THE Maya AND THE CONCEPTION OF Mixbaal

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Fascinating, mysterious and magnificent, this is how the Maya culture can be defined. Established in the South East of Mexico — the Yucatan peninsula and its nearest regions, parts of Guatemala, Belice and Honduras, it is considered as one of the greatest civilizations in America, and in the world. The Maya are undoubtably an extraordinary example of human beings' capacity and intelligence.

The Maya times are basically divided into three periods which cover about twelve centuries: the Old Empire (AD 317–987) namely the origin of their civilization, its rise, first florescence and first decline; the New Empire (987–1527) including its renaissance and final decline; and finally, the Spanish conquest (1527–1697). Throughout those periods, the Maya devised an advanced society. Their architectural techniques, hieroglyphs and arts but also their precise understanding of the Universe and ideas of the Cosmos — part of their religion — were largely described by sixteeth century Spanish writers. Educated Maya who were taught by Catholic missionaries to write their language in Spanish alphabet, to facilitate their instruction in the Catholic faith, gave even more accurate summaries of their ancient history too.

The Conception of Zero

Equally impressive, and perhaps less known, are their arithmetic and numerical system. And indeed, the Maya created the concept of zero, from which the principle of positional numeration derives, some 1,000 years before the Hindus and 2,000 years before the Arabs introduced the positional system and zero in the Western Europe.

Sylvanus Morley, a scientist and researcher specialised in the Maya culture said the following on the latter: *The ancient Maya priests devised a simple numerical system which even today, more than two thousand years later, stands as one of the most brilliant achievements of human mind.*

For the Maya, zero was the seed of numbers and they called it *He* (meaning egg) or *Mixbaal*. According to their cosmical religion, they thought that zero was synonymous with Chaos, creator of the Universe and of the planet Sun, seed of human life. They did not associate the notion of zero with that of emptiness. Indeed they believed that absolute emptiness was impossible.

Positional numeration

The creation of *Mixbaal* meant an important step in the history of numbers. It allowed the Maya to have a positional numerical system. Recall that such a system consists of a fixed number of numerical signs depending on the base number adopted. There is also a sign expressing nullity which is used to indicate the absence of units within an order and the passage to a superior order.

The Maya's system was based upon 20 as its unit of progression instead of 10, that is to say vigesimal instead of decimal. It required a total of 20 different signs (each representing the numbers 0, 1, ..., 19). The signs are formed with three basic symbols: a dot (with value one), a bar (with value five) and a shell (meaning zero) represented at that time by beans, straws (probably from the image of five crossed beans) and shells (see Figure 1). Although it appears a funny system, it allowed them to express abstract ideas.

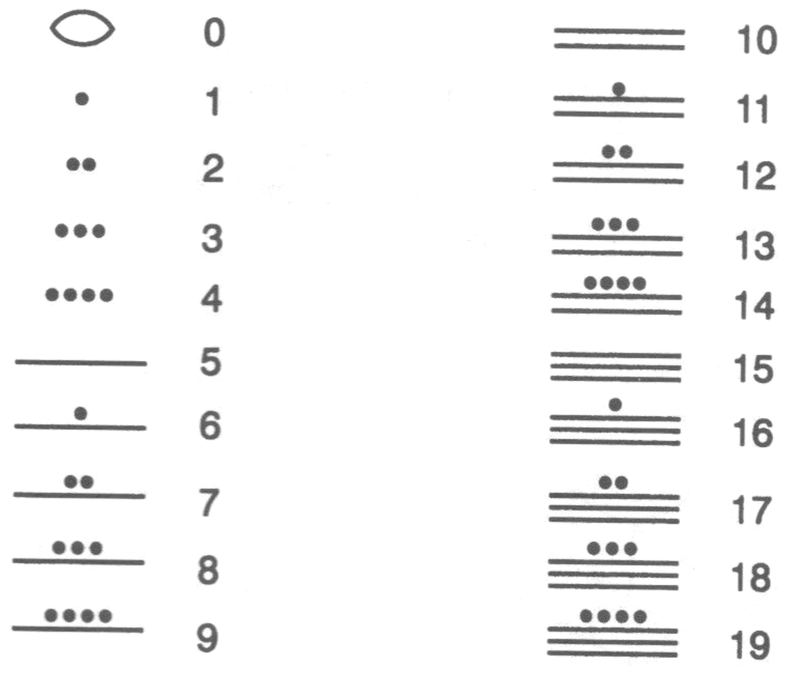

Fig. 1

From Figure 1, it is readily apparent as S. Morley wrote that *the Maya bar-and-dot notation was superior to Roman notation in two respects. To write the numbers from I to XIX inclusive in Roman notation, it is necessary to employ three symbols — the letter I, V, and X — and two arithmetical processes — addition and subtraction: VI is V plus I, but IV is V minus I. On the other hand, in Maya bar-and-dot notation, in order to write the numbers from I to XIX inclusive, it is necessary to employ only two symbols — the bar and the dot —*

and only one arithmetical process, namely addition. In other words, Maya bar-and-dot notation used not only one symbol less to write the numbers from I to XIX inclusive than Roman notation requires, but also one arithmetical process less.

How did they use those symbols? The Maya conceived a simple table formed by a column of cells to express any quantity Q. In each cell, there was only one sign. Each sign had a value according to its form and its place in the table. In the decimal system, the positions to the left of the decimal point increase by tens from right to left — units, tens, hundreds, thousands, etc. In the Maya positional system the values of positions increase by twenties from bottom to top.

So, if we label each cell as described above with numbers 0, 1, 2, ..., the corresponding value V of sign S in the ith-cell, is given by multiplying 20^i times the number represented by S. Hence, the value of quantity Q is obtained by adding all the values Vs (see Figure 2(a), (b) and (c)).

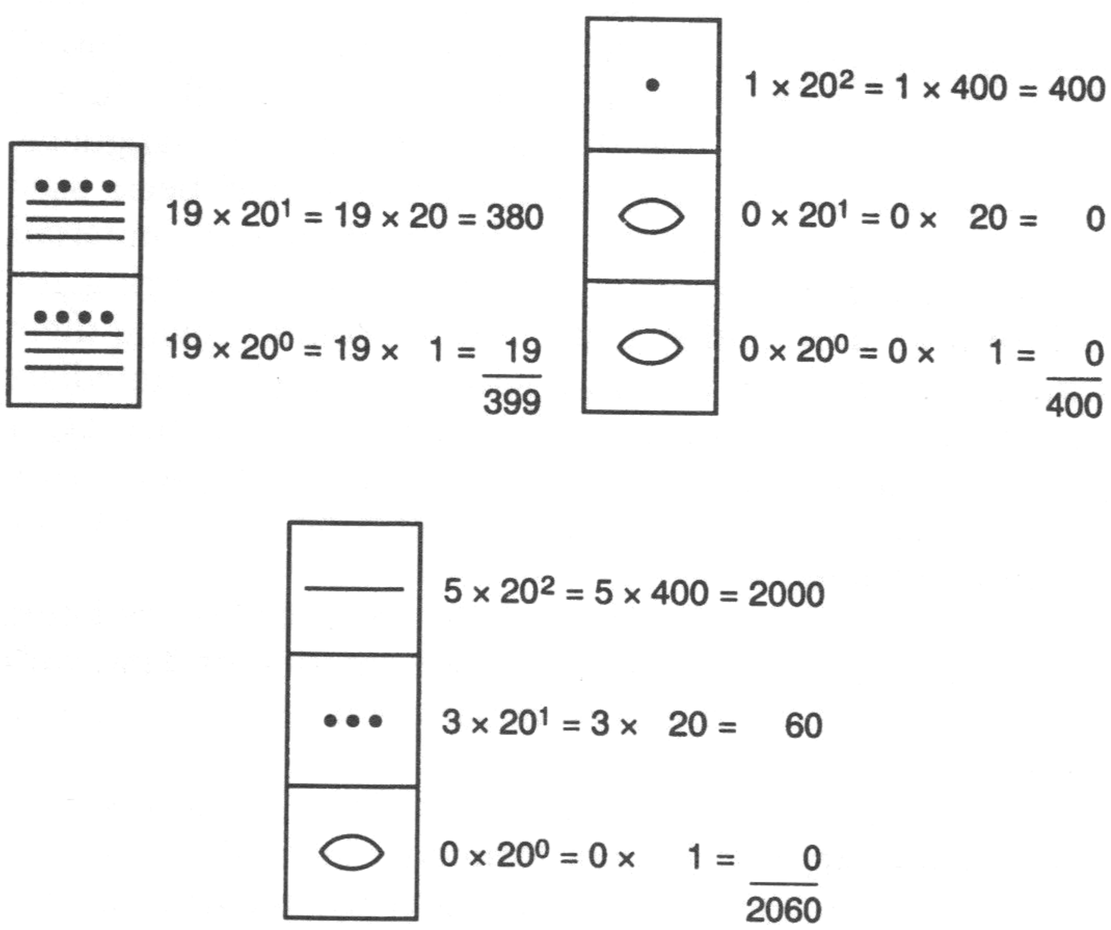

Fig. 2

The Sum and the Product

We shall show the way the Maya compute the sum and the product. As we already mentioned, the Maya used a simple table consisting of columns and rows, in which they could easily accumulate beans, straws and shells. Now, let us see how they proceeded.

The sum procedure (Bux-Xoci): First, write each quantity in one column; then, put all the dots (beans) and the bars (straws) together in the first cell of each row. Finally, delete every four bars in the same cell and substitute them by a dot in the above cell. The result is given by the quantity obtained in the first column (see Figure 3).

The product procedure (Dzaac-Xox): Write the two factors A and B outside the table. One in the left-hand-side by putting each numeral in one row (in a vertical order as used by the Maya) and the other in the above part by putting each numeral in one column (in a horizontal order as we do it in our system base 10). Now, multiply each numeral of A by each numeral of B. Write down the partial result in the cell which is in the intersection of the corresponding row and column. Substitute every four bars of each partial result by a dot in the above cell. For each diagonal (from left to right) of the table, proceed to add up the quantities of the corresponding cells (here again substitute every four bars of each diagonal sum by a dot in the above diagonal sum); the numerals obtained by those quantities give the final result (see Figure 4).

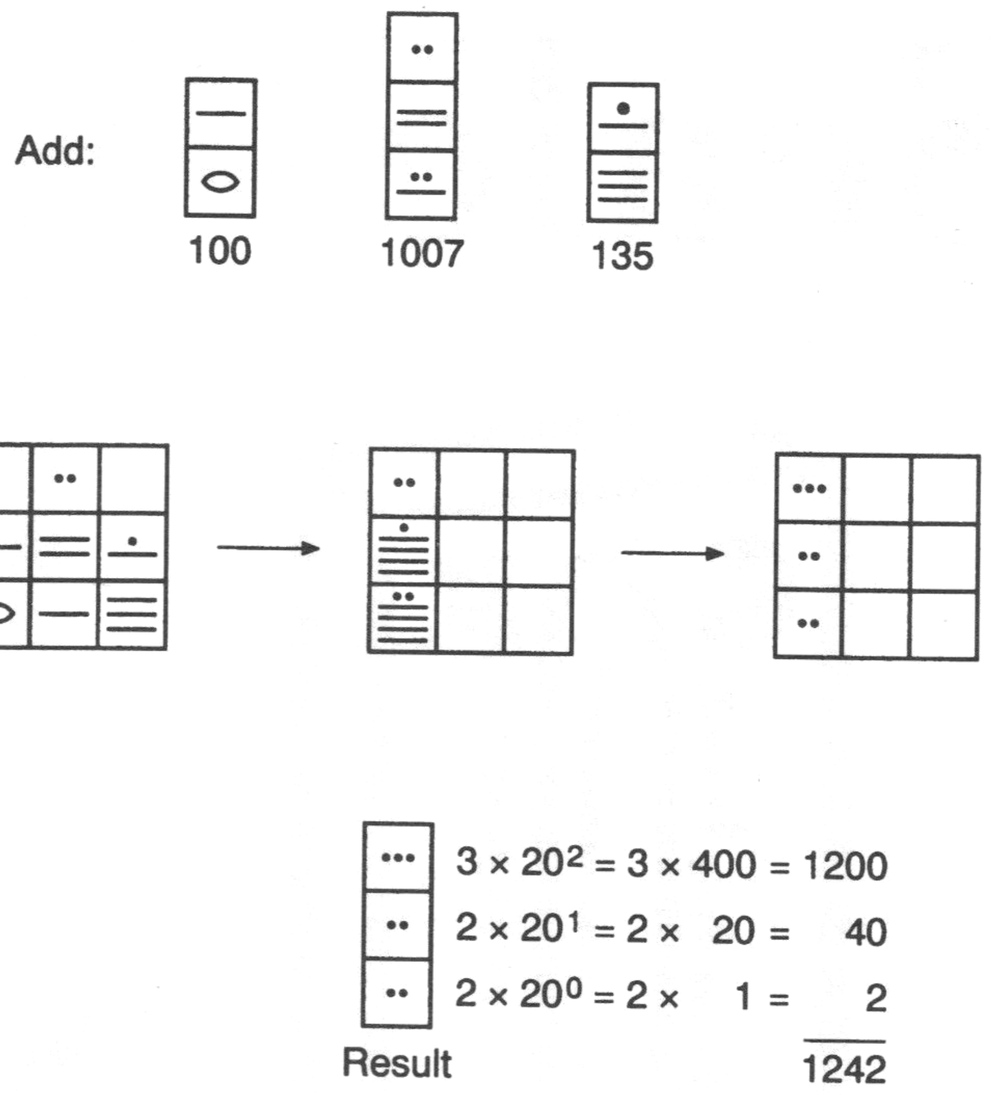

Fig. 3

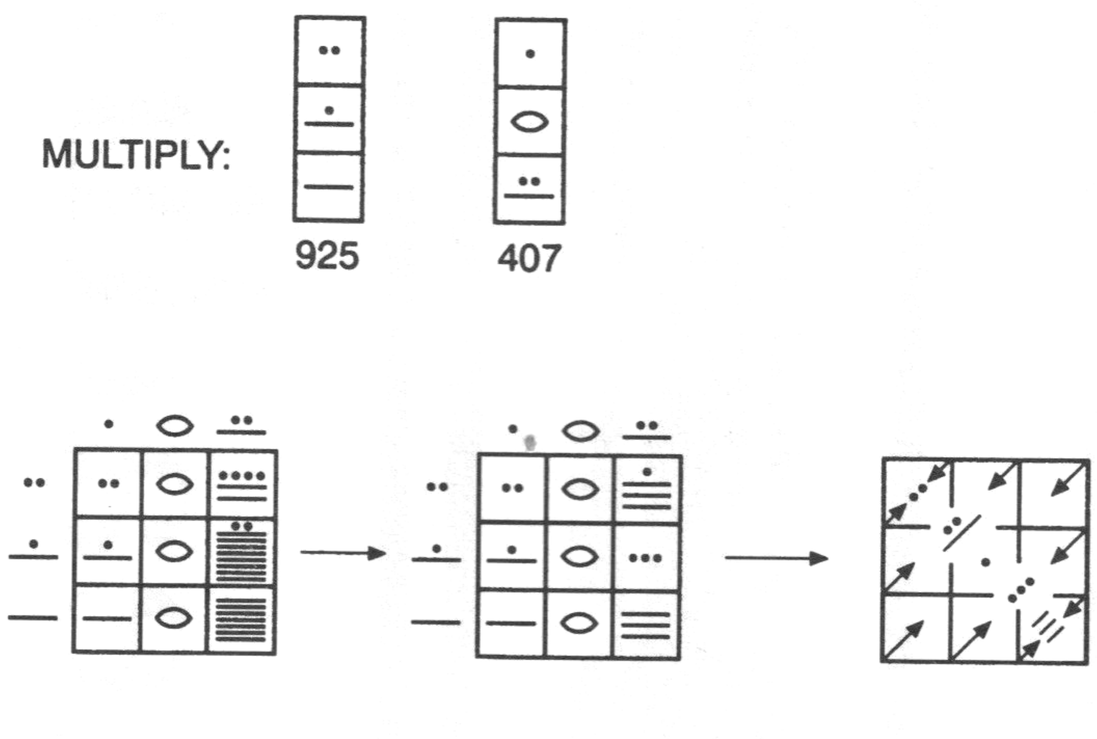

Fig. 4

Here we are, the ancient Maya made use of a simple but efficient arithmetic for the first time in the history of human life. As S. Morley said: *Some time during the fourth or third century before Christ, the Maya priests for the first time in the history of the human race, devised a system of numeration by position, involving the conception and use of the mathematical quantity of zero, a tremendous abstract intellectual accomplishment.*

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