Some references

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The classical electromagnetic field is described by the four vectors $\mathcal{E}$, $\mathcal{D}$, $\mathcal{H}$, and $\mathcal{B}$ which are functions of the position $\mathbf{x} \in \mathbb{R}^3$ and of the time $t \in \mathbb{R}$. The vectors $\mathcal{E}$ and $\mathcal{H}$ are referred to as the electric and magnetic field, while $\mathcal{D}$ and $\mathcal{B}$ are the electric and magnetic displacements, respectively.
The Faraday law of induction states

$$\int_{\partial\Sigma} \mathcal{E} \cdot ds = -\frac{d}{dt} \int_{\Sigma} \mathcal{B} \cdot \mathbf{n}$$

The Ampère law says

$$\int_{\partial\Sigma} \mathcal{H} \cdot ds = \frac{d}{dt} \int_{\Sigma} \mathcal{D} \cdot \mathbf{n} + \int_{\Sigma} \mathcal{J} \cdot \mathbf{n}$$

where $\mathcal{J}$ denotes the current density vector.

It can be noticed that the fields $\mathcal{E}$, $\mathcal{H}$, $\mathcal{B}$, and $\mathcal{D}$ have a different nature. Indeed, the first two are integral 1-forms, while the latter two are integral 2-forms.
This remark is of fundamental importance for the design of finite element schemes.

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The differential forms of Faraday and Ampère laws read

$$\frac{\partial \mathcal{B}}{\partial t} + \mathbf{curl}\,\mathcal{E} = 0$$
$$\frac{\partial \mathcal{D}}{\partial t} - \mathbf{curl}\,\mathcal{H} = -\mathcal{J}$$

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which are usually referred to as Maxwell's equations, together with the two Gauss Laws

$$\mathrm{div}\, \mathcal{D} = \rho$$
$$\mathrm{div}\, \mathcal{B} = 0$$

where ρ denotes the charge density function.

The time-harmonic Maxwell system is considered, for instance, when the Fourier transform in time is used or when the propagation of electromagnetic waves at a given frequency is studied. Then, given a frequency ω, we consider the *ansatz*:

$$\mathcal{E}(\mathbf{x},t) = \Re\left(e^{-i\omega t}\mathbf{E}(\mathbf{x})\right)$$
$$\mathcal{D}(\mathbf{x},t) = \Re\left(e^{-i\omega t}\mathbf{D}(\mathbf{x})\right)$$
$$\mathcal{H}(\mathbf{x},t) = \Re\left(e^{-i\omega t}\mathbf{H}(\mathbf{x})\right)$$
$$\mathcal{B}(\mathbf{x},t) = \Re\left(e^{-i\omega t}\mathbf{B}(\mathbf{x})\right)$$

where $\Re$ denotes the *real part*. We define also

$$\mathcal{J}(\mathbf{x},t) = \Re\left(e^{-i\omega t}\mathbf{J}(\mathbf{x})\right)$$
$$\rho(\mathbf{x},t) = \Re\left(e^{-i\omega t}r(\mathbf{x})\right)$$

Standard constitutive equations for linear media read

$$\mathbf{D} = \varepsilon \mathbf{E} \qquad \mathbf{B} = \mu \mathbf{H}$$

where ε and μ denote the electric permittivity and the magnetic permeability, respectively. For general inhomogeneous, anisotropic materials ε and μ are 3×3 positive definite matrix functions.

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where ε and μ denote the electric permittivity and the magnetic permeability, respectively. For general inhomogeneous, anisotropic materials ε and μ are 3×3 positive definite matrix functions. Inserting constitutive relations into the Maxwell's equations and considering the time-harmonic assumptions, we get the time harmonic Maxwell's equations

$$\begin{aligned} &\mathbf{curl}\, \mathbf{E} - i\omega\mu\mathbf{H} = \mathbf{0} \\ &\mathrm{div}(\varepsilon\mathbf{E}) = r \\ &\mathbf{curl}\, \mathbf{H} + i\omega\varepsilon\mathbf{E} = \mathbf{J} \\ &\mathrm{div}(\mu\mathbf{H}) = 0 \end{aligned}$$

It is a standard procedure to eliminate one variable and to write the time harmonic Maxwell's system as a second order system. Eliminating for instance the field $\mathbf{H}$, we get

$$\mathbf{curl}(\mu^{-1}\,\mathbf{curl}\,\mathbf{E}) - \omega^2\varepsilon\mathbf{E} = \mathbf{F}$$

where $\mathbf{F}$ is given by $i\omega\mathbf{J}$, together with the divergence condition (which follows from the equation)

$$-\omega^2\,\mathrm{div}(\varepsilon\mathbf{E}) = \mathrm{div}\,\mathbf{F}.$$

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The equation is usually equipped with suitable boundary conditions. The simplest one is the perfect conducting boundary condition which reads

$$\mathbf{E} \times \mathbf{n} = 0$$

where $\mathbf{n}$ is the outward unit vector.