Design of optimal maintenance strategies using piecewise deterministic Markov processes

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Outline

Maintenance optimization

Equipments

- with several components
- subject to random degradation and failures

Maintenance optimization problem: find some optimal balance between

- repairing/changing components too often
- do nothing and wait for the total failure of the system

Optimize some criterion

- minimize a cost: repair, maintenance, unavailability penalty, failure penalty, ...
- maximize a reward: availability, production, ...

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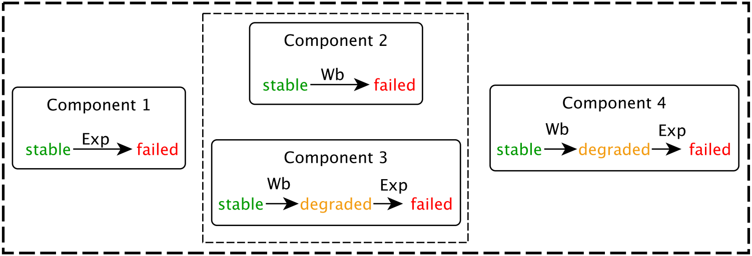
Equipment model

Typical model with 4 components

- Component 1: 2 states – stable $\xrightarrow{\text{Exponential}}$ failed
- Component 2: 2 states – stable $\xrightarrow{\text{Weibull}}$ failed
- Components 3 and 4: 3 states

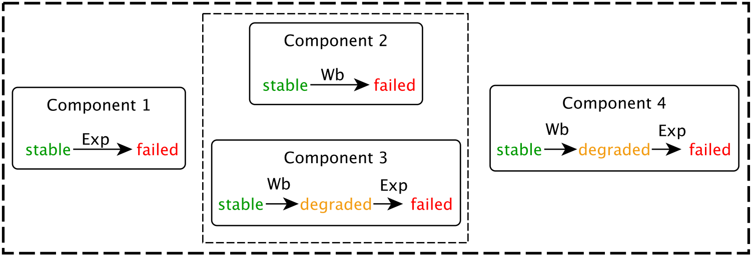
stable $\xrightarrow{\text{Weibull}}$ degraded $\xrightarrow{\text{Exponential}}$ failed



Maintenance operations

Possible maintenance operations

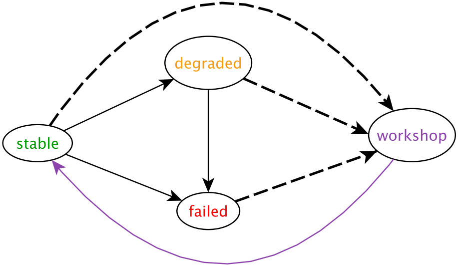
- All components, all states: do nothing
- Components 1 and 2: change in failed state
- Components 3 and 4: change in degraded or failed states, repair only in degraded state



Criterion to optimize

Minimize the **unavailability + maintenance** costs

- unavailability cost proportional to time spend in failed state
- fixed cost for going to the workshop + repair < change costs



Aim and roadmap

Objective

- design a feasible maintenance policy that minimizes the average unavailability + maintenance costs

Resolution roadmap

- model the system dynamics as a Piecewise deterministic Markov process (PDMP) and the maintenance optimization problem as an impulse control problem for PDMPs
- implement a simulation-based algorithm to solve the impulse control problem

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PDMP model of the equipment

- Euclidean variables: 5 time variables
  - functioning time of components 2, 3 and 4
  - calendar time
  - time spent in the workshop

- Discrete variables: 225 modes
  - state of the components / maintenance operations

Exact simulation of the PDMP for reference policies

Implementation of an exact simulator for reference policies to serve as benchmark

- Policy 1: do nothing
- Policy 2: send equipment to workshop 1 day after failure, repair all degraded components, change all failed ones
- Policy 3: send equipment to workshop 1 day after degradation, repair all degraded components, change all failed ones

Policy	1	2	3
Mean cost	18140	13060	10270

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Approximation of the optimal cost

Algorithm from [dSD 12]: many parameters to tune

- Number of point in the discretized spaces : one different grid at each jump time of the process
- Number of iterations of the algorithm $\simeq$ allowed number of jumps + interventions
- Time discretization step $\simeq$ minimum lag between interventions

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Discretizations

- replace the continuous state space by a finite one using optimal quantization
- select a finite number of starting points after a maintenance operation
- compromise between precision and complexity

Number of starting points after an intervention – Tests on policy 3

Grid	Number of points	relative error
$3 \times 3 \times 3 \times 5$	246	0.1034
$4 \times 4 \times 4 \times 5$	331	0.0241
$5 \times 5 \times 5 \times 5$	592	0.0062
$3 \times 3 \times 3 \times 11$	615	0.0341
$4 \times 4 \times 4 \times 11$	923	0.0819
$5 \times 5 \times 5 \times 11$	1855	0.0186
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$4 \times 4 \times 4 \times 21$	1899	0.0170
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Number of allowed jumps + interventions

Number of iterations

- up to 5 for Policy 1
- up to 30 for Policy 2 (mean 6)
- up to 25 for Policy 3 (mean 6)

	Pol. 1	Pol. 2	Pol. 3	
Mean cost	18140	13060	10270	
Number of iterations	6	10	15	20
Approximation of the minimal cost	10140	7340	7190	7160

30% relative gain compared to Policy 3 (intervention gap: 11 days)

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Construction of an optimal policy

- (approximate) optimal operations at each point of the discretized space and each time step can be obtained as a by-product of the computation of the optimal cost
- Simulation of optimally controlled trajectories: after each jump
  1. project the true value onto the corresponding quantization grid
  2. retrieve the optimal intervention date and corresponding action
  3. if no natural jump occurs before the set intervention date, perform intervention at set date
  4. otherwise allow natural jump and go back to step 1
- evaluate cost through Monte Carlo simulations

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Performance of the optimal policy

- simulation of optimally controlled trajectories
  $\Rightarrow$ optimal to do nothing if short time left as unavailability cost $\ll$ repair/replace costs
- Policy 4: same as Policy 3 but do nothing if short time left

	Pol. 1	Pol. 2	Pol. 3	Pol. 4	Optimal
Mean cost	18140	13060	10270	7560	7160

only 5% relative gain compared to Policy 4: within the approximation error

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Performance of the optimal policy – new parameters

Divide by 1000 the maintenance cost so that maintenance and unavailability costs are of the same magnitude

	Pol. 1	Pol. 2	Pol. 3	Value function	Optimal
Mean cost	18140	28.26	26.54	17.64	18.27

- no apparent structure in the optimal policy
- mean number of visits to the workshop decreased by 7%
- mean unavailability cost decreased by 42%

An example of optimally controlled trajectory

- starting point: (s,s,s,s) at calendar time 0 prop: never
- natural jump: (s,s,d,s) at 1314.43 prop: in 1 day
- no jump before planned intervention date: impulse: (s,s,r,s) at 1315.4 out of workshop (s,s,s,s) at 1350.43 prop: never
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Conclusion

Numerical method to derive a feasible optimal strategy

- rigorously validated, with general error bounds for the approximation of the value function [dSD 12, dSDG 17]
- numerically demanding but viable in low dimensional examples
- consistent results on the use case with different parameter values

References

[CD 89] **O. Costa, M. Davis** *Impulse control of piecewise-deterministic processes*
[Davis 93] **M. Davis**, *Markov models and optimization*
[dSD 12] **B. de Saporta, F. Dufour** *Numerical method for impulse control of piecewise deterministic Markov processes*
[dSDG 17] **B. de Saporta, F. Dufour, A. Geeraert** *Optimal strategies for impulse control of piecewise deterministic Markov processes*
[dSDZ 14] **B. de Saporta, F. Dufour, H. Zhang** *Numerical methods for simulation and optimization of PDMPs: application to reliability*
[P 98] **G. Pagès** *A space quantization method for numerical integration*
[PPP 04] **G. Pagès, H. Pham, J. Printems** *An optimal Markovian quantization algorithm for multi-dimensional stochastic control problems*