

About Michel's conjectures and others

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Combinatorial Geometries: matroids, oriented matroids and
applications

Marseille-Luminy, April 3, 2013

- Combinatorics and geometry : oriented matroids, 1991 (M. Las Vergnas, G. Ziegler)



- Combinatorics and geometry : oriented matroids, 1999 (R. Cordovil, M. Las Vergnas, R.A.)

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- Combinatorics and geometry : oriented matroids, 1999 (R. Cordovil, M. Las Vergnas, R.A.)
- Combinatorial geometries and applications : oriented matroids and matroids, 2005 (R. Cordovil, M. Las Vergnas, R.A.)
- Combinatorial geometries : matroids, oriented matroids and applications, 2013 (E. Gioan, R.A.)

Number of entries at MathSciNet : 87

Sole author : 46

Co-author : 41 (where 19 one of the co-authors is a former PhD student)

B. Acharya

D. Forge

C. Berge

J.-C. Fournier

J.-C. Bermond

E. Gioan

R.G. Bland

A. Guedes de Oliveira

F. Bry

Y. Hamidoune

R.Cordovil

P. Hansen

B. Devadas

A. Mandel

P. Duchet

H. Meyniel

G. Etienne

P. Schuchert

Related to graphs : 31

First published paper on graphs : M. Las Vergnas, Une propriété forte de connexité en théorie des graphes *C. R. Acad. Sci. Paris Sér. A-B*, **266** (1968), A561-A563

Related to matroids : 56

First published paper on matroids : M. Las Vergnas, Sur un théorème de Rado *C. R. Acad. Sci. Paris Sér. A-B*, **270** (1970), A733-A735

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- Matroids
- Oriented matroids
- Tutte polynomial

Some subjects :

- Duality
- Principal extensions
- Products on matroids
- Binary matroides
- Activities of elements in bases
- Orientability
- Signature of bases
- Convexity and applications
- Shannon switching game
- Linear programming
- Tutte polynomial (special evaluations)
- Polynomials associated to morphism of matroids
- Generalization to 3 variables polynomial

Las Vergnas Cube conjecture

M. Las Vergnas, J.-P. Roudneff, I Saläun, Regular polytopes, preprint 1989, 12pp., unpublished.

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Theorem (J. Bokowski, A. Guedes de Oliveira, U. Thiemann, A. Veloso da Costa 1996) Las Vergnas conjecture has positive answer for $n \leq 7$.

Las Vergnas simplex conjecture

M. Las Vergnas, Convexity on oriented matroids, *J. ComB. Th. Ser B* **29** (1980), 231-243.

Conjecture (Las Vergnas 1980) Every loopless oriented matroid has at least one acyclic reorientation with exactly r extreme points.

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Theorem (Shannon 1979) If M is realizable then there exists at least n simplexes.

Tutte polynomial

Michel Las Vergnas, Extensions normales d'un matroïdes, polynôme de Tutte d'un morphisme, *C.R. Acad. Sci. Paris Sér. A*, **280** (1975), A1479-A1482

Tutte polynomial

Michel Las Vergnas, Extensions normales d'un matroïdes, polynôme de Tutte d'un morphisme, *C.R. Acad. Sci. Paris Sér. A*, **280** (1975), A1479-A1482

Introduced a 3-variable polynomial associated with two matroids M and M' related by a **matroid perspective** (every circuit of M is the union of circuits of M').

$$t(M, M'; x, y, z) = \sum_{X \subseteq E} (x-1)^{r(M')-r_{M'}(X)} (y-1)^{|X|-r_M(X)} z^{r(M)-r(M')-(r_M(X)-r_{M'}(X))}$$

Tutte polynomial

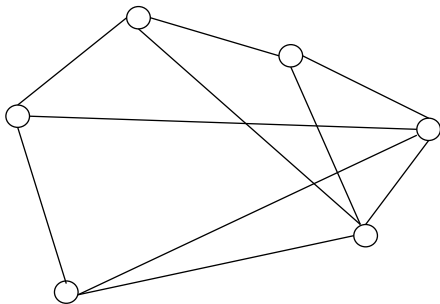
Michel Las Vergnas, Extensions normales d'un matroïdes, polynôme de Tutte d'un morphisme, *C.R. Acad. Sci. Paris Sér. A*, **280** (1975), A1479-A1482

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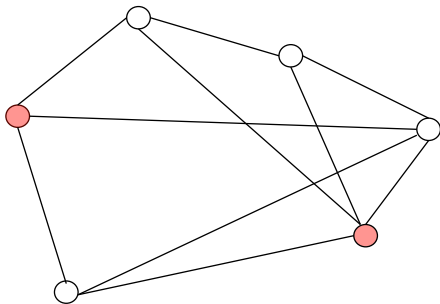
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If $M = M'$ then $t(M, M'; x, y, z) = t(M; x, y)$.

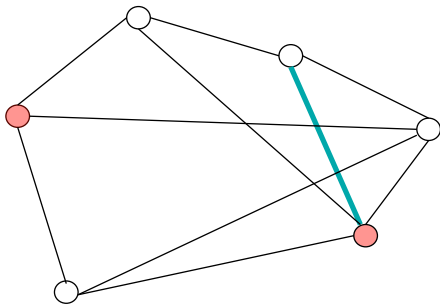
Shannon switching game



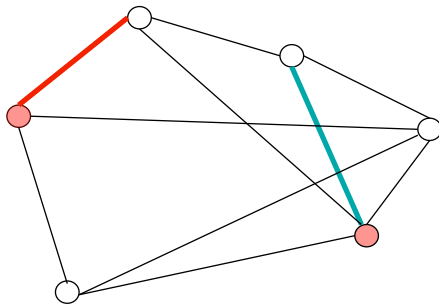
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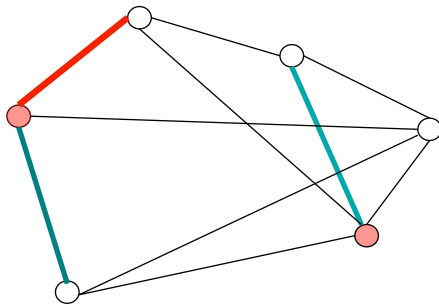
Shannon switching game



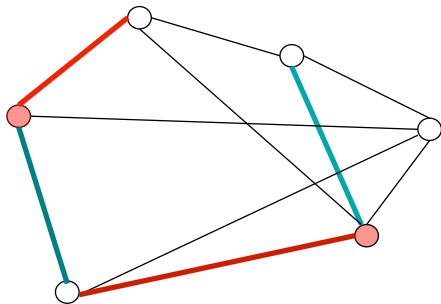
Shannon switching game



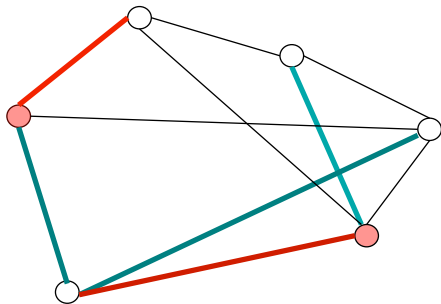
Shannon switching game



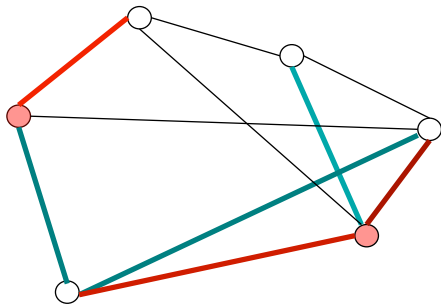
Shannon switching game



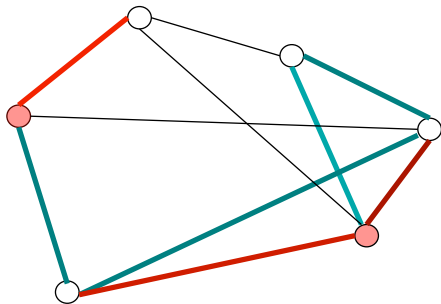
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Shannon switching game



Shannon switching game



Shannon switching game

Theorem (Lehman 1964) Solution of Shannon switching game in a wider context of matroids

- (1) polynomial characterizations of winning and losing games and polynomial strategies for graphs and matroids
- (2) Any position can be decided in polynomial time

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Introduced directed version of Shannon game (**directed** path).

They proved that White has a winning strategy if and only if there exist two edge-disjoint trees on a same set of vertices containing the two distinguished vertices (however the strategy is more elaborated).

Shannon switching game

MICHEL'S QUESTION :

Can the positional version be decided in polynomial time?

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Introduced the arborescence game (a spanning arborescence rooted at a given vertex).

Shannon switching game

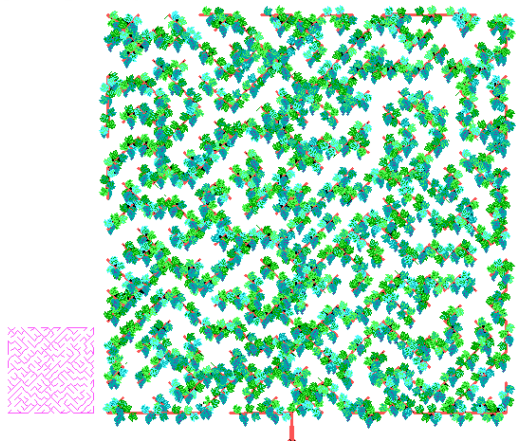
MICHEL'S QUESTION :

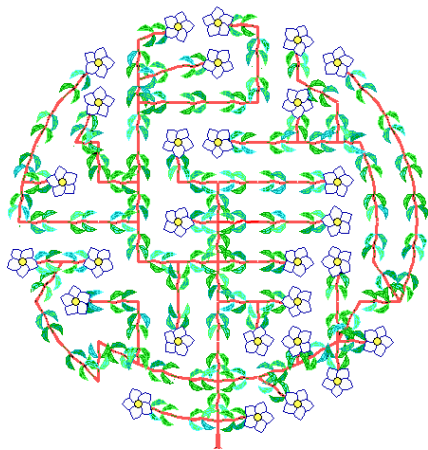
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Introduced the arborescence game (a spanning arborescence rooted at a given vertex).

They characterized winning positions when the graph is a disjoint union of two spanning trees.





Vals, Ariège



Vals Church



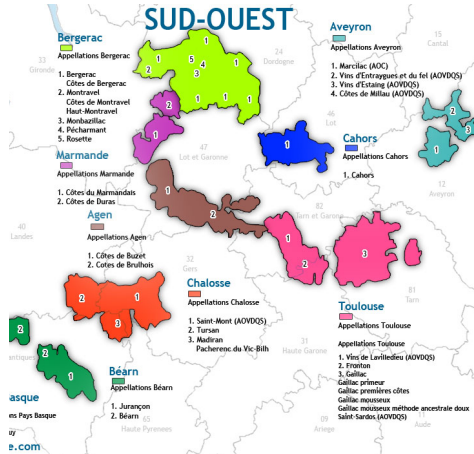
Confit de canard



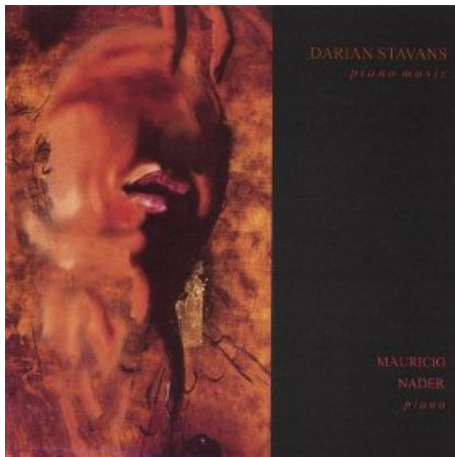
Cassoulet



Vin : Cahors et Madiran



Compositor : Darian Stavans, Piano player : Mauricio Nader



MERCI

