\theta \to J\big((\mathrm{Id}+\theta)\Omega_0\big)$ at 0.

Hadamard structure theorem: the shape derivative of $J(\Omega)$ can always be written (in a distributional sense)

$$J'(\Omega_0)(\theta) = \int_{\partial\Omega_0} \theta(x)\cdot n(x)\, j(x)\, ds$$

where $j(x)$ is an integrand depending on the state u and an adjoint p.

We choose the velocity $V = \theta \cdot n$ such that $J'(\Omega_0)(\theta) \leq 0$.

Example: for the compliance, $j(x) = -Ae(u)\cdot e(u)$

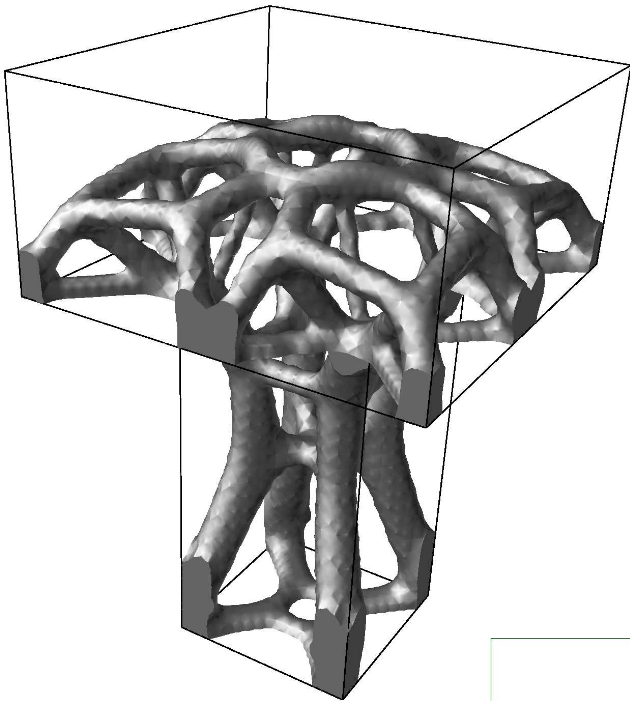
NUMERICAL ALGORITHM

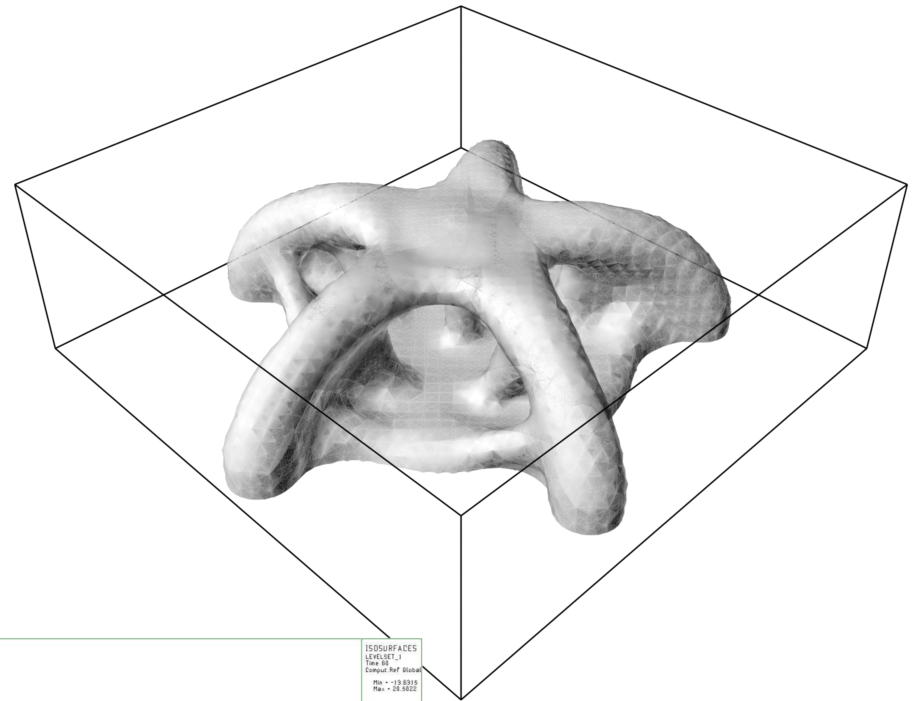
1. Initialization of the level set function ψ_0 (including holes).
2. Iteration until convergence for $k \geq 1$:
   (a) Compute the elastic displacement u_k for the shape ψ_k.
   Deduce the shape gradient = normal velocity = V_k
   (b) Advect the shape with V_k (solving the Hamilton Jacobi equation) to obtain a new shape ψ_{k+1}.

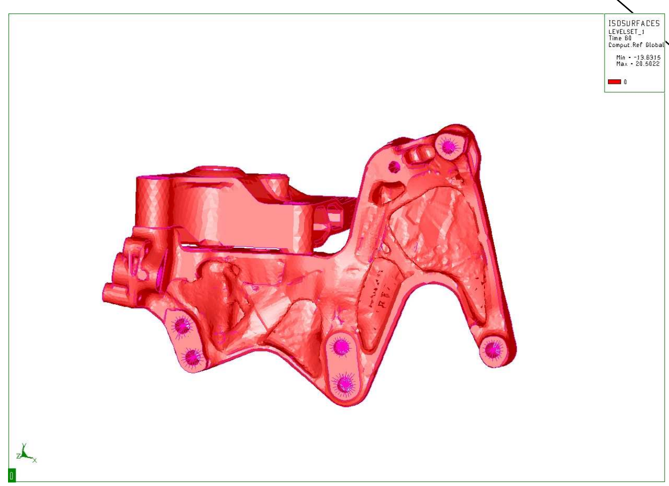
For numerical examples, see the web page:

http://www.cmap.polytechnique.fr/~optopo/level_en.html

Examples of results with complex topologies







-II- THICKNESS CONSTRAINTS

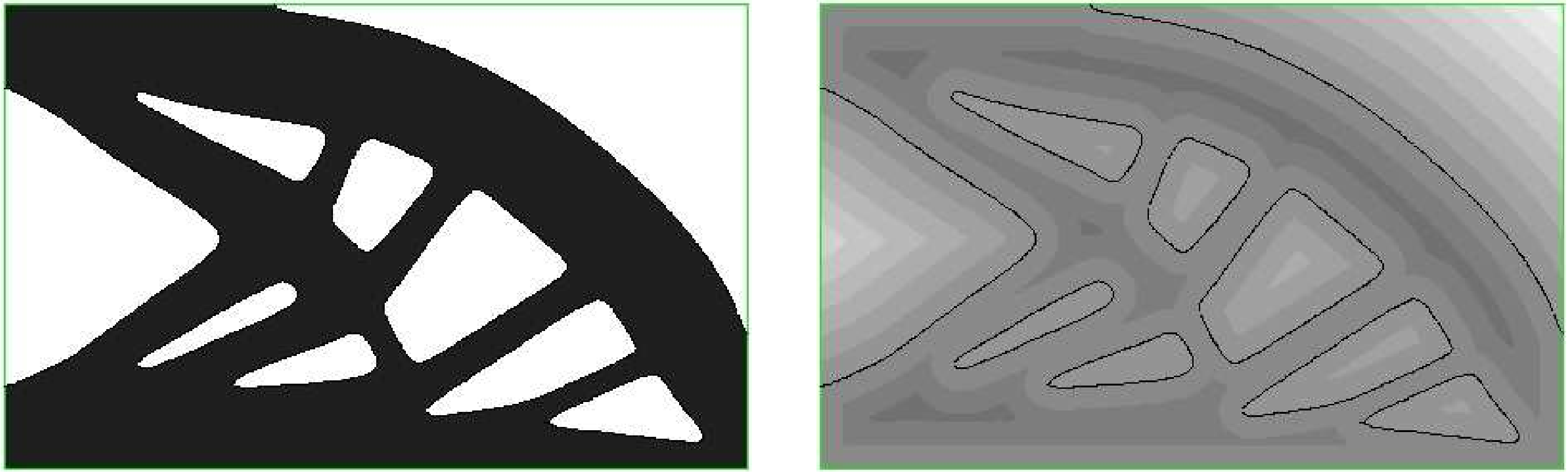
We (Allaire-Jouve-Michailidis) focus on thickness control because of

- manufacturability,
- uncertainty in the microscale (MEMS design),
- robust design (fatigue, buckling, etc.).

Previous works:

- Several approaches in the framework of the **SIMP** method to ensure minimum length scale (Sigmund, Poulsen, Guest, etc.).
- In the **level-set** framework: Chen, Wang and Liu implictly control the feature size by adding a "line" energy term to the objective function ; Alexandrov and Santosa kept a fixed topology by using offset sets.
- Many works in **image processing**.

Signed-distance function



Definition. Let $\Omega \subset \mathbb{R}^d$ be a bounded domain. The **signed distance function** to Ω is the function $\mathbb{R}^d \ni x \mapsto d_\Omega(x)$ defined by :

$$d_\Omega(x) = \begin{cases} -d(x, \partial\Omega) & \text{if } x \in \Omega \\ 0 & \text{if } x \in \partial\Omega \\ d(x, \partial\Omega) & \text{if } x \in \mathbb{R}^d \setminus \Omega \end{cases}$$

where $d(\cdot, \partial\Omega)$ is the usual Euclidean distance.

Constraint formulations

Maximum thickness.

Let $d_{\max}$ be the maximum allowed thickness. The constraint reads:

$$d_\Omega(x) \geq -d_{\max}/2 \quad \forall x \in \Omega$$

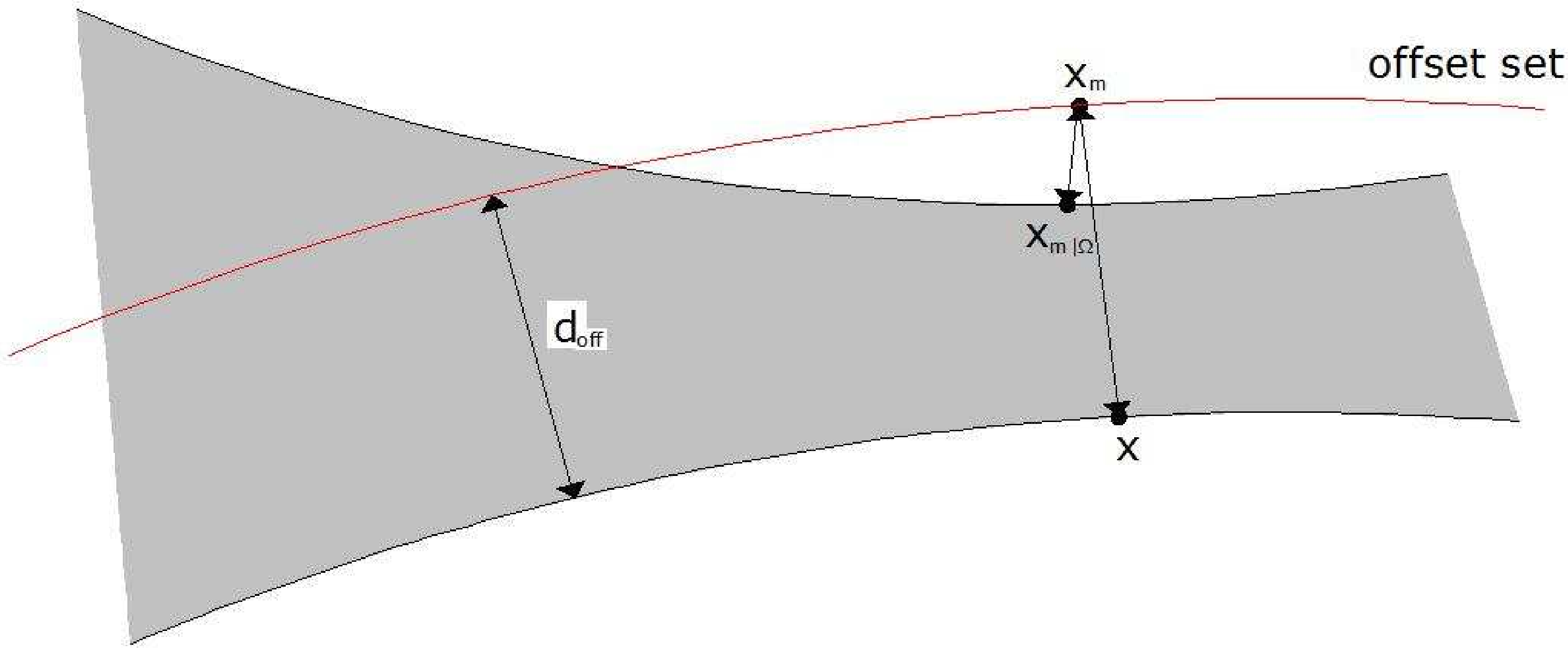
Minimum thickness

Let $d_{\min}$ be the minimum allowed thickness. The constraint reads:

$$d_\Omega\left(x - d_{\text{off}} n(x)\right) \leq 0 \quad \forall x \in \partial\Omega,\ \forall d_{\text{off}} \in [0, d_{\min}]$$

Remark: similar constraints for the thickness of holes.

Offset sets



For minimum thicknes we rely on the classical notion of **offset sets** of the boundary of a shape, defined by

$$\{x - d_{\text{off}}\, n(x) \quad \text{such that } x \in \partial\Omega\}$$

Quadratic penalty method

We reformulate the pointwise constraint into a global one denoted by $P(\Omega)$.

Maximum thickness

$$P(\Omega) = \int_{\Omega} \left[\left(d_{\Omega}(x) + d_{\max}/2 \right)^{-} \right]^2 dx$$

Minimum thickness

$$P(\Omega) = \int_{\partial\Omega} \int_{0}^{d_{\min}} \left[\left(d_{\Omega}\left(x - d_{\text{off}}\, n(x) \right) \right)^{+} \right]^2 dx\, dd_{\text{off}}$$

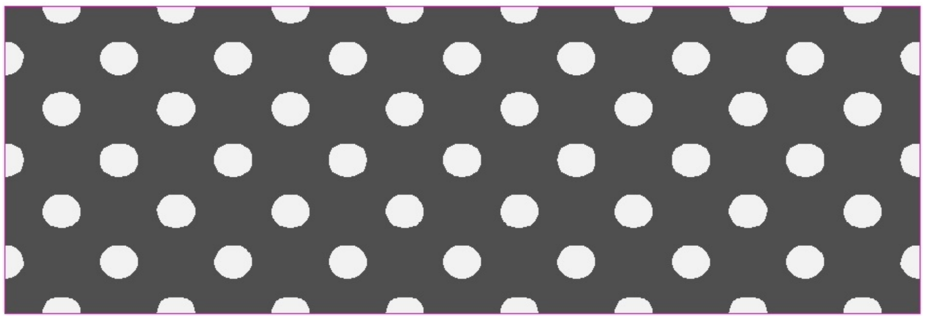
where $f^{+} = \max(f, 0)$ and $f^{-} = \min(f, 0)$.

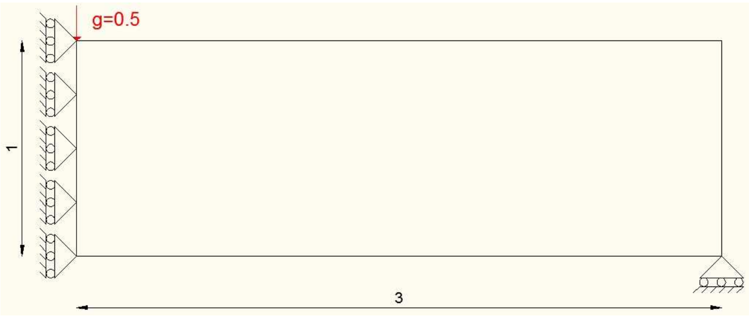
Then, we compute shape derivatives of the constraints.

NUMERICAL RESULTS

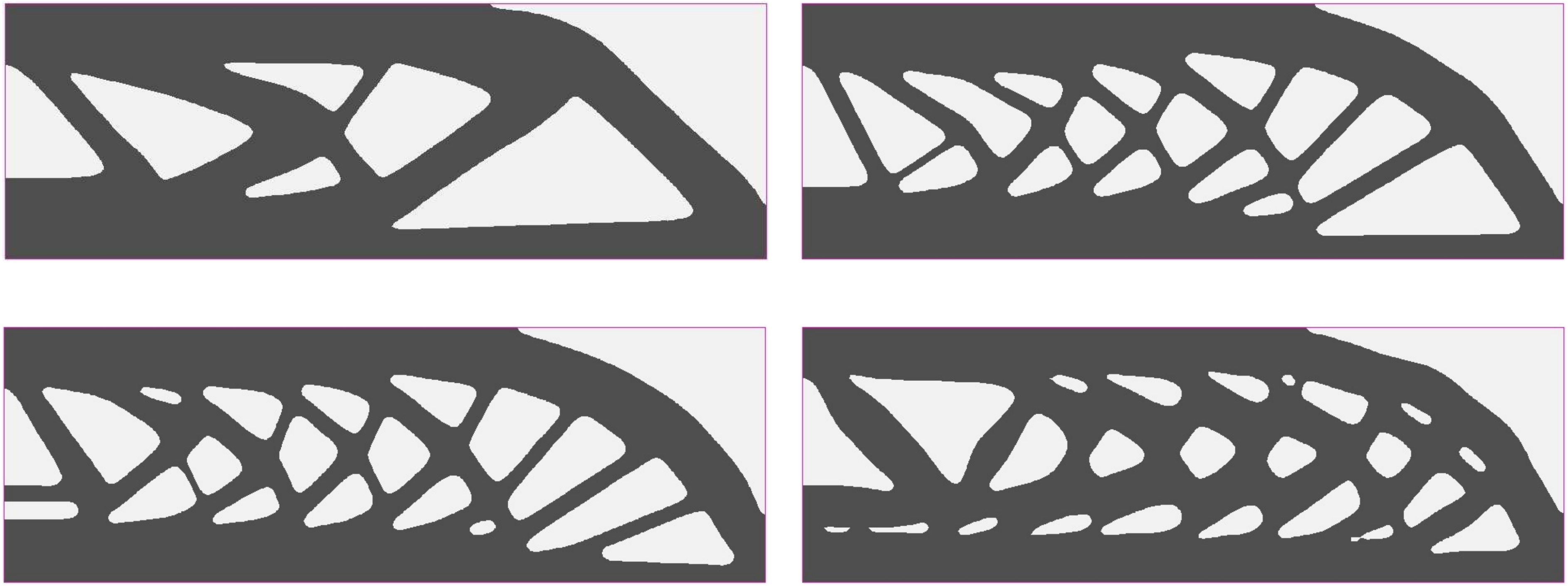
- ☞ All the geometrical computations (skeleton, offset, projection, etc.) are standard and very cheap (compared to the elasticity analysis).
- ☞ All our numerical examples are for compliance minimization (except otherwise mentioned).
- ☞ At convergence, the geometrical constraints are exactly satisfied.
- ☞ All results have bee obtained with our software developped in the finite element code SYSTUS of ESI group.

Maximum thickness (MBB, solution without constraint)

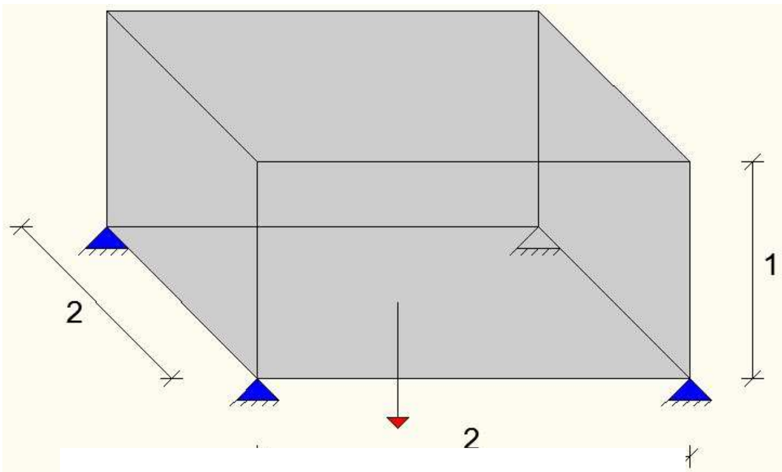


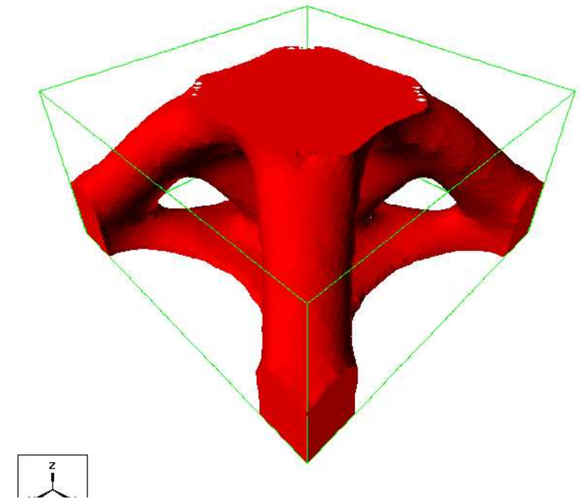


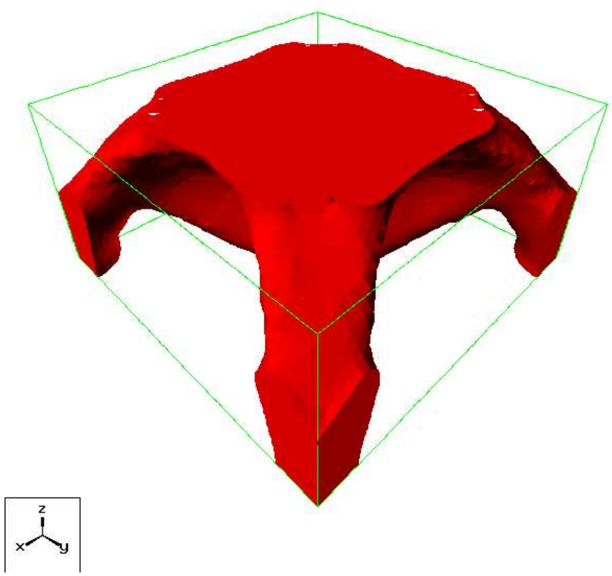
Maximum thickness (solution with increasing constraint)

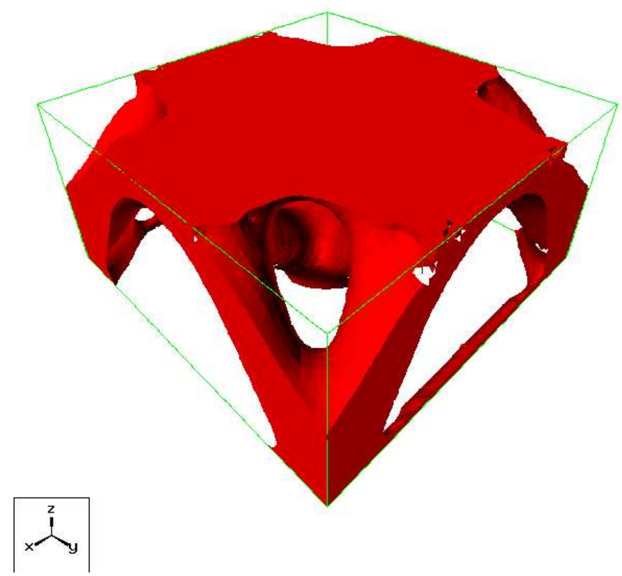


Maximum thickness (3d Box)

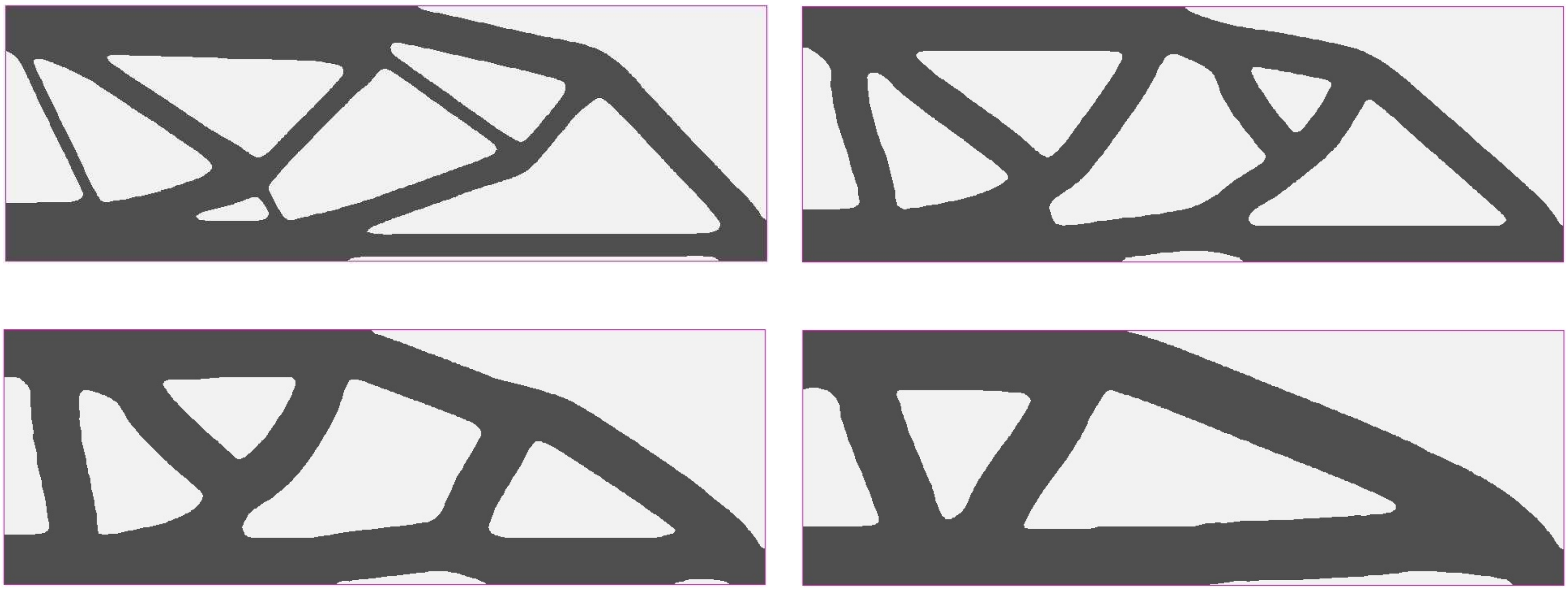




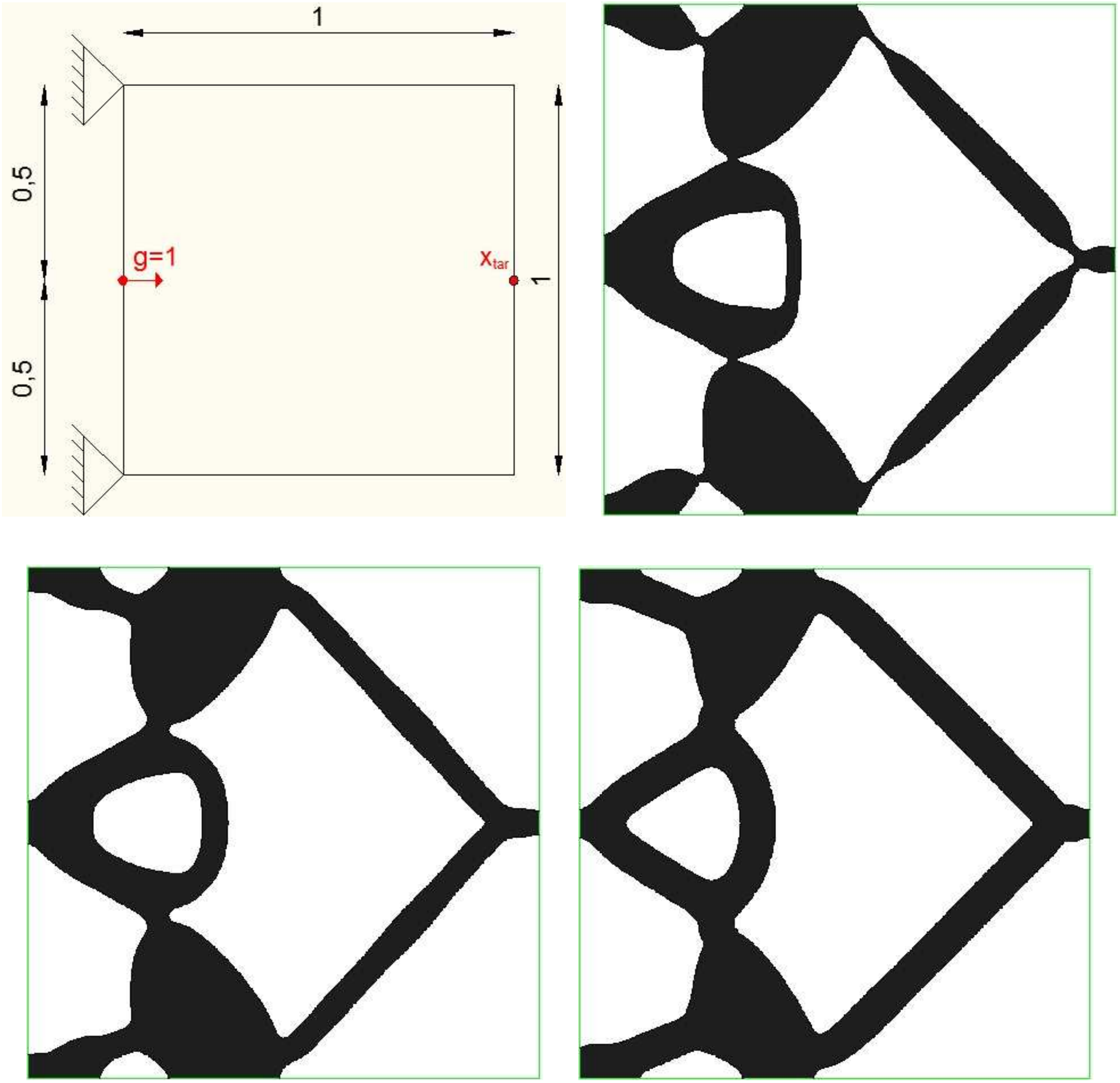




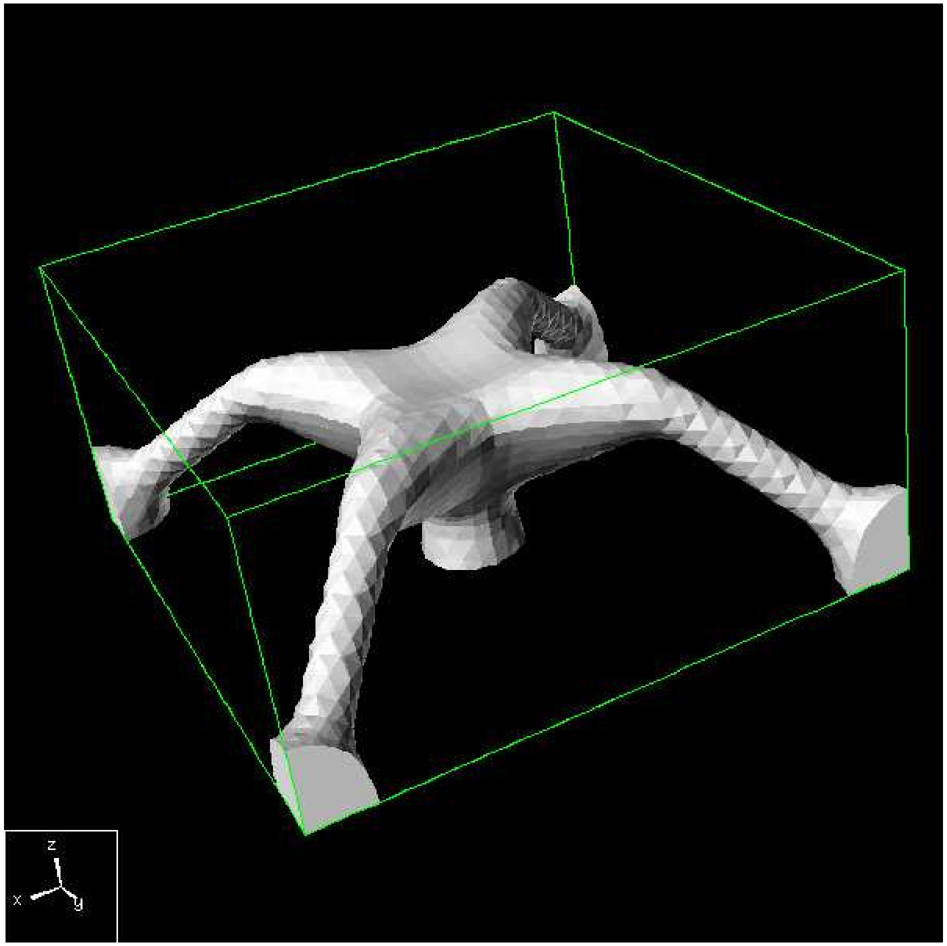
Minimum thickness (MBB beam)

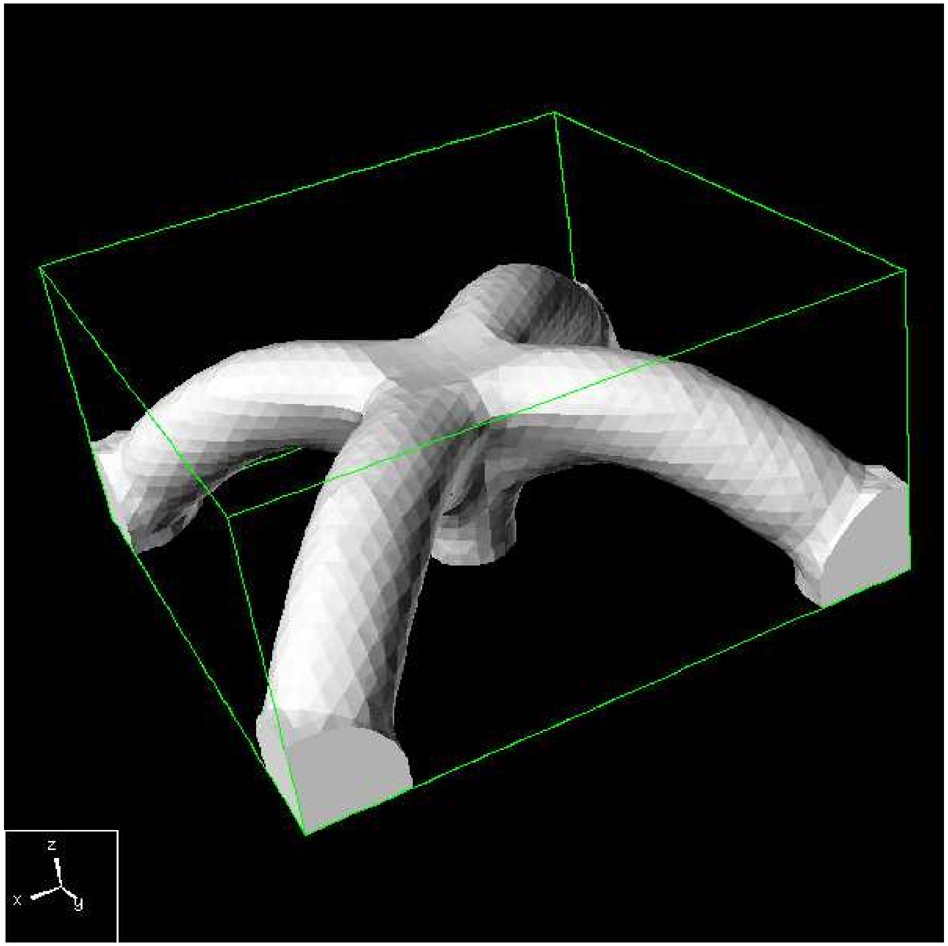


Minimum thickness (force inverter)



Minimum thickness (3d)





-III- UNCERTAINTIES AND WORST-CASE DESIGN

Uncertainties on:

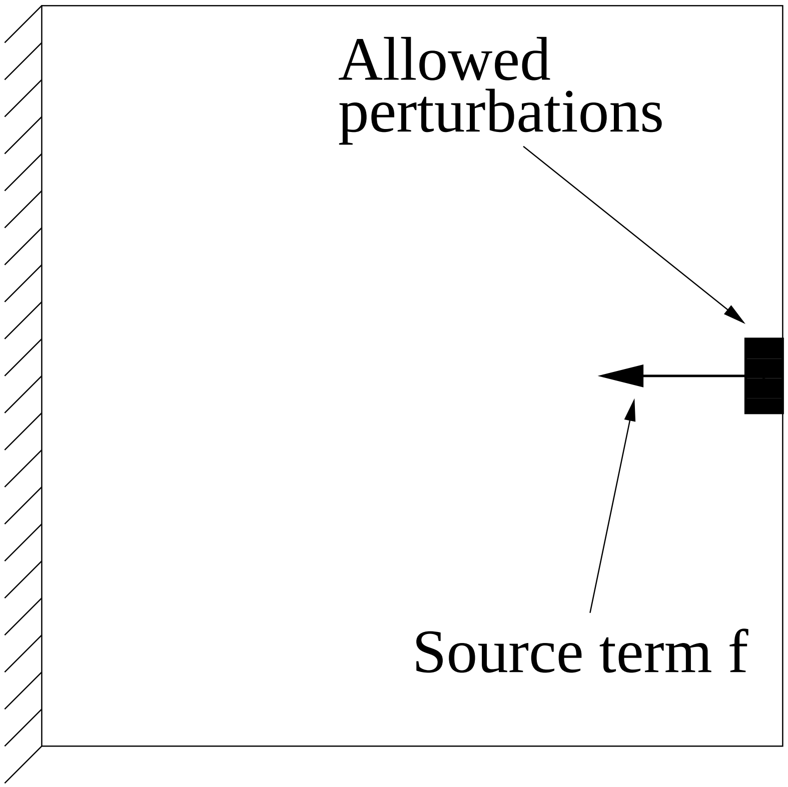
☞ location, magnitude and orientation of the body forces or surface loads

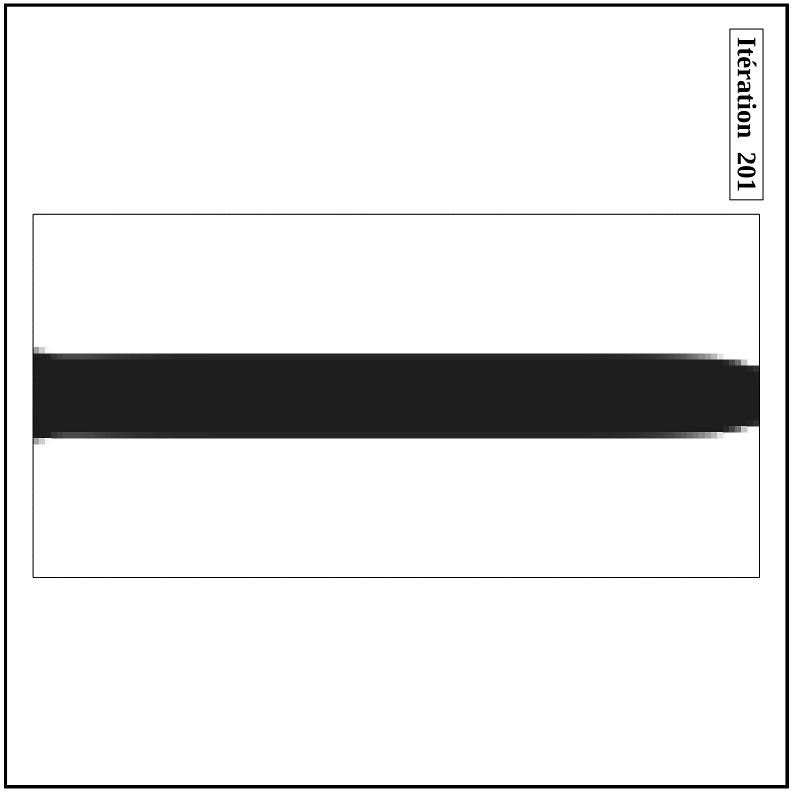
☞ elastic material's properties

☞ geometry of the shape

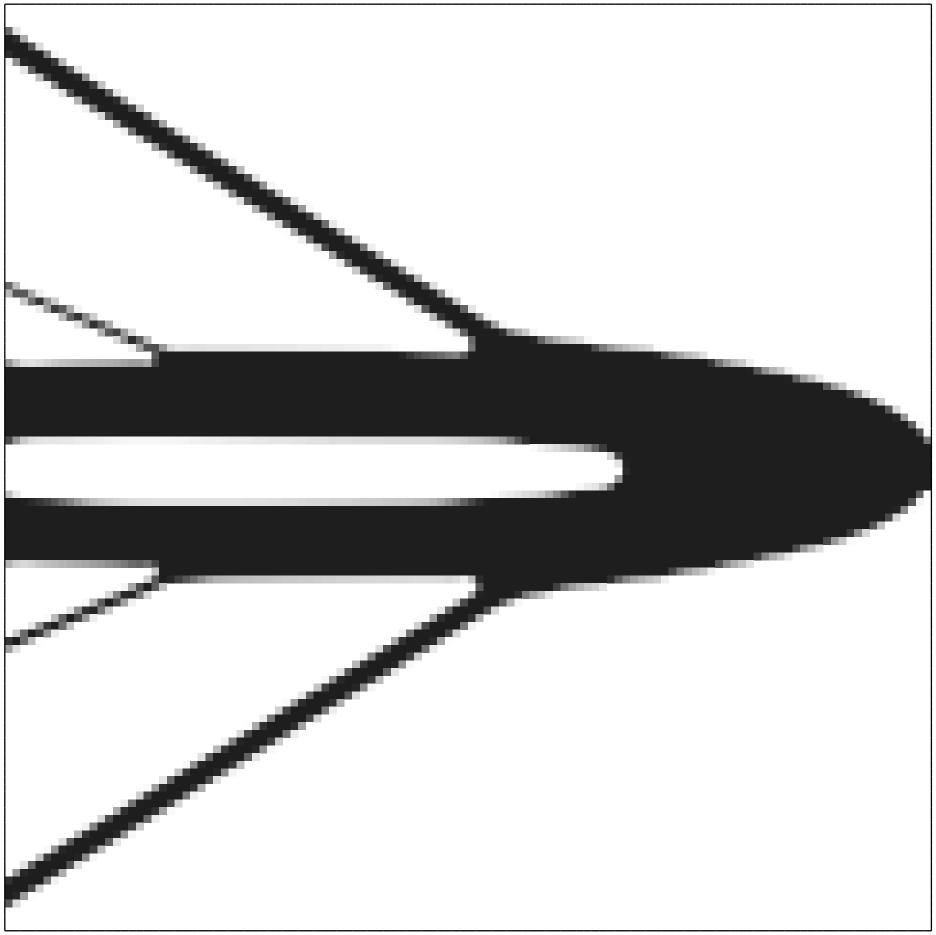
Crucial issue: optimal structures are so optimal for a given set of loads that they cannot sustain a different load !

Example: minimal weight and minimal compliance





Optimal design with load uncertainties



State of the art

☞ Probabilistic approach (Choi et al. 2007, Frangopol-Maute 2003, Kalsi et al. 2001...)

- Monte-Carlo methods
- Polynomial chaos, Karhunen-Loève expansions...
- First-Order Reliability-based Methods (FORM)

☞ Worst case approach

- Robust compliance: Cherkaev-Cherkaeva (1999, 2003), de Gournay-Allaire-Jouve (2008).
- Present work (Allaire-Dapogny).

Worst case design

Example in the case of force uncertainties.

The force is the sum $f + \delta$ where f is known and δ is unknown.

The only information is the location of δ and its maximal magnitude $m > 0$ such that $\|\delta\| \le m$.

We replace the standard objective function $J(\Omega, f + \delta)$ by its worst case version $\mathcal{J}(\Omega, f)$.

Worst case design optimization problem:

$$\min_{\Omega} \mathcal{J}(\Omega, f) = \min_{\Omega} \max_{\|\delta\| \le m} J(\Omega, f + \delta)$$

ABSTRACT (AND FORMAL) SETTING

- Designs $h \in \mathcal{H}$, perturbations $\delta \in \mathcal{P}$
- State equation $\mathcal{A}(h)u(h) = b$
- Perturbed state equation $\mathcal{A}(h)u(h,\delta) = b(\delta)$
- Worst case objective function

$$\inf_{h \in \mathcal{H}} \left\{ \mathcal{J}(h) = \sup_{\substack{\delta \in \mathcal{P} \\ ||\delta||_{\mathcal{P}} \leq m}} J(u(h,\delta)) \right\}$$

- Assume that the perturbations are small, i.e., $m << 1$, and linearize

$$\mathcal{J}(h) \approx \widetilde{\mathcal{J}}(h) = \sup_{\substack{\delta \in \mathcal{P} \\ ||\delta||_{\mathcal{P}} \leq m}} \left(J(u(h)) + \frac{dJ}{du}(u(h)) \frac{\partial u}{\partial \delta}(h,0)(\delta) \right)$$

- Introduce an adjoint, $\mathcal{A}(h)^T p(h) = \frac{dJ}{du}(u(h))$,

$$\widetilde{\mathcal{J}}(h) = J(u(h)) + m \left|\left| \frac{db}{d\delta}(0) \cdot p(h) \right|\right|_{\mathcal{P}^*}$$

First case: loading uncertainties.

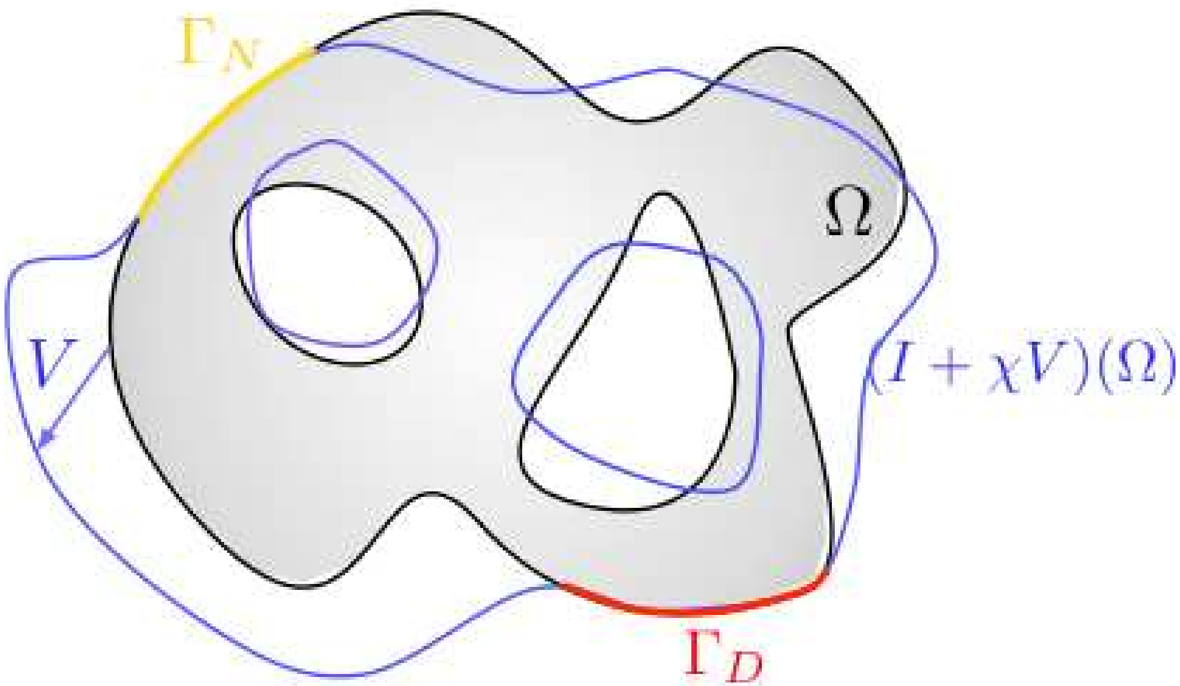
Given load $f \in L^2(\mathbb{R}^d)^d$. Unknown load $\delta \in L^2(\mathbb{R}^d)^d$ with small norm $\|\delta\|_{L^2(\mathbb{R}^d)^d} \leq m$. Solution $u_{\Omega,\delta}$ of

$$\begin{cases} -\operatorname{div}\left(A\,e(u_{\Omega,\delta})\right) = f + \delta & \text{in } \Omega \\ u_{\Omega,\delta} = 0 & \text{on } \Gamma_D \\ \big(A\,e(u_{\Omega,\delta})\big)n = g & \text{on } \Gamma_N \\ \big(A\,e(u_{\Omega,\delta})\big)n = 0 & \text{on } \Gamma \end{cases}$$

Many variants are possible (δ may be localized, or parallel to a fixed vector, or restricted to Γ_N, etc.)

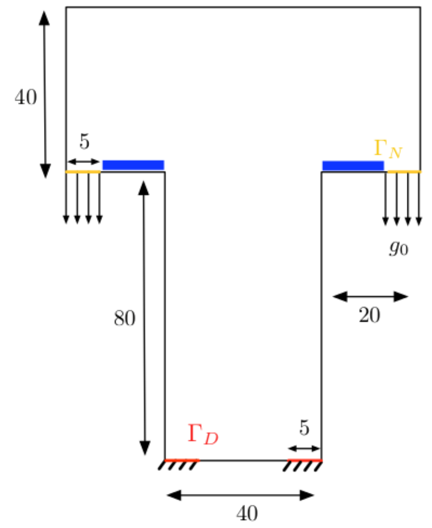
Second case: geometric uncertainties

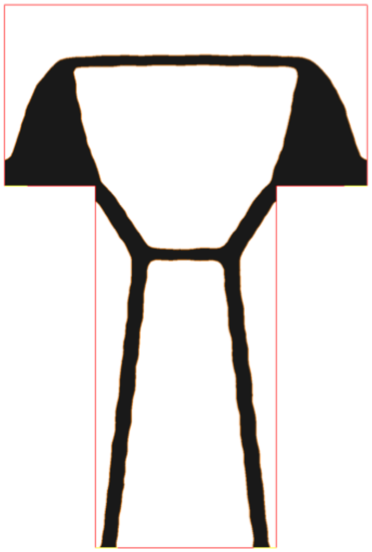
Perturbed shapes $(I + \chi V)(\Omega), \quad V \in W^{1,\infty}(\mathbb{R}^d, \mathbb{R}^d), \ ||V||_{L^\infty(\mathbb{R}^d)^d} \leq m.$

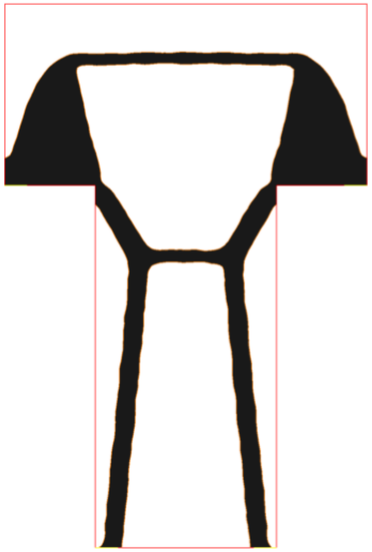


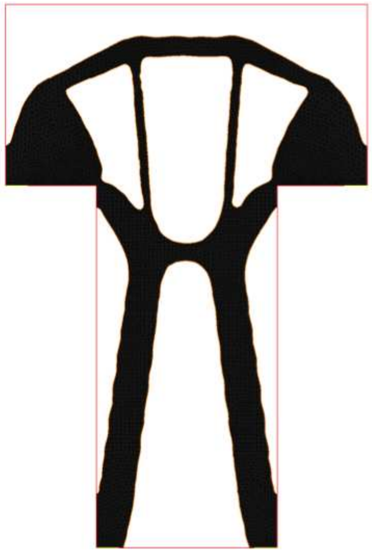
χ is a smooth localizing function such that $\chi \equiv 0$ on $\Gamma_D \cup \Gamma_N$.

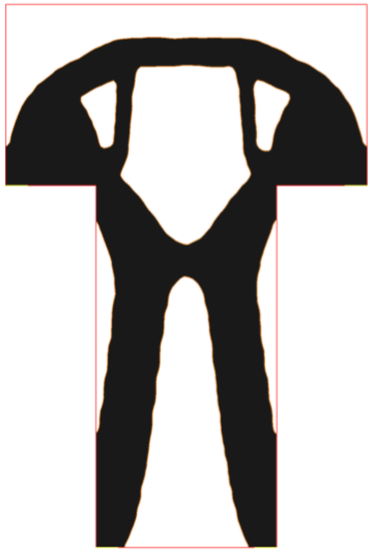
Load uncertainties in geometric optimization (compliance)



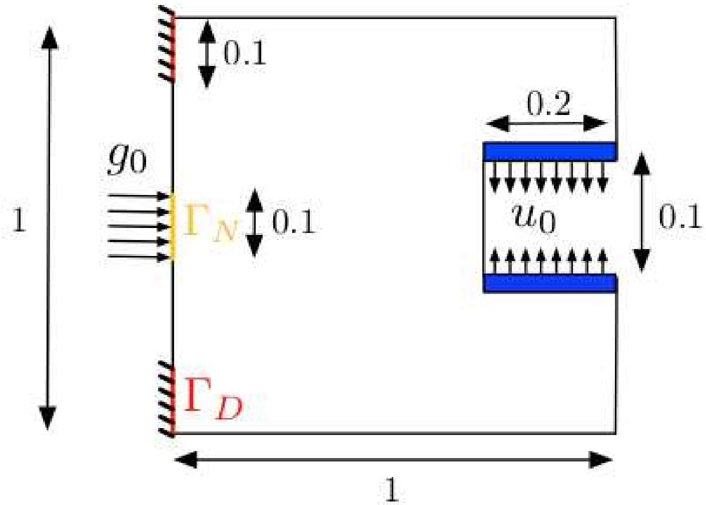


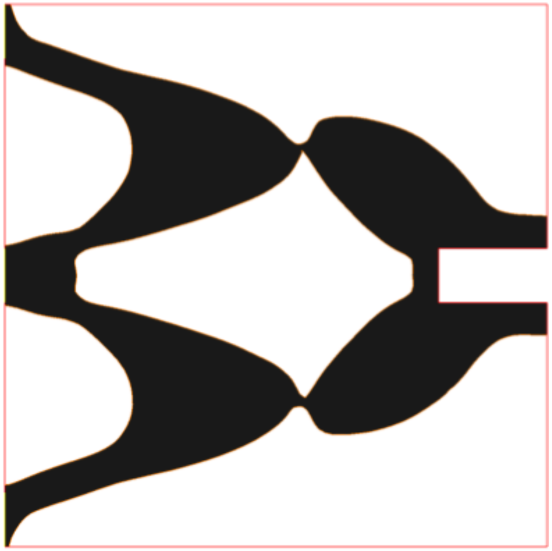


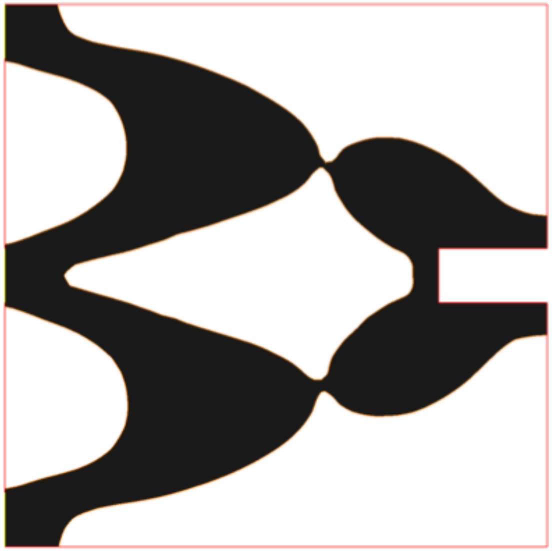


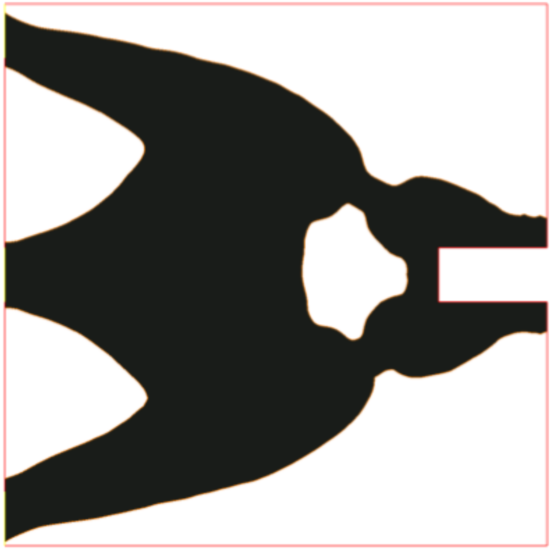


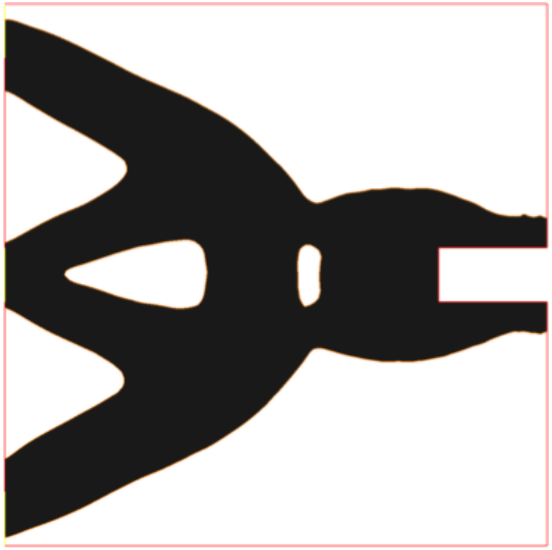
Geometric uncertainties in geometric optimization



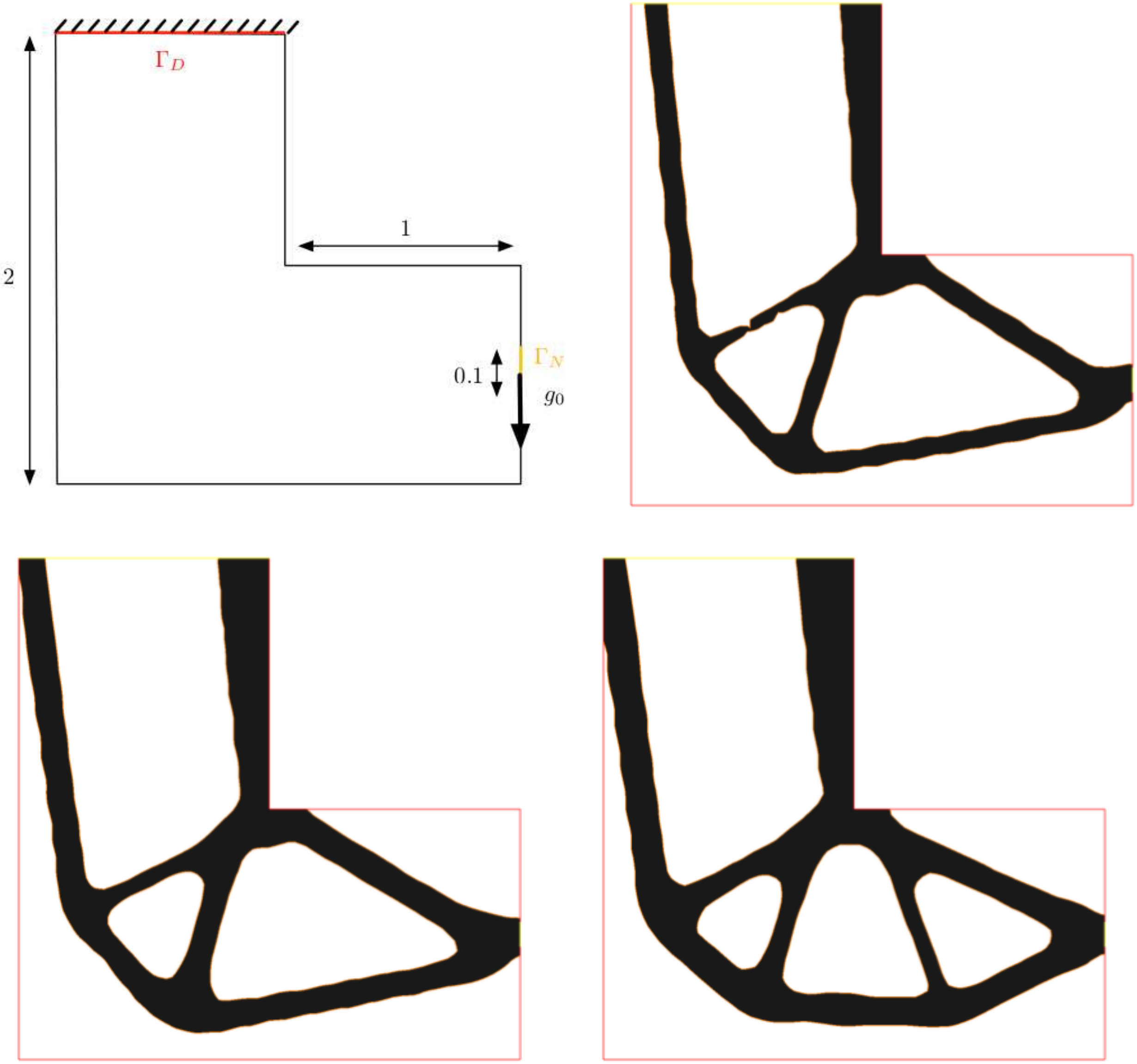






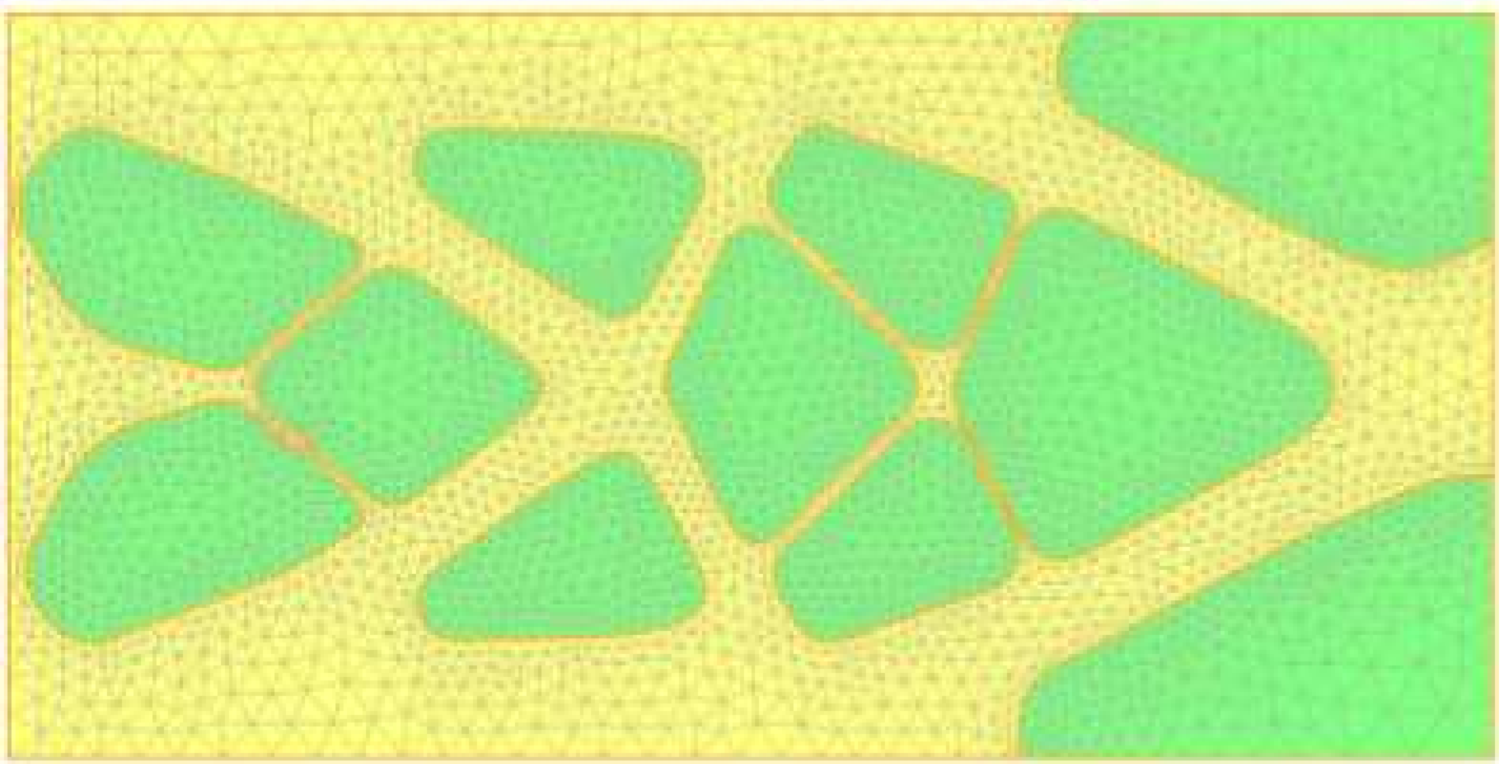


Geometric uncertainties (stress minimization)



-IV- A MESH EVOLUTION METHOD

Main idea: rather than using a fixed (regular) mesh and **capturing** the shape with a level set method, use a moving (simplicial) mesh, **tracking** the shape.



Principle of the method (with C. Dapogny and P. Frey)

☞ The shape is **exactly** meshed at each optimization iteration.

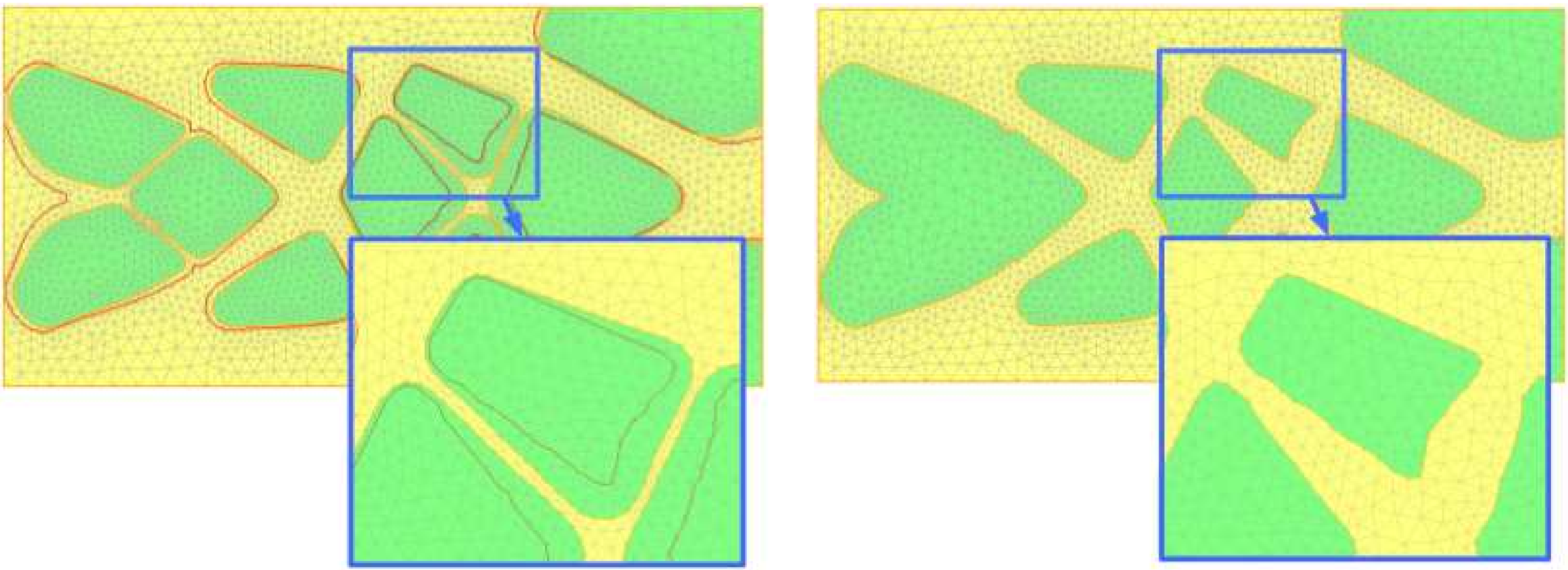
☞ Only the interior mesh is used for the elasticity analysis: **no erstaz material in the holes**.

☞ Use the full mesh (interior and exterior) to advect the shape's boundary, again using the **level set algorithm**.

Two key ingredients:

1. Advect a level set function on a simplicial mesh: characteristic algorithm for a linearization of the Hamilton-Jacobi equation (J. Strain, JCP 1999).
2. Build a new simplicial mesh which contains the zero level set in its faces (or edges in 2-d).

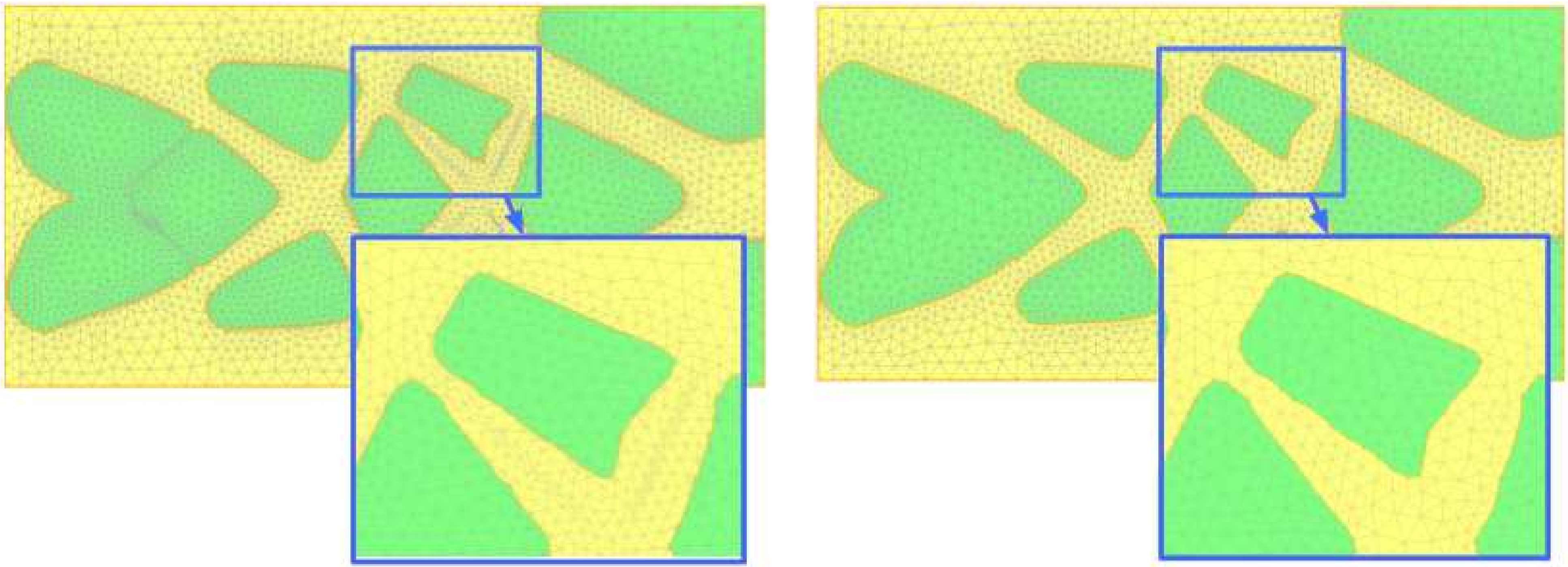
Principle of the method



Before remeshing (left), after remeshing (right).

Yellow = interior mesh, green = exterior mesh, red line = zero level set.

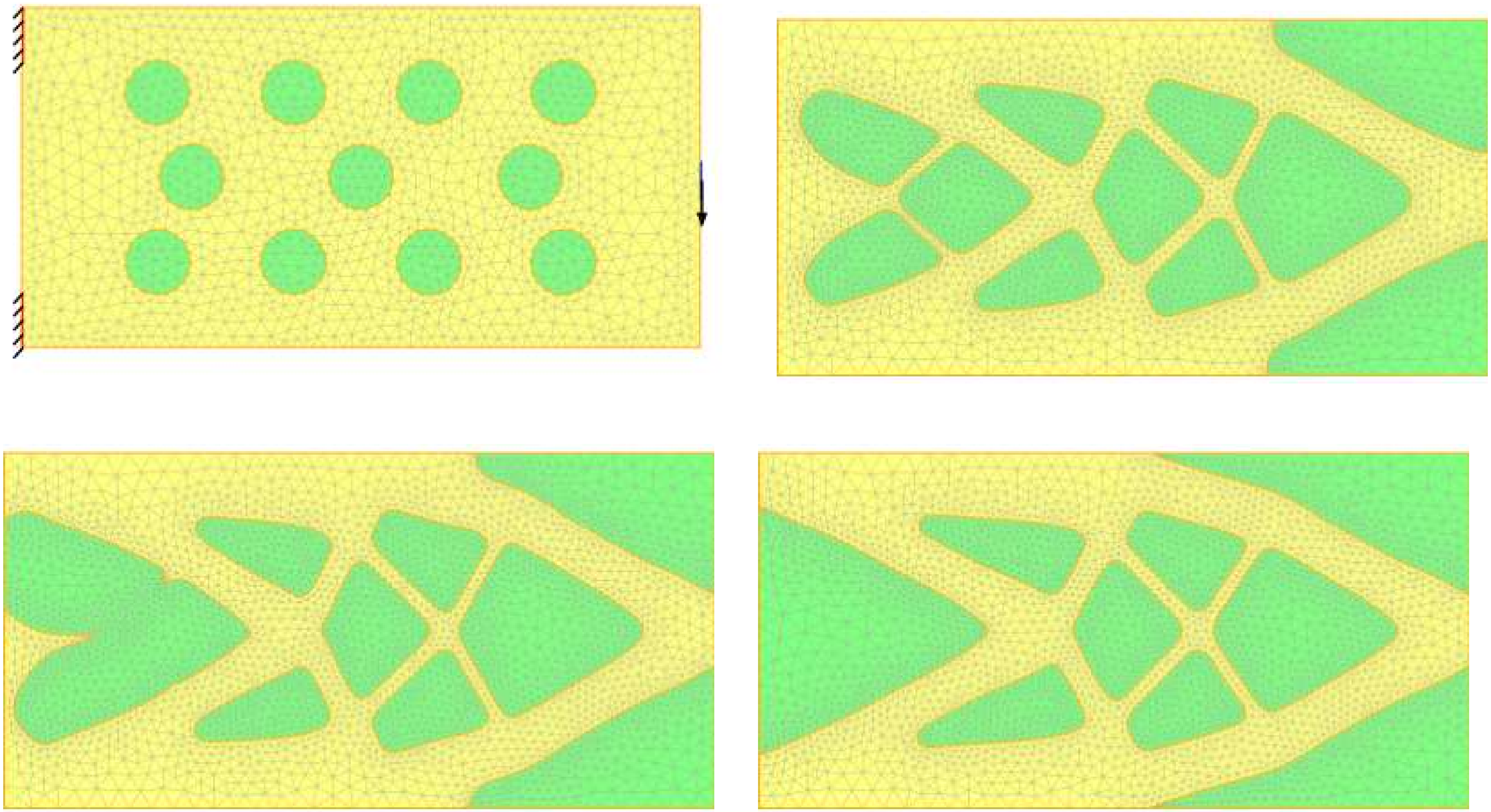
Some technical details about remeshing



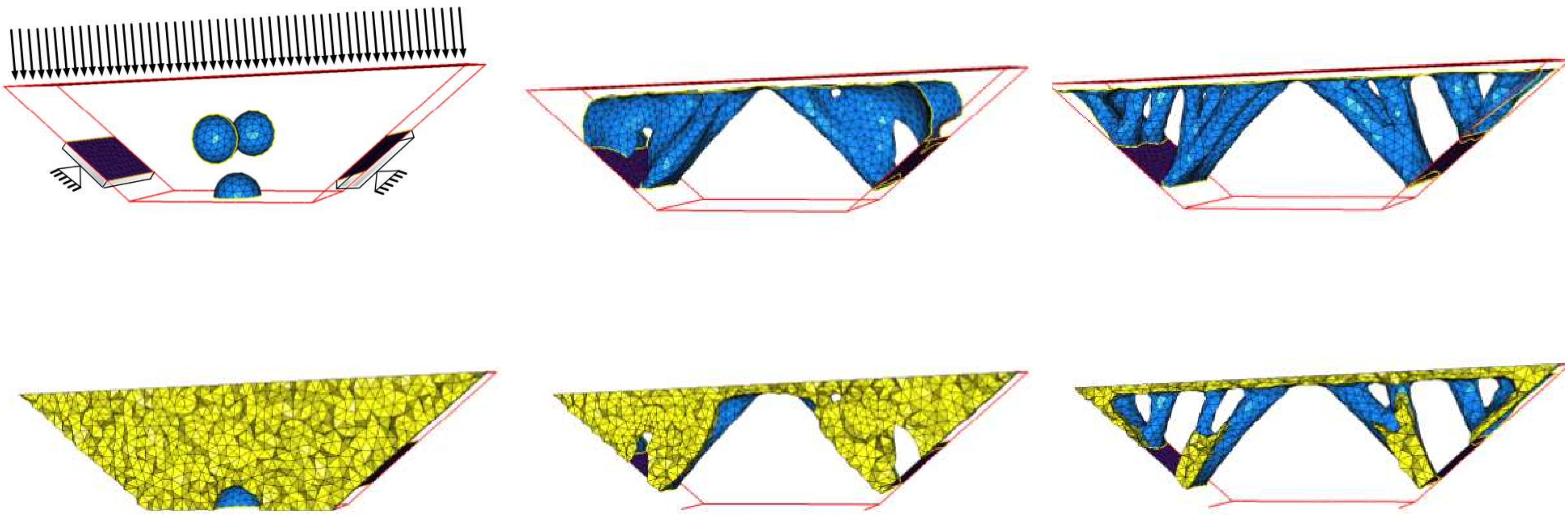
Left: bad mesh incorporating the zero level set (easy part).

Right: nice mesh after local smoothing operations, split, swap, collapse of edges, vertex relocation (hard part).

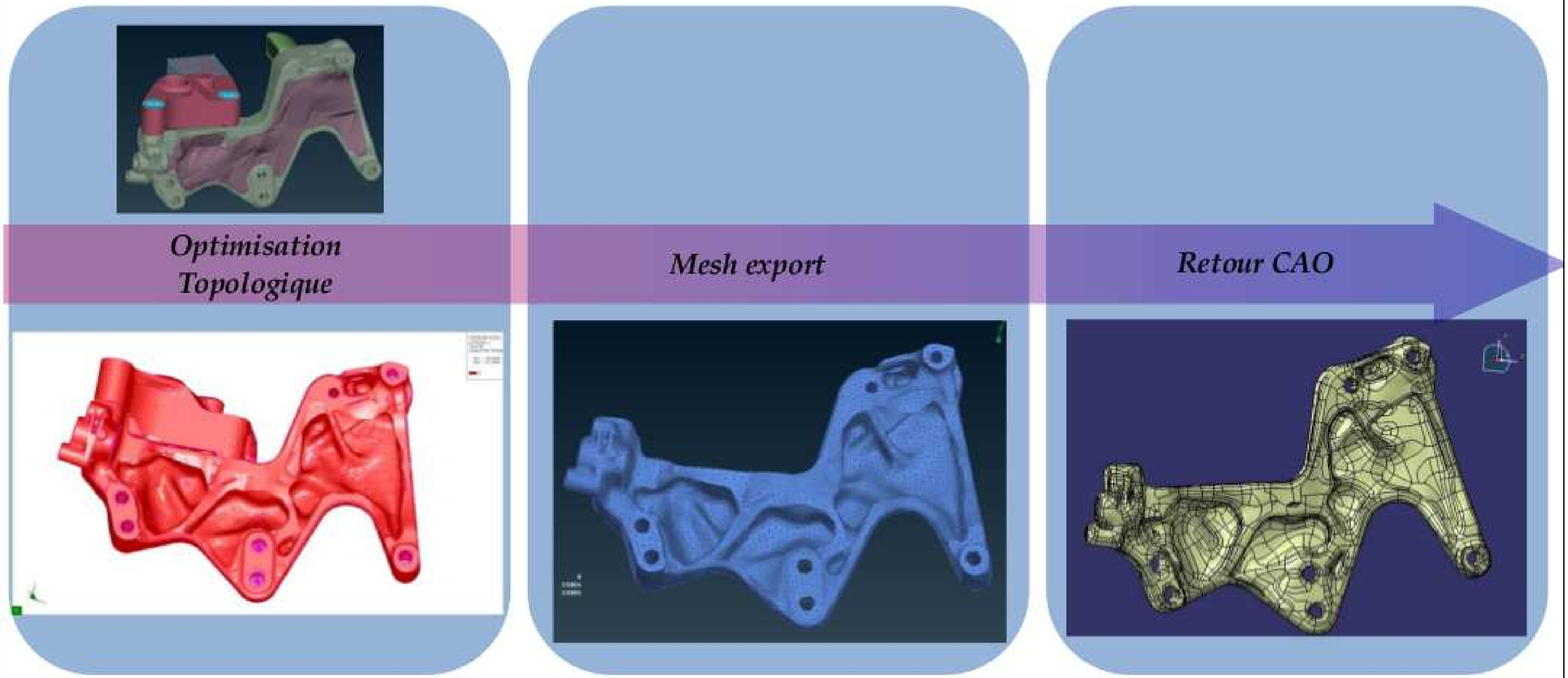
Minimal compliance cantilever



Minimal compliance bridge



Quelques résultats



Confidentiel Renault

Conclusion

Three issues addressed in this talk:

1. Thickness constraints.

2. Uncertainties and linearized worst-case design.

3. A level set based mesh evolution method.

Other studies in the RODIN project:

☞ Second-order optimization algorithms (Jean-Léopold Vié).

☞ Contact and plasticity (Aymeric Maury).

☞ Composite panel optimization (Gabriel Delgado).

☞ Molding and casting constraints.

☞ Average and variance of optimal designs under random uncertainties.

☞ Export to CAD environment.

☞ Converting input and output files for other mechanical softwares.