Supporting Information

Table of Contents

A Mathematical derivation

In this section, we provide the mathematical derivation of our results. In Section A.1 we treat the case of one habitat. In Section A.2 we provide our derivations for the case of two habitats. Finally in Section A.3 we show how our method can be used to study model (2).

A.1 One population

A.1.1 Derivation of our *first approximation* in absence of migration

Our *first approximation* is based on the computation of the terms $u_0(z)$ and $v_0(z)$. Based on such computations we can provide an approximation of the population's total density $N^*_{\varepsilon,0}$ and the phenotypic density $n^*_{\varepsilon,0}(z)$ in the following form

$$N^*_{\varepsilon,0} \approx N_0 + \varepsilon K_0, \qquad n^*_{\varepsilon,0}(z) \approx \frac{1}{\sqrt{2\pi\varepsilon}} \exp\Big(\frac{u_0(z) + \varepsilon v_0(z)}{\varepsilon}\Big). \tag{A.1}$$

Indeed we neglect the error term in (9) since when ε is small, in view of (7), it has only small contribution to the phenotypic density $n^*_{\varepsilon,0}(z)$.

We will prove in what follows that $N_0 = N^*_0$, with N^*_0 the total population size at the demographic equilibrium of the ESS z_0. We will also compute the other terms of the expansions K_0, $u_0(z)$ and $v_0(z)$.

Using (5) and $\varepsilon = \sqrt{V_m}$, the equilibrium $n^*_{\varepsilon,0}(z)$ solves:

$$0 = U\varepsilon^2 \frac{\partial^2 n^*_{\varepsilon,0}(z)}{\partial z^2} + n^*_{\varepsilon,0}(z)\left(r_0(z) - \kappa_0 N^*_{\varepsilon,0}\right). \tag{A.2}$$

Replacing (7) in the above equation we obtain:

$$0 = U\varepsilon \frac{\partial^2 u_{\varepsilon,0}(z)}{\partial z^2} + U|\frac{\partial}{\partial z} u_{\varepsilon,0}(z)|^2 + r_0(z) - \kappa_0 N^*_{\varepsilon,0}. \tag{A.3}$$

This equation is derived using the following equalities:

$$\frac{\partial}{\partial z} n^*_{\varepsilon,0}(z) = \Big(\frac{\partial}{\partial z} u_{\varepsilon,0}(z)\Big)\frac{n^*_{\varepsilon,0}(z)}{\varepsilon}, \qquad \frac{\partial^2}{\partial z^2} n^*_{\varepsilon,0}(z) = \Big(\varepsilon\frac{\partial^2}{\partial z^2} u_{\varepsilon,0}(z) + |\frac{\partial}{\partial z} u_{\varepsilon,0}(z)|^2\Big)\frac{n^*_{\varepsilon,0}(z)}{\varepsilon^2}.$$

We then replace the ansatz (9) in (A.3). We first keep the zero order terms with respect to ε (the ones in front of which there is no ε, corresponding to the dominant terms) to obtain the following equation on $u_0(z)$:

$$0 = U|\partial_z u_0(z)|^2 + r_0(z) - \kappa_0 N_0.$$

Note also that to have a finite but positive size of population, we should have

$$\max_{z \in \mathbb{R}} u_0(z) = 0.$$

Otherwise, in view of (7), the total population size whether becomes infinite as $\varepsilon \to 0$ (if $\max_{z \in \mathbb{R}} u_0(z) > 0$) or it goes to 0 (if $\max_{z \in \mathbb{R}} u_0(z) < 0$).

At the maximum point $z_{\max}$ of u_0, we have $\partial_z u_0(z_{\max}) = 0$ and hence

$$r_0(z_{\max}) - \kappa_0 N_0 = 0.$$

For all other traits z

$$r_0(z) - \kappa_0 N_0 = -U|\partial_z u_0(z)|^2 \leq 0.$$

We deduce that $z_{\max}$ is the maximum point of $r_0(z)$, that is $z_{\max} = z_0$. In other words, u takes its maximum at the ESS point z_0 and the zero order term N_0 in the approximation of the population size is given by

$$N_0 = \frac{r_0(z_0)}{\kappa_0}. \tag{A.4}$$

This corresponds indeed to the total population size N_0^* at the demographic equilibrium of the ESS z_0. We gather our results on u_0 in the following form Perthame and Barles 2008; Barles et al. 2009: u_0 is indeed the unique solution to the following Hamilton-Jacobi equation

$$\begin{cases} 0 = U|\partial_z u_0(z)|^2 + w(z; N_0^*), \\ \max_z u_0(z) = u_0(z_0) = 0, \end{cases} \tag{A.5}$$

where we recall that $w(z; N_0^*) = r_0(z) - \kappa_0 N_0^*$. This equation can be solved explicitly. The solution $u_0(z)$ is given by

$$u_0(z) = -\frac{1}{\sqrt{U}} \Big| \int_{z_0}^{z} \sqrt{-w(y, N_0^*)} dy \Big|. \tag{A.6}$$

The reader can verify that $u_0(z)$, given by the formula above, is smooth and solves (A.5). Note that the absolute values are necessary since the upper limit of the integral z can be smaller or larger than the lower limit z_0.

We then keep the terms of order ε

$$-U \frac{\partial^2}{\partial z^2} u_0(z) = 2U \frac{\partial}{\partial z} v_0(z) \frac{\partial}{\partial z} u_0(z) - \kappa_0 K_0. \tag{A.7}$$

An evaluation of this equation at the point z_0 gives

$$K_0 = \frac{U}{\kappa_0} \frac{\partial^2}{\partial z^2} u_0(z_0). \tag{A.8}$$

The function $v_0(z)$ can also be computed thanks to (A.7), that is by integrating the following quantity

$$\frac{\partial}{\partial z} v_0(z) = \frac{-U \frac{\partial^2}{\partial z^2} u_0(z) + \kappa_0 K_0}{2U \frac{\partial}{\partial z} u_0(z)}. \tag{A.9}$$

Note that to compute $v_0(z)$ we also need to choose the value of $v_0(z_0)$. This value is fixed in a way such that

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\varepsilon}} \exp\Big(\frac{u_0(z) + \varepsilon v_0(z)}{\varepsilon}\Big) dz = N_0^* + \varepsilon K_0. \tag{A.10}$$

Example of quadratic growth rate (10)**.** In this example the ESS, which is indeed the maximum point of $r_0(z)$, is given by $z_0 = \theta_0$. Considering the specific fitness function (10) in (A.4) we first obtain that

$$N_0^* = \frac{r_{\max,0}}{\kappa_0}.$$

4

Using (A.6) we then obtain that

$$u_0(z) = -\frac{1}{\sqrt{U}} \Big| \int_{\theta_0}^z \sqrt{s_0(y-\theta_0)^2} dy \Big| = -\frac{\sqrt{s_0}}{2\sqrt{U}}(z-\theta_0)^2.$$

We also obtain from (A.8) that $K_0 = -\frac{\sqrt{s_0 U}}{\kappa_0}$. Moreover, from (A.9) we obtain that $\frac{\partial}{\partial z}v_0(z) = 0$ which means that $v_0(z)$ is a constant. Combining these informations with (A.10) we obtain

$$N_{\varepsilon,0}^* \approx \frac{1}{\kappa_0}(r_{\max,0}-\varepsilon\sqrt{s_0 U}), \quad n_{\varepsilon,0}^*(z) \approx \frac{1}{\sqrt{2\pi\varepsilon}}\exp\Big(\frac{u_0(z)+\varepsilon v_0(z)}{\varepsilon}\Big) = \frac{N_{\varepsilon,0}^* s_0^{1/4}}{\sqrt{2\pi\varepsilon\sqrt{U}}}\exp\Big(-\frac{\sqrt{s_0}}{2\varepsilon\sqrt{U}}(z-\theta_0)^2\Big).$$

In other words, our *first approximation* of the phenotypic density $n_{\varepsilon,0}^*(z)$ is given by (8) and (11). We note finally that the Gaussian distribution obtained above solves (A.2) and hence it is indeed an exact solution.

Example of non-symmetric growth rate (12). In this example similarly to the previous example the ESS z_0 is given by $z_0 = \theta_0$ and

$$N_0^* = \frac{r_{\max,0}}{\kappa_0}.$$

The expression of $u_0(z)$ is however different, and it is given thanks to (A.6) by

$$u_0(z) = -\frac{1}{\sqrt{U}} \Big| \int_{\theta_0}^z \sqrt{s_0(y-\theta_0)^2(a+(y-\theta_0-b)^2)} dy \Big|. \tag{A.11}$$

From this expression we can then compute K_0 and $v_0(z)$ similarly to above using (A.8) and (A.9).

A.1.2 Derivation of our *second approximation* in absence of migration

In this section, we provide the main idea to obtain explicit formula for the moments of the population's distribution. The computation of explicit formula for the moments of the population's distribution is based on the observation that, when ε is small, the phenotypic density $n_{\varepsilon,0}^*(z)$ is exponentially small far from the ESS point, since $u_0(z)$ takes negative values at those points. Therefore, only the values of $u_0(z)$ and $v_0(z)$ around the ESS point matter. We indeed use the Taylor expansions of $u_0(z)$ and $v_0(z)$ around the ESS point to compute such analytic formula.

We first provide our analytic formula for the moments of the population's distribution. We next show how to compute such approximations.

Analytic formula for the moments of the population's distribution.
In order to provide an explicit approximation of the moments of the population's distribution, we compute the third order approximation of $u_0(z)$ around the ESS z_0:

$$u_0(z) = -\frac{A}{2}(z - z_0)^2 + B(z - z_0)^3 + O(z - z_0)^4, \tag{A.12}$$

and the first order approximation of $v_0(z)$ around z_0:

$$v_0(z) = \log(\sqrt{A}N_0^*) + D(z - z_0) + O(z - z_0)^2. \tag{A.13}$$

Such coefficients can be computed thanks to (A.6) and (A.9). To obtain the zero order term in the expansion for $v_0(z)$ we use the fact that, as the mutation's variance vanishes ($\varepsilon \to 0$), the total population size $N_{\varepsilon,0}^*$ tends to N_0^* which corresponds to the demographic equilibrium at the ESS. The above approximation allows us to estimate the moments of the population's distribution:

$$\begin{cases} \mu_{\varepsilon,0}^* = \frac{1}{N_{\varepsilon,0}^*}\int z n_{\varepsilon,0}^*(z)dz = z_0 + \varepsilon(\frac{3B}{A^2} + \frac{D}{A}) + O(\varepsilon^2), \\ \sigma_{\varepsilon,0}^{*\,2} = \frac{1}{N_{\varepsilon,0}^*}\int (z - \mu_{\varepsilon,0}^*)^2 n_{\varepsilon,0}^*(z)dz = \frac{\varepsilon}{A} + O(\varepsilon^2), \\ \psi_{\varepsilon,0}^* = \frac{1}{N_{\varepsilon,0}^*}\int (z - \mu_{\varepsilon,0}^*)^3 n_{\varepsilon,0}^*(z)dz = \frac{6B}{A^3}\varepsilon^2 + O(\varepsilon^3). \end{cases} \tag{A.14}$$

Derivation of the analytic formula. We next show how to compute such approximations. We can indeed use the expressions in (A.12) and (A.13) to compute for any integer $k \geq 1$,

$$\begin{aligned} \int(z - z_0)^k n_{\varepsilon,0}^*(z)dz &= \frac{\varepsilon^{\frac{k}{2}}\sqrt{A}N_0^*}{\sqrt{2\pi}}\int_{\mathbb{R}}(y^k e^{-\frac{A}{2}y^2}\left(1 + \sqrt{\varepsilon}(By^3 + Dy) + O(\varepsilon)\right)dy \\ &= \varepsilon^{\frac{k}{2}}N_0^*\left(\omega_k(\tfrac{1}{A}) + \sqrt{\varepsilon}\left(B\omega_{k+3}(\tfrac{1}{A}) + D\omega_{k+1}(\tfrac{1}{A})\right)\right) + O(\varepsilon^{\frac{k+2}{2}}), \end{aligned}$$

where $\omega_k(\sigma^2)$ corresponds to the k-th order central moment of a Gaussian distribution with vari-

ance σ^2. Note that to compute the integral terms above we have performed a change of variable $z - z_0 = \sqrt{\varepsilon}y$, therefore each term $z - z_0$ can be considered as of order $\sqrt{\varepsilon}$ in the integrations. Note also that since the term v is multiplied by ε in (9), a first order expansion of v is enough, while a third order expansion of u is required to obtain the above approximation. The above integrations are the main ingredients to obtain the approximations given in (A.14), i.e. our *second approximation*.

Example of non-symmetric growth rate (12). Using (A.11) and (A.9) we can compute the coefficients in the Taylor expansions of $u_0(z)$ and $v_0(z)$, that is (A.12) and (A.13), to obtain

$$A = \frac{\sqrt{s_0(a + b^2)}}{\sqrt{U}}, \quad B = \frac{\sqrt{s_0}\, b}{3\sqrt{U(a + b^2)}}, \quad D = \frac{b}{a + b^2}.$$

Then the expressions of $\mu^*_{\varepsilon,0}$, $\sigma^{*2}_{\varepsilon,0}$ and $\psi^*_{\varepsilon,0}$ given in the main text can be derived thanks to (A.14).

A.2   Two populations

This section is devoted to the mathematical derivation of our results in the case of two habitats. In Subsection A.2.1 we provide the mathematical derivation of our result in the general case. In Subsection A.2.2 we treat the extreme case where there is no migration from habitat 2, that is $m_2 = 0$.

A.2.1   A general case (where $m_1 > 0$ and $m_2 > 0$)

In Subsection A.2.1.1, we provide the details of our results in the Adaptive Dynamics framework. In Subsection A.2.1.2 we present the analysis to obtain our *first approximation*. In Subsection A.2.1.3 we provide the derivation of our *second approximation*.

A.2.1.1   Adaptive dynamics in presence of migration   In this section, we provide the conditions for a global evolutionary stable strategy. To be able to characterize the ESS one should first characterize the demographic equilibrium corresponding to a set of traits. Because there are only two habitats, at most two distinct traits can co-exist. Therefore, we only need to consider two scenarios where the phenotypic distribution is either monomorphic (with phenotype z^M) or dimorphic (with

phenotypes z_I^D and z_{II}^D, where the subscripts I and II indicate that the phenotype is best adapted to habitat 1 and 2, respectively).

The monomorphic equilibrium is given by $n_i^M(z) = N_i^M \delta(z - z^M)$ where $\delta(.)$ is the dirac delta function, $\left(N_1^M, N_2^M\right)^T$ is the right eigenvector associated with the dominant eigenvalue $W(z^M; N_1^M, N_2^M) = 0$ of $\mathcal{A}(z^M; N_1^M, N_2^M)$. In a similar way the dimorphic equilibrium is characterized by: $n_i^D(z) = \nu_{I,i} \delta(z - z_I^D) + \nu_{II,i} \delta(z - z_{II}^D)$, where $\nu_{I,i} + \nu_{II,i} = N_i^D$ and $(\nu_{k,1}, \nu_{k,2})^T$ are the right eigenvectors associated with the largest eigenvalues $W(z_k^D; N_1^D, N_2^D) = 0$ (for $k = I, II$) of $\mathcal{A}(z_k^D; N_1^D, N_2^D)$.

The evolutionary stability of a resident strategy z^{M*} can be studied with the analysis of the invasion of a new mutant strategy z_m at the demographic equilibrium $\left(N_1^{M*}, N_2^{M*}\right)$ set by the resident strategy. The monomorphic strategy z^{M*} is an evolutionary stable strategy if for any mutant $z_m \neq z^{M*}$, the effective fitness is negative: $W(z_m; N_1^{M*}, N_2^{M*}) < 0$. In a similar way, the dimorphic strategy $\{z_I^{D*}, z_{II}^{D*}\}$ is an evolutionary stable strategy if for any mutant $z_m \notin \{z_I^{D*}, z_{II}^{D*}\}$, the effective fitness is negative: $W(z_m; N_1^{D*}, N_2^{D*}) < 0$.

To determine the global ESS, we first define

$$z^{D*} = \sqrt{\theta^2 - \frac{m_1 m_2}{4\theta^2 s_1 s_2}}, \qquad N_1^{D*} = \frac{\frac{m_1 m_2}{4\theta^2 s_2} + r_{\max,1} - m_1}{\kappa_1}, \qquad N_2^{D*} = \frac{\frac{m_1 m_2}{4\theta^2 s_1} + r_{\max,2} - m_2}{\kappa_2}.$$

Theorem A.1 (Mirrahimi 2017) *There exists a unique global ESS.*
(i) The ESS is dimorphic if

$$\frac{m_1 m_2}{4 s_1 s_2 \theta^4} < 1, \tag{A.15}$$

$$0 < m_2 N_2^{D*} + (w_1(-z^{D*}; N_1^{D*}) - m_1) N_1^{D*}, \tag{A.16}$$

and

$$0 < m_1 N_1^{D*} + (w_2(z^{D*}; N_2^{D*}) - m_2) N_2^{D*}. \tag{A.17}$$

Then the dimorphic equilibrium is given by

$$n_i^{D*} = \nu_{\mathrm{I},i}\delta(z + z^{D*}) + \nu_{\mathrm{II},i}\delta(z - z^{D*}), \quad \nu_{\mathrm{I},i} + \nu_{\mathrm{II},i} = N_i^{D*}, \quad i = 1, 2,$$

with $\nu_{k,i}$ given in Section B.1.

(ii) If the above conditions are not satisfied then the ESS is monomorphic. In the case where condition (A.15) is verified but the r.h.s. of (A.16) (respectively (A.17)) is negative, the fittest trait belongs to the interval $(-\theta, -z^{D*})$ (respectively (z^{D*}, θ)). If (A.15) is satisfied but (A.16) (respectively (A.17)) is an equality then the monomorphic ESS is given by $\{-z^{D*}\}$ (respectively $\{z^{D*}\}$).

If the habitats are symmetric, then the second and the third conditions (A.16)–(A.17) above are always satisfied and the dimorphism occurs under the only condition (A.15). In other words, if migration is weak with respect to the selection or the difference of the optimal traits in the two habitats, then the ESS will be dimorphic. When the habitats are non-symmetric the extra conditions (A.16) and (A.17) appear which are conditions of mutual invasibility. Condition (A.16) (respectively condition (A.17)) means indeed that a mutant trait of type z^{D*} (respectively $-z^{D*}$) can invade a monomorphic resident population of type $-z^{D*}$ (respectively z^{D*}) which is at it's demographic equilibrium (see Mirrahimi 2017-Proposition 3.4).

We can indeed rewrite conditions (A.16) and (A.17) respectively as below

$$\eta_1 < \beta_2 r_{\max,2} - \alpha_1 r_{\max,1}, \qquad \eta_2 < \beta_1 r_{\max,1} - \alpha_2 r_{\max,2},$$

with η_i, α_i and β_i constants depending on m_1, m_2, g_1, g_2, κ_1, κ_2 and θ (see Section B.2 for the expressions of these coefficients). These conditions, which are given in the main text in the general non-symmetric scenario, are indeed a measure of asymmetry between the habitats. They appear from the fact that even if condition (A.15), which is the only condition for dimorphism in symmetric habitats, is satisfied, while the quality of the habitats are very different, the ESS cannot be dimorphic. In this case, the population will be able to adapt only to one of the habitats and it will be maladapted to the other one (see Figure 5). To avoid lengthy and technical computations, the proof of Theorem (A.1) is given in Mirrahimi 2017–Section 4.

A.2.1.2   Derivation of our *first approximation* in presence of migration   Similarly to Subsection A.1.1 our *first approximation* is based on the computation of the terms $u_i(z)$ and $v_i(z)$. Based on such computations we can provide an approximation of the population's total density $N^*_{\varepsilon,i}$ and the phenotypic density $n^*_{\varepsilon,i}(z)$ in the following form

$$N^*_{\varepsilon,i} \approx N_i + \varepsilon K_i, \qquad n^*_{\varepsilon,i}(z) \propto \frac{1}{\sqrt{2\pi\varepsilon}} \exp\Big(\frac{u_i(z) + \varepsilon v_i(z)}{\varepsilon}\Big). \tag{A.18}$$

Note that the equilibrium $(n^*_{\varepsilon,1}, n^*_{\varepsilon,2})$ solves

$$\begin{cases} 0 = U\varepsilon^2 \frac{\partial^2 n^*_{\varepsilon,1}(z)}{\partial z^2} + n^*_{\varepsilon,1}(z)\big(r_1(z) - \kappa_1 N^*_{\varepsilon,1}\big) + m_2 n^*_{\varepsilon,2}(z) - m_1 n^*_{\varepsilon,1}(z), \\[4mm] 0 = U\varepsilon^2 \frac{\partial^2 n^*_{\varepsilon,2}(z)}{\partial z^2} + n^*_{\varepsilon,2}(z)\big(r_2(z) - \kappa_2 N^*_{\varepsilon,2}\big) + m_1 n^*_{\varepsilon,1}(z) - m_2 n^*_{\varepsilon,2}(z). \end{cases} \tag{A.19}$$

We first let $\varepsilon \to 0$ in the above equation to obtain that $n^*_{\varepsilon,i}(z) \to n_i(z)$ and $N^*_{\varepsilon,i} \to N_i$ with

$$\begin{cases} 0 = n_1(z)\big(r_1(z) - \kappa_1 N_1\big) + m_2 n_2(z) - m_1 n_1(z), \\[6mm] 0 = n_2(z)\big(r_2(z) - \kappa_2 N_2\big) + m_1 n_1(z) - m_2 n_2(z), \\[6mm] N_i = \int_{-\infty}^{\infty} n_i(z) dz, \end{cases}$$

which is equivalent with

$$\mathcal{A}(z, N_1, N_2) \begin{pmatrix} n_1(z) \\ n_2(z) \end{pmatrix} = 0, \qquad N_i = \int_{-\infty}^{\infty} n_i(z) dz,$$

with $\mathcal{A}(z, N_1, N_2)$ given by (15). This means that (N_1, N_2) corresponds to the sizes of the populations 1 and 2 at the demographic equilibrium $(n_1(z), n_2(z))$, in absence of mutations. We will show that this equilibrium corresponds indeed to a global evolutionary stable strategy and hence $N_i = N^*_i$. To

this end, we replace (16) in (A.19) and obtain

$$
\begin{cases}
0 = \varepsilon U \frac{\partial^2 u_{\varepsilon,1}(z)}{\partial z^2} + U |\frac{\partial}{\partial z} u_{\varepsilon,1}(z)|^2 + r_1(z) - \kappa_1 N^*_{\varepsilon,1} + m_2 \exp(\frac{u_{\varepsilon,2}(z) - u_{\varepsilon,1}(z)}{\varepsilon}) - m_1, \\[4mm]
0 = \varepsilon U \frac{\partial^2 u_{\varepsilon,2}(z)}{\partial z^2} + U |\frac{\partial}{\partial z} u_{\varepsilon,2}(z)|^2 + r_2(z) - \kappa_2 N^*_{\varepsilon,2} + m_1 \exp(\frac{u_{\varepsilon,1}(z) - u_{\varepsilon,2}(z)}{\varepsilon}) - m_2.
\end{cases}
\tag{A.20}
$$

Similarly to above, this system is derived using the following equalities

$$
\frac{\partial}{\partial z} n^*_{\varepsilon,i}(z) = \left(\frac{\partial}{\partial z} u_{\varepsilon,i}(z)\right) \frac{n^*_{\varepsilon,i}(z)}{\varepsilon}, \qquad \frac{\partial^2}{\partial z^2} n^*_{\varepsilon,i}(z) = \left(\varepsilon \frac{\partial^2}{\partial z^2} u_{\varepsilon,i}(z) + |\frac{\partial}{\partial z} u_{\varepsilon,i}(z)|^2\right) \frac{n^*_{\varepsilon,i}(z)}{\varepsilon^2}.
$$

We can determine $u_i(z)$, $v_i(z)$ from the above equation and (17).

Note that the exponential terms in (A.20) suggest that, when $m_i > 0$ for $i = 1, 2$, as $\varepsilon \to 0$ $u_{\varepsilon,1}(z)$ and $u_{\varepsilon,2}(z)$ converge to the same limit $u(z)$. Otherwise, one of these exponential terms tends to infinity while the other terms are bounded. Keeping the zero order terms (the ones in front of which there is no ε, corresponding to the dominant terms) we obtain

$$
\begin{cases}
0 = U |\frac{\partial}{\partial z} u(z)|^2 + r_1(z) - \kappa_1 N_1 + m_2 \exp(v_2(z) - v_1(z)) - m_1, \\[4mm]
0 = U |\frac{\partial}{\partial z} u(z)|^2 + r_2(z) - \kappa_2 N_2 + m_1 \exp(v_1(z) - v_2(z)) - m_2.
\end{cases}
\tag{A.21}
$$

We then multiply the first line by $\exp(v_1(z))$ and the second line by $\exp(v_2(z))$ and write the system in the matrix form to obtain, using (14),

$$
\begin{pmatrix} w_1(z; N_1) - m_1 & m_2 \\ m_1 & w_2(z; N_2) - m_2 \end{pmatrix} \begin{pmatrix} \exp(v_1(z)) \\ \exp(v_2(z)) \end{pmatrix} = -U |\frac{\partial}{\partial z} u(z)|^2 \begin{pmatrix} \exp(v_1(z)) \\ \exp(v_2(z)) \end{pmatrix}.
$$

Note that the matrix in the l.h.s. is nothing but $\mathcal{A}(z, N_1, N_2)$ given by (15). The equality above means that $-U |\frac{\partial}{\partial z} u(z)|^2$ is indeed the principal eigenvalue of $\mathcal{A}(z; N_1, N_2)$, that is

$$
-U |\frac{\partial}{\partial z} u(z)|^2 = W(z; N_1, N_2).
$$

Similarly to Subsection A.1.1, to have a finite but positive size of population, we should have

$$\max_{z \in \mathbb{R}} u(z) = 0.$$

Otherwise, in view of (16), the total population size whether becomes infinite as $\varepsilon \to 0$ (if $\max_{z \in \mathbb{R}} u(z) > 0$) or it goes to 0 (if $\max_{z \in \mathbb{R}} u(z) < 0$). Similarly we obtain

$$\text{supp} \, n_i \subset \{z \, | \, u(z) = 0\},$$

where $\text{supp} \, n_i$ is the set of traits z such that the density $n_i(z)$ is positive. The above property holds since $u(z_0) < 0$ implies that $\lim_{\varepsilon \to 0} n_{\varepsilon,i}^*(z_0) = 0$.

Let $\bar{z}$ be such that $u(\bar{z}) = 0$ which means that it is a maximum point of $u(z)$. Then, $\frac{\partial}{\partial z} u(\bar{z}) = 0$ and hence

$$W(\bar{z}; N_1, N_2) = 0.$$

Moreover in all the points $z \in \mathbb{R}$, we have

$$W(z; N_1, N_2) = -U |\frac{\partial}{\partial z} u(z)|^2 \leq 0.$$

This implies that

if $z \in \text{supp} \, n_1 = \text{supp} \, n_2$ then $W(z; N_1, N_2) = 0$,

if $z \notin \text{supp} \, n_1 = \text{supp} \, n_2$ then $W(z; N_1, N_2) \leq 0$.

In other words $(n_1(z), n_2(z))$ corresponds to the demographic equilibrium corresponding to the global ESS and hence $n_i(z) = n_i^*(z)$ and $N_i = N_i^*$, with $n_i^*(z)$ and N_i^* given in Subsection A.2.1.1.

We gather the informations that we obtained on $u(z)$:

$$\begin{cases} -U |\frac{\partial}{\partial z} u(z)|^2 = W(z; N_1^*, N_2^*), \\ \max_{z \in \mathbb{R}} u(z) = 0, \end{cases} \tag{A.22}$$

with the maximum points of $u(z)$ attained at the ESS points.

This property allows us to provide explicit formula for $u(z)$.

In the case of monomorphic ESS, $u(z)$ is given by

$$u(z) = -\frac{1}{\sqrt{U}} \Big| \int_{z^{M*}}^{z} \sqrt{-W(x; N_1^{M*}, N_2^{M*})} dx \Big|. \tag{A.23}$$

The reader can verify that $u(z)$, given by the formula above, is smooth and solves (A.22) with its maximum point at z^{M*}.

In the case of dimorphic ESS, $u(z)$ is given by

$$u(z) = \max\Big(-\frac{1}{\sqrt{U}} \Big| \int_{z_{\mathrm{I}}^{D*}}^{z} \sqrt{-W(x; N_1^{M*}, N_2^{M*})} dx \Big|, -\frac{1}{\sqrt{U}} \Big| \int_{z_{\mathrm{II}}^{D*}}^{z} \sqrt{-W(x; N_1^{M*}, N_2^{M*})} dx \Big| \Big). \tag{A.24}$$

The reader can also verify that the above function is smooth at all points except at the point where the two functions in the maximum operator intersect. Moreover, $u(z)$ solves (A.22) at the smooth points and it attains its maximum at the ESS points z_{I}^{D*} and z_{II}^{D*}. See Mirrahimi 2017 for the details on why this is indeed the solution obtained at the limit $\varepsilon \to 0$.

Computation of the next order terms $v_i(z)$:

The derivation of the next order terms $v_i(z)$ follows also similar arguments as in Section A.1.1. However, since here we have a system the computations are less straight forward. We present the main ingredients to compute these terms.

From (A.21) and (A.22) we can compute $v_2(z) - v_1(z)$ thanks to the following formula

$$v_2(z) - v_1(z) = \log\Big(\frac{1}{m_2} \big(W(z, N_1^{M*}, N_2^{M*}) - w(z, N_1^{M*}) + m_1 \big) \Big).$$

We next keep the first order terms in (A.20), i.e. the terms with an ε in front of them. To do so, we need to go further than (17) in the approximation of $u_{\varepsilon,i}(z)$ and also keep the term of order ε^2, $l_i(z)$:

$$u_{\varepsilon,i}(z) = u(z) + \varepsilon v_i(z) + \varepsilon^2 l_i(z) + O(\varepsilon^3).$$

13

Then, keeping the first order terms in (A.20) we obtain

$$\begin{cases} 0 = U\frac{\partial^2}{\partial z^2}u(z) + 2U\frac{\partial}{\partial z}u(z)\frac{\partial}{\partial z}v_1(z) - \kappa_1 K_1 + m_2\exp(v_2(z) - v_1(z))(l_2(z) - l_1(z)), \\ 0 = U\frac{\partial^2}{\partial z^2}u(z) + 2U\frac{\partial}{\partial z}u(z)\frac{\partial}{\partial z}v_2(z) - \kappa_2 K_2 + m_2\exp(v_1(z) - v_2(z))(l_1(z) - l_2(z)). \end{cases} \quad \text{(A.25)}$$

Using the above equalities and by evaluating them at the ESS points we can compute $v_i(z)$ and K_i for $i = 1, 2$. See Mirrahimi 2017–Section 3.3 for the details of such computations.

A.2.1.3 Derivation of our *second approximation* in presence of migration

Explicit formula for the moments of order k, with $k \geq 1$, of the phenotypic distribution, called our *second approximation*, can be derived following similar arguments as in Section A.1.2. However, in presence of migration we should consider two cases of monomorphic and dimorphic population. The case of monomorphic ESS can be treated exactly as in Section A.1.2. The dimorphic case is slightly different and we provide the additional elements to compute the moments of the phenotypic distribution in this case.

Let's suppose that the model has a dimorphic ESS $\{z_{\mathrm{I}}^{D*}, z_{\mathrm{II}}^{D*}\}$ with $N_i^{D*} = \nu_{\mathrm{I,i}} + \nu_{\mathrm{II,i}}$. We first compute the local moments of the phenotypic density, that is, for $k = \mathrm{I}, \mathrm{II}$,

$$\begin{cases} \nu_{\varepsilon,k,i}^{D*} = \int_{\mathcal{O}_k} n_{\varepsilon,i}^{D*}(z)dz, \\ \mu_{\varepsilon,k,i}^{D*} = \frac{1}{\nu_{\varepsilon,k,i}}\int_{\mathcal{O}_k} z n_{\varepsilon,i}^{D*}(z)dz, \\ \sigma_{\varepsilon,k,i}^{D*\,2} = \frac{1}{\nu_{\varepsilon,k,i}}\int_{\mathcal{O}_k}(z - \mu_{\varepsilon,k,i}^{D*})^2 n_{\varepsilon,i}^{D*}(z)dz, \\ \psi_{\varepsilon,k,i}^{D*} = \frac{1}{\nu_{\varepsilon,k,i}^{D*}}\int_{\mathcal{O}_k}(z - \mu_{\varepsilon,k,i}^{D*})^3 n_{\varepsilon,i}^{D*}(z)dz \end{cases}$$

with $\mathcal{O}_{\mathrm{I}} = (-\infty, 0)$ and $\mathcal{O}_{\mathrm{II}} = (0, \infty)$. We can next compute the global moments of the population's distribution from the above local moments.

Since in this case we also need to compute the local population sizes $\nu_{\varepsilon,k,i}^{D*}$, we need to go an order further in the Taylor expansions and also use the value of the second order term l_i at the ESS points

z_k^{D*}:

$$u(z) = -\frac{A_k}{2}(z - z_k^{D*})^2 + B_k(z - z_k^{D*})^3 + C_k(z - z_k^{D*})^4 + O(z - z_k^{D*})^5,$$

$$v_i(z) = \log(\sqrt{A_k}\nu_{k,i}) + D_{k,i}(z - z_k^{D*}) + E_{k,i}(z - z_k^{D*})^2 + O(z - z_k^{D*})^3, \qquad l_i(z_k^{D*}) = F_{k,i}.$$

Note that we can compute the Taylor expansions of u and v thanks to the expression of u, given by (A.24), and the expression of v_i, obtained following Section A.2.1.2. Moreover, we can derive the value of $F_{k,i}$ thanks to (A.25).

The above approximation allows us, similarly to Subsection A.1.2, to estimate the local moments of the population's distribution:

$$\begin{cases} \nu_{\varepsilon,k,i}^{D*} = \nu_{k,i}(1 + \varepsilon K_{k,i}) + O(\varepsilon^2), \\[2mm] \mu_{\varepsilon,k,i}^{D*} = z_k^{D*} + \varepsilon(\frac{3B_k}{A_k^2} + \frac{D_{k,i}}{A_k}) + O(\varepsilon^2), \\[2mm] \sigma_{\varepsilon,k,i}^{D*\,2} = \frac{\varepsilon}{A_k} + O(\varepsilon^2), \\[2mm] \psi_{\varepsilon,k,i}^{D*} = \frac{6B_k}{A_k^3}\varepsilon^2 + O(\varepsilon^3), \end{cases} \qquad (A.26)$$

with

$$K_{k,i} = F_{k,i} + \frac{E_{k,i} + 0.5 D_{k,i}^2}{A_k} + \frac{3(C_k + B_k D_{k,i})}{A_k^2} + \frac{7.5 B_k^2}{A_k^3}.$$

Derivation of the analytic formula (A.26)**:**

The above analytic formula for the moments of order $1 \leq k \leq 3$ can be derived following similar arguments as in Section A. To derive the explicit formula for $\nu_{\varepsilon,k,i}^{D*}$ we also use similar type of arguments. We can indeed use the Taylor expansions above to compute

$$\begin{aligned} \nu_{\varepsilon,k,i}^{D*} &= \int_{\mathcal{O}_k} n_{\varepsilon,i}(z)dz \\ &= \frac{\sqrt{A_k}\nu_{k,i}}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-\frac{A_k}{2}y^2}\left(1 + \sqrt{\varepsilon}(B_k y^3 + D_{k,i} y) + \varepsilon(0.5(B_k y^3 + D_{k,i} y)^2 + C_k y^4 + E_{k,i} y^2 + F_{k,i}) + O(\varepsilon^2)\right)dy \\ &= \nu_{k,i}(1 + \varepsilon(F_{k,i} + \frac{E_{k,i} + 0.5 D_{k,i}^2}{A_k} + \frac{3(C_k + B_k D_{k,i})}{A_k^2} + \frac{7.5 B_k^2}{A_k^3})) + O(\varepsilon^2), \end{aligned}$$

Note that to compute the integral terms above we have performed a change of variable $z - z^{M*} = \sqrt{\varepsilon}y$, therefore each term $z - z^{M*}$ can be considered as of order $\sqrt{\varepsilon}$ in the integrations.

A.2.2 The extreme source and sink case (where $m_1 > 0$ and $m_2 = 0$)

In this section, we provide the derivation of our approximations in the extreme source and sink scenario. In Subsection A.2.2.1 we provide our derivation in the Adaptive Dynamics framework and in Subsection A.2.2.2 we present the analysis for our *first approximation*. We do not provide the derivation of our *second approximation* in the extreme source and sink scenario since it is similar to the general case.

A.2.2.1 Adaptive dynamics in the extreme source and sink scenario

We first recall the evolutionary outcome in the first habitat which depends only on selection acting in habitat 1: the ESS is $-\theta$ and

$$N_1^* = \frac{r_{\max,1} - m_1}{\kappa_1}.$$

In the second habitat, a part of the population 1, with trait $-\theta$ and with size $\frac{m_1(r_{\max,1}-m_1)}{\kappa_1}$, is present thanks to migration. The growth rate of any other trait z in this habitat is given by $w_2(z; N_2^*) = r_{\max,2} - s_2(z - \theta)^2 - \kappa_2 N_2^*$. Therefore the only trait, in addition to the trait $z = -\theta$, that can be present at the ESS is the maximum point of $w_2(z; N_2^*)$, that is $z = \theta$. This means that the ESS at the second habitat, is whether monomorphic and given by $\{-\theta\}$ or dimorphic and given by $\{-\theta, \theta\}$. The dimorphism in the second habitat occurs if the trait θ is strong enough to compete with the maladapted trait $-\theta$ coming from the first habitat. This occurs under the following condition:

$$\frac{m_1(r_{\max,1} - m_1)}{\kappa_1} < \frac{4s_2\theta^2 r_{\max,2}}{\kappa_2}. \tag{A.27}$$

Under this condition, there exists indeed a positive demographic equilibrium for the set of traits $\{-\theta, \theta\}$:

$$n_2^{D*} = \alpha\delta(z + \theta) + \beta\delta(z - \theta), \quad N_2^{D*} = \alpha + \beta = \frac{r_2}{\kappa_2},$$

such that

$$m_1 N_1^* + w_2(-\theta; N_2^{D*})\alpha = 0, \qquad w_2(\theta; N_2^{D*})\beta = 0.$$

We can indeed compute the densities α, β and N_2^{D*}:

$$N_2^{D*} = \frac{r_2}{\kappa_2}, \qquad \alpha = \frac{m_1(r_{\max,1} - m_1)}{4s_2\theta^2\kappa_1}, \qquad \beta = \frac{r_{\max,2}}{\kappa_2} - \frac{m_1(r_{\max,1} - m_1)}{4s_2\theta^2\kappa_1}.$$

When the condition (A.27) is not satisfied, then there is no such demographic equilibrium. The ESS is then monomorphic with trait $-\theta$ and the demographic equilibrium is given by

$$n_2^{M*} = N_2^* \, \delta(z + \theta), \qquad m_1 N_1^* + w_2(\theta; N_2^{M*})N_2^{M*} = 0,$$

leading to

$$N_2^{M*} = \frac{1}{2\kappa_2}\left(r_2 - 4s_2\theta^2 + \sqrt{(r_{\max,2} - 4s_2\theta^2)^2 + 4\frac{\kappa_2}{\kappa_1}m_1(r_{\max,1} - m_1)}\right).$$

To understand the relation with Section A.2.1.1, note that the above computations are equivalent with finding an ESS for the effective growth rate $W(z; N_1^*, N_2^*)$, corresponding to the principal eigenvalue of $\mathcal{A}(z; N_1^*, N_2^*)$ which is given by

$$W(z; N_1^*, N_2^*) = \max\left(w_1(z; N_1^*) - m_1, w_2(z; N_2^*)\right),$$

as a consequence of $m_2 = 0$.

A.2.2.2 Derivation of the *first approximation* in the extreme source and sink scenario

The population's phenotypic density $n_{\varepsilon,1}^*$ can be computed explicitly as in the one population scenario, the example of quadratic growth rate: $n_{\varepsilon,1}^* = N_{\varepsilon,1}^* f_\varepsilon$, where $N_{\varepsilon,1}^* = \frac{r_{\max,1} - m_1 - \varepsilon\sqrt{Us_1}}{\kappa_1}$ and f_ε is the probability density of a normal distribution $\mathcal{N}(-\theta, \frac{\varepsilon\sqrt{U}}{\sqrt{s_1}})$ (see (11)). This allows us to compute $u_{\varepsilon,1}$, given by (16):

$$u_{\varepsilon,1} = -\frac{\sqrt{s_1}(z + \theta)^2}{2\sqrt{U}} + \varepsilon \log\left(s_1^{\frac{1}{4}}\left(N_1^* - \varepsilon\frac{\sqrt{Us_1}}{\kappa_1}\right)\right),$$

and hence

$$u_{\varepsilon,1} = u_1 + \varepsilon v_1 + \varepsilon^2 l_1 + o(\varepsilon^2), \qquad N_{\varepsilon,1} = N_1^* + \varepsilon K_1 + o(\varepsilon),$$

with

$$u_1 = -\frac{\sqrt{s_1}(z+\theta)^2}{2\sqrt{U}}, \qquad v_1 \equiv \log\left(s_1^{\frac{1}{4}} N_1^*\right), \qquad l_1 \equiv -\frac{\sqrt{Us_1}}{\kappa_1 N_1^*}, \qquad K_1 = -\frac{\sqrt{Us_1}}{\kappa_1}. \tag{A.28}$$

To estimate the phenotypic density in the second habitat we study the equation on $u_{\varepsilon,2}$ given by (16) and (A.20):

$$0 = \varepsilon U \frac{\partial^2 u_{\varepsilon,2}(z)}{\partial z^2} + U|\frac{\partial}{\partial z} u_{\varepsilon,2}(z)|^2 + r_2(z) - \kappa_2 N_{\varepsilon,2}^* + m_1 \exp\left(\frac{u_{\varepsilon,1}(z) - u_{\varepsilon,2}(z)}{\varepsilon}\right). \tag{A.29}$$

Note that here we have used the fact that $m_2 = 0$. As above we expect to have an asymptotic expansion for $u_{\varepsilon,2}(z)$ and $N_{\varepsilon,2}^*$:

$$u_{\varepsilon,2}(z) = u_2(z) + \varepsilon v_2(z) + \varepsilon^2 l_2(z) + o(\varepsilon^2), \qquad N_{\varepsilon,2}^* = N_2^* + \varepsilon K_2.$$

Derivation of $u_2(z)$.

We replace the above expansion in (A.29). From the exponential term in (A.29) we deduce that

$$u_2(z) \geq u_1(z).$$

Otherwise this exponential term will tend to infinity. However, unlike Section A.2.1.2, here we don't have necessarily that $u_1(z) \geq u_2(z)$, which means that we don't have necessarily $u_1(z) = u_2(z)$. This is because $m_2 = 0$ and there is no such exponential term in the equation on $u_{\varepsilon,1}(z)$. This is why the source and sink case is a degenerate case comparing to what we studied in Section A.2.1.2.

In order to identify $u_2(z)$ we first notice that keeping the ε^0 order terms in (A.29) and using the positivity of the exponential term in (A.29), we obtain that, for all $z \in \mathbb{R}$,

$$-U|u_2'(z)|^2 \geq r_2(z) - \kappa_2 N_2^*. \tag{A.30}$$

Next, we consider two cases:

(i) Let z be such that $u_1(z) < u_2(z)$. In this case, $\exp\left(\frac{u_{\varepsilon,1}(z) - u_{\varepsilon,2}(z)}{\varepsilon}\right)$ tends to 0, and keeping the ε^0

order terms in (A.29) we obtain

$$-U|u_2'(z)|^2 = r_2(z) - \kappa_2 N_2^*. \tag{A.31}$$

(ii) Let z be such that $u_1(z) = u_2(z)$. Assuming that the equality holds on an open interval including z, we can write

$$-U|u_2'(z)|^2 = -U|u_1'(z)|^2 = r_1(z) - \kappa_1 N_1^* - m_1. \tag{A.32}$$

Thanks to (A.30) and (A.32) we deduce in particular that if at some trait z_0,

$$r_1(z_0) - \kappa_1 N_1^* - m_1 < r_2(z_0) - \kappa_2 N_2^*,$$

then $u_1(z_0) \neq u_2(z_0)$ and hence u_2 solves (A.31) at z_0.

Combining the above properties we can determine the equation on u_2. To this end, we consider three cases:

(1) Let $s_2 < s_1$. In this case, there exists (z_1, z_2) such that $z_1 < -\theta < z_2 < \theta$ and

$$r_1(z) - \kappa_1 N_1^* - m_1 \geq r_2(z) - \kappa_2 N_2^*, \qquad \text{for } z \in [z_1, z_2],$$

and

$$r_1(z) - \kappa_1 N_1^* - m_1 < r_2(z) - \kappa_2 N_2^*, \qquad \text{for } z \in (-\infty, z_1) \cup (z_2, \infty).$$

This leads to the following equation on $u_2(z)$:

$$-U|u_2'(z)|^2 = g_1(z) := \max\left(r_2(z) - \kappa_2 N_2^*, r_1(z) - \kappa_1 N_1^* - m_1\right).$$

(2) Let $s_1 < s_2$. In this case, there exists (z_1, z_2) such that $-\theta < z_1 < \theta < z_2$ and

$$r_1(z) - \kappa_1 N_1^* - m_1 \leq r_2(z) - \kappa_2 N_2^*, \qquad \text{for } z \in [z_1, z_2],$$

and

$$r_1(z) - \kappa_1 N_1^* - m_1 > r_2(z) - \kappa_2 N_2^*, \qquad \text{for } z \in (-\infty, z_1) \cup (z_2, \infty).$$

This leads to the following equation on $u_2(z)$:

$$-U|u_2'(z)|^2 = g_2(z),$$

with $g_2(z)$ given by

$$g_2(z) = \begin{cases} r_1(z) - \kappa_1 N_1^* - m_1, & \text{if } z \leq z_1, \\ r_2(z) - \kappa_2 N_2^*, & \text{if } z > z_1. \end{cases}$$

(3) Let $s_1 = s_2$. In this case, there exists a point z_1 such that $-\theta < z_1 < \theta$ and

$$r_1(z) - \kappa_1 N_1^* - m_1 \geq r_2(z) - \kappa_2 N_2^*, \qquad \text{for } z \in (-\infty, z_1],$$

and

$$r_1(z) - \kappa_1 N_1^* - m_1 < r_2(z) - \kappa_2 N_2^*, \qquad \text{for } z \in (z_1, \infty).$$

This leads to the following equation on $u_2(z)$:

$$-U|u_2'(z)|^2 = g_3(z),$$

with

$$g_3(z) = g_1(z) = \max \left(r_2(z) - \kappa_2 N_2^*, r_1(z) - \kappa_1 N_1^* - m_1 \right).$$

From the above formula we can determine $u_2(z)$:

Monomorphic case: Under condition

$$\frac{4 s_2 \theta^2 r_{\max,2}}{\kappa_2} < \frac{m_1 (r_{\max,1} - m_1)}{\kappa_1},$$

the functions $g_i(z)$ have a unique maximum point at $-\theta$. This means in particular that the ESS is monomorphic (see condition (21)). This leads to the following formula for $u_2(z)$

$$u_2(z) = -\frac{1}{\sqrt{U}} \Big| \int_{-\theta}^{z} \sqrt{-g_i(x)} dx \Big|^2, \tag{A.33}$$

where we use g_i, with $i = 1, 2, 3$, for case (i).

Dimorphic case: Under condition

$$\frac{4s_2\theta^2 r_{\max,2}}{\kappa_2} > \frac{m_1(r_{\max,1} - m_1)}{\kappa_1},$$

the functions $g_i(z)$ have two maximum points at $-\theta$ and θ. This means that the ESS is dimorphic and $z_1^{D*} = -z_2^{D*} = -\theta$. Moreover, the function $u_2(z)$ can be computed as below

$$u_2(z) = \max\left(-\frac{1}{\sqrt{U}}\Big|\int_{-\theta}^{z}\sqrt{-g_i(x)}dx\Big|^2, -\frac{1}{\sqrt{U}}\Big|\int_{\theta}^{z}\sqrt{-g_i(x)}dx\Big|^2\right), \tag{A.34}$$

where we use g_i, with $i = 1, 2, 3$, for case (i).

Derivation of $v_2(z)$.

To compute the next order term $v_2(z)$, we also consider two cases:

(i) Let z be such that $u_1(z) < u_2(z)$. As above, in this case, $\exp\left(\frac{u_{\varepsilon,1}(z) - u_{\varepsilon,2}(z)}{\varepsilon}\right)$ tends to 0, and keeping the ε order terms in (A.29) we obtain

$$-U\, u_2''(z) = 2U\, u_2'(z)v_2'(z) - \kappa_2 K_2.$$

Since we have already computed the expression for $u_2(z)$ ((A.33) and (A.34)) we can compute $v_2'(z)$ (and hence $v_2(z)$) from the above formula. In particular, in the dimorphic case, the computations are straight forward. In this case $u_2'(z)$ is given by (A.31) with $r_2(z) - \kappa_2 N_2^* = -s_2(z - \theta)^2$. Therefore,

$$u_2'(z) = -\frac{\sqrt{s_2}}{\sqrt{U}}(z - \theta), \qquad u_2''(z) = -\frac{\sqrt{s_2}}{\sqrt{U}}.$$

Combining the above equalities we obtain that $v_2'(z) = 0$, and hence $v_2(z)$ is constant in this zone:

$$v_2(z) = v_2(\theta), \qquad K_2 = -\frac{\sqrt{U s_2}}{\kappa_2}.$$

21

(ii) Let z be such that $u_1(z) = u_2(z)$. Then, keeping the ε^0 order terms in (A.29) we obtain

$$-U|u_2'(z)|^2 = r_2(z) - \kappa_2 N_2^* + m_1 \exp(v_1(z) - v_2(z)).$$

Since $u_2(z)$ and $v_1(z)$ are already given by (A.33), (A.34) and (A.28), the above formula allows us to compute $v_2(z)$:

$$v_2(z) = v_1(z) - \log\left(\frac{r_1(z) - \kappa_1 N_1^* - m_1 - r_2(z) + \kappa_2 N_2^*}{m_1}\right).$$

The above computations are relevant for z far from the point $\bar{z}_0$ such that

$$r_1(\bar{z}_0) - \kappa_1 N_1^* - m_1 = r_2(\bar{z}_0) - \kappa_2 N_2^*.$$

From the expression of $v_2(z)$ above we notice indeed that there is a singularity at the point $\bar{z}_0$ since the value inside the logarithmic term vanishes at $\bar{z}_0$. This shows that such approximation does not hold close to the point $\bar{z}_0$. In our numerical simulations in Figure 6 we use indeed a linear approximation of the formula above for $v_2(z)$ which provides already convincing results.

A.3 Derivation of a Hamilton-Jacobi equation in the case of model (2)

Our approach can also be used to study the more general model (2). The objective would be to provide an approximation of the equilibrium solution n_i^* when the variance of the mutation distribution is small. We assume indeed that the variance of the mutation distribution K_ε scales as $\varepsilon^2 V_0$. More precisely, we assume that $K_\varepsilon(y)dy = K(\frac{y}{\varepsilon})\frac{dy}{\varepsilon}$ (for instance a Gaussian distribution with mean 0 and variance $\varepsilon^2 \sigma^2$ has such form). Then, the stationary version of (2) may be written as

$$
\begin{aligned}
0 &= U\left(\int_{-\infty}^{+\infty} n_{\varepsilon,i}^*(z - \varepsilon y)K(y)dy - n_{\varepsilon,i}^*(z)\right) + n_{\varepsilon,i}^*(z)\left(r_i(z) - \kappa_i \int_{-\infty}^{+\infty} n_{\varepsilon,i}^*(t,y)dy\right) \\
&\quad + m_j n_{\varepsilon,j}^*(z) - m_i n_{\varepsilon,i}^*(z).
\end{aligned}
$$

Next, analogously to our work in the case of (3), we use the ansatz (16):

$$n_{\varepsilon,i}^* = \frac{1}{2\pi\varepsilon}\exp\left(\frac{u_{\varepsilon,i}}{\varepsilon}\right)$$

22

and postulate an expansion for $u_{\varepsilon,i}$ in terms of ε:

$$u_{\varepsilon,i} = u_i + \varepsilon v_i + O(\varepsilon^2).$$

The computation of the above terms allows us to provide approximations of the population phenotypic densities $n_{\varepsilon,i}^*(z)$ and their moments. To compute these terms, analogously to what we presented for the diffusion case, thanks to the combination of the above equalities, we derive some equations satisfied by u_i and v_i. The resolution of such equations, which is less straight forward comparing to the diffusion case, allows us to compute these terms. We provide here, the equation satisfied by the zero order term u_i. To this end, we replace (16) in the above equation to obtain

$$\begin{cases} -U \int_{-\infty}^{+\infty} \left(e^{\frac{u_{\varepsilon,1}(z-\varepsilon y)-u_{\varepsilon,1}(z)}{\varepsilon}} - 1\right)K(y)dy = r_{\max,1} - \kappa_1 N_{\varepsilon,1}^* + m_2 \exp(\frac{u_{\varepsilon,2}-u_{\varepsilon,1}}{\varepsilon}) - m_1, \\[4mm] -U \int_{-\infty}^{+\infty} \left(e^{\frac{u_{\varepsilon,2}(z-\varepsilon y)-u_{\varepsilon,2}(z)}{\varepsilon}} - 1\right)K(y)dy = r_{\max,2} - \kappa_2 N_{\varepsilon,2}^* + m_1 \exp(\frac{u_{\varepsilon,1}-u_{\varepsilon,2}}{\varepsilon}) - m_2. \end{cases}$$

Similarly to Subsection A.2.1.2, the exponential terms, coming from the migration terms, suggest that when $m_i > 0$ for $i = 1, 2$, as $\varepsilon \to 0$, $u_{\varepsilon,1}$ and $u_{\varepsilon,2}$ converge to the same limit u. The limit u solves the following Hamilton-Jacobi equation

$$\begin{cases} -U \int_{-\infty}^{+\infty} \left(e^{-y\frac{\partial}{\partial z}u(z)} - 1\right)K(y)dy = W(z; N_1^*, N_2^*), \\[4mm] \max_{z\in\mathbb{R}} u(z) = 0, \end{cases} \tag{A.35}$$

where (N_1^*, N_2^*) is the demographic equilibrium corresponding to the ESS and W is the largest eigenvalue of matrix $\mathcal{A}$ given by (15). See Barles et al. 2009 where the details of such computations are provided in the case of a homogeneous environment.

B Some expressions for the case studies

B.1 Local densities in the dimorphic case for general and symmetric scenarios

In this section, we provide the expressions of the local densities in the dimorphic case in the Adaptive Dynamics framework and for the general non-symmetric scenario with $m_i > 0$. The expressions of the local densities in the symmetric case can be obtained from the same formula, using $m_1 = m_2 = m, \kappa_1 = \kappa_2 = \kappa, s_1 = s_2 = g, r_{\max,1} = r_{\max,2} = r_{\max}$.

To this end, we first recall the values of the global densities at the ESS:

$$N_1^{D*} = \frac{\frac{m_1 m_2}{4\theta^2 s_2} + r_{\max,1} - m_1}{\kappa_1}, \qquad N_2^{D*} = \frac{\frac{m_1 m_2}{4\theta^2 s_1} + r_{\max,2} - m_2}{\kappa_2}.$$

Then the local densities $\nu_{k,i}$, for $k = \mathrm{I}, \mathrm{II}$ and $i = 1, 2$, are given by

$$\begin{pmatrix} \nu_{\mathrm{I},1} \\ \nu_{\mathrm{I},2} \end{pmatrix} = \frac{m_1 N_1^{D*} + (w_2(z^{D*}; N_2^{D*}) - m_2) N_2^{D*}}{m_1 m_2 - \left(w_1(-z^{D*}; N_1^{D*}) - m_1\right)\left(w_2(z^{D*}; N_2^{D*}) - m_2\right)} \begin{pmatrix} m_2 \\ -w_1(-z^{D*}; N_1^{D*}) + m_1 \end{pmatrix},$$

$$\begin{pmatrix} \nu_{\mathrm{II},1} \\ \nu_{\mathrm{II},2} \end{pmatrix} = \frac{m_2 N_2^{D*} + (w_1(-z^{D*}; N_1^{D*}) - m_1) N_1^{D*}}{m_1 m_2 - \left(w_1(-z^{D*}; N_1^{D*}) - m_1\right)\left(w_2(z^{D*}; N_2^{D*}) - m_2\right)} \begin{pmatrix} -w_2(z^{D*}; N_2^{D*}) + m_2 \\ m_1 \end{pmatrix}.$$

B.2 Condition for dimorphism in a general non-symmetric scenario

We provide below the expressions of the constants α_i, β_i and η_i which appear in the condition for dimorphism in the main text in the general non-symmetric scenario (see also Subsection A.2.1.1):

$$\alpha_1 = \frac{2s_1\theta}{\kappa_1}\left(\theta - \sqrt{\theta^2 - \frac{m_1 m_2}{4\theta^2 s_1 s_2}}\right), \qquad \alpha_2 = \frac{2s_2\theta}{\kappa_2}\left(\theta - \sqrt{\theta^2 - \frac{m_1 m_2}{4\theta^2 s_1 s_2}}\right),$$

$$\beta_1 = \frac{m_1}{\kappa_1}, \qquad \beta_2 = \frac{m_2}{\kappa_2},$$

$$\eta_1 = \frac{m_2^2}{\kappa_2} + \frac{2s_1\theta}{\kappa_1}\left(\theta - \sqrt{\theta^2 - \frac{m_1 m_2}{4\theta^2 s_1 s_2}}\right)\left(\frac{m_1 m_2}{4\theta^2 s_2} - m_1\right) - \frac{m_1 m_2^2}{4\theta^2 s_1 \kappa_2},$$

$$\eta_2 = \frac{m_1^2}{\kappa_1} + \frac{2s_2\theta}{\kappa_2}\left(\theta - \sqrt{\theta^2 - \frac{m_1 m_2}{4\theta^2 s_1 s_2}}\right)\left(\frac{m_1 m_2}{4\theta^2 s_1} - m_2\right) - \frac{m_1^2 m_2}{4\theta^2 s_2 \kappa_1}.$$

B.3 Analytic formula for the moments of the dimorphic phenotypic distribution in the symmetric case

We provide below our analytic approximations of the local moments of the phenotypic distribution, in the dimorphic case, for both the symmetric scenario and the source and sink scenario. Such local moments are defined in Subsection A.2.1.3). One can compute the global moments of the population's distribution from such local moments.

For the symmetric scenario, we define $\gamma = \sqrt{1 - \frac{m^2}{4s^2\theta^4}}$. Our approximation is then given by

$$
\begin{cases}
N_{\varepsilon,1}^{D*} = N_{\varepsilon,2}^{D*} = \frac{1}{\kappa}\left(\frac{m^2}{4s\theta^2} + r_{\max} - m\right) - \varepsilon\frac{\sqrt{U}s\gamma}{\kappa} + O(\varepsilon^2), \\[2mm]
\mu_{\varepsilon,\mathrm{I},1}^{D*} = z_{\mathrm{I}}^{D*} + \varepsilon\sqrt{U}\left(\frac{m^2}{4s^2\sqrt{s}\theta^5\gamma^2} - \frac{4s^2\theta^4(1-\gamma)^2}{\theta\sqrt{s}\gamma(4s^2\theta^4(1-\gamma)^2+m^2)}\right) + O(\varepsilon^2), \\[2mm]
\mu_{\varepsilon,\mathrm{I},2}^{D*} = z_{\mathrm{I}}^{D*} + \varepsilon\sqrt{U}\left(\frac{m^2}{4s^2\sqrt{s}\theta^5\gamma^2} + \frac{m^2}{\theta\sqrt{s}\gamma(4s^2\theta^4(1-\gamma)^2+m^2)}\right) + O(\varepsilon^2), \\[2mm]
\mu_{\varepsilon,\mathrm{II},1}^{D*} = z_{\mathrm{II}}^{D*} - \varepsilon\sqrt{U}\left(\frac{m^2}{4s^2\sqrt{s}\theta^5\gamma^2} + \frac{m^2}{\theta\sqrt{s}\gamma(4s^2\theta^4(1-\gamma)^2+m^2)}\right) + O(\varepsilon^2), \\[2mm]
\mu_{\varepsilon,\mathrm{II},2}^{D*} = z_{\mathrm{II}}^{D*} - \varepsilon\sqrt{U}\left(\frac{m^2}{4s^2\sqrt{s}\theta^5\gamma^2} - \frac{4s^2\theta^4(1-\gamma)^2}{\theta\sqrt{s}\gamma(4s^2\theta^4(1-\gamma)^2+m^2)}\right) + O(\varepsilon^2), \\[2mm]
\sigma_{\varepsilon,\mathrm{I},1}^{D*\,2} = \sigma_{\varepsilon,\mathrm{I},2}^{D*\,2} = \sigma_{\varepsilon,\mathrm{II},1}^{D*\,2} = \sigma_{\varepsilon,\mathrm{II},2}^{D*\,2} = \frac{\varepsilon\sqrt{U}}{\sqrt{s}\gamma} + O(\varepsilon^2), \\[2mm]
\psi_{\varepsilon,\mathrm{I},i}^{D*} = \frac{m^2U\varepsilon^2}{4s^3\theta^5\gamma^3} + O(\varepsilon^3), \\[2mm]
\psi_{\varepsilon,\mathrm{II},i}^{D*} = -\frac{m^2U\varepsilon^2}{4s^3\theta^5\gamma^3} + O(\varepsilon^3),
\end{cases}
$$

where the subscripts I and II indicate that the local moments are in the sets $\mathcal{O}_{\mathrm{I}} = (-\infty, 0)$ and $\mathcal{O}_{\mathrm{II}} = (0, \infty)$ which include respectively z_{I}^{D} and z_{II}^{D}.

B.4 Some expressions for the source and sink scenario

When the condition (21) holds, the evolutionary equilibrium is monomorphic and the evolutionary stable strategy is $z^* = -\theta$ and the total population size is given by

$$
N_2^* = \frac{1}{2\kappa_2}\left(r_{\max,2} - 4s_2\theta^2 + \sqrt{(r_{\max,2} - 4s_2\theta^2)^2 + 4\frac{\kappa_2}{\kappa_1}m_1(r_{\max,1} - m_1))}\right).
$$

There is indeed a population of size N_2^* in the second habitat which is of type $z^* = -\theta$: this population is very maladapted.

Our *second approximation* provides analytic formula for the moments of the phenotypic distribution in the sink (see Figure 6 for a comparison between the first and the second approximations):

$$
\begin{cases}
N_{\varepsilon,2}^{M*} = N_2^* - \varepsilon \frac{\sqrt{U}\, m_1 N_1^* N_2^*}{m_1 N_1^* + \kappa_2 N_2^{*2}} \Big(-\frac{\sqrt{s_1}}{\kappa_1 N_1^*} - \frac{N_2^* \sqrt{s_1}}{m_1 N_2^*} + \frac{(s_2-s_1)(r_{\max,2}-\kappa_2 N_2^*)+4s_2(s_1+3s_2)\theta^2}{(\kappa_2 N_2^*-r_{\max,2}+4s_2\theta^2)^2 \sqrt{s_1}} \Big) + O(\varepsilon^2), \\
\mu_{\varepsilon,2}^{M*} = -\theta + \varepsilon \frac{4\sqrt{U}\, s_2\theta}{\sqrt{s_1}\big(\kappa_2 N_2^* - r_{\max,2}+4s_2\theta^2\big)} + O(\varepsilon^2), \\
\sigma_{\varepsilon,2}^{M*2} = \frac{\varepsilon\sqrt{U}}{\sqrt{s_1}} + O(\varepsilon^2), \\
\psi_{\varepsilon,2}^{M*} = O(\varepsilon^3).
\end{cases}
$$

When the condition (21) is not satisfied, the evolutionary equilibrium is dimorphic in the second habitat:

$$
n_2^* = \nu_{\mathrm{I},2}\delta(z+\theta) + \nu_{\mathrm{II},2}\delta(z-\theta),
$$

with

$$
\nu_{\mathrm{I},2} = \frac{m_1(r_{\max,1}-m_1)}{4s_2\theta^2\kappa_1}, \qquad \nu_{\mathrm{II},2} = \frac{r_{\max,2}}{\kappa_2} - \frac{m_1(r_{\max,1}-m_1)}{4s_2\theta^2\kappa_1}), \qquad \nu_{\mathrm{I},2} + \nu_{\mathrm{II},2} = N_2^* = \frac{r_{\max,2}}{\kappa_2}.
$$

Then, the moments of the population's distribution in this habitat can be approximated as below:

$$
\begin{cases}
N_{\varepsilon,2}^{D*} = N_2^* - \varepsilon \frac{\sqrt{U s_2}}{\kappa_2} + O(\varepsilon^2), \\
\mu_{\varepsilon,\mathrm{I},2}^{D*} = -\theta + \varepsilon \frac{\sqrt{U}}{\theta\sqrt{s_1}} + O(\varepsilon^2), \\
\mu_{\varepsilon,\mathrm{II},2}^{D*} = \theta + O(\varepsilon^2), \\
\sigma_{\varepsilon,\mathrm{I},2}^{D*2} = \frac{\varepsilon\sqrt{U}}{\sqrt{s_1}} + O(\varepsilon^2), \\
\sigma_{\varepsilon,\mathrm{II},2}^{D*2} = \frac{\varepsilon\sqrt{U}}{\sqrt{s_2}} + O(\varepsilon^2), \\
\psi_{\varepsilon,\mathrm{I},2}^{D*} = \psi_{\varepsilon,\mathrm{II},2}^{D*} = O(\varepsilon^3).
\end{cases}
$$

References

Barles, G., Mirrahimi, S., and Perthame, B. (2009). Concentration in Lotka-Volterra parabolic or integral equations: a general convergence result. *Methods Appl. Anal.*, 16(3):321–340.

Mirrahimi, S. (2017). A Hamilton-Jacobi approach to characterize the evolutionary equilibria in heterogeneous environments. *Math. Models Methods Appl. Sci.*, 27(13):2425–2460.