Semicontinuity properties of Kazhdan-Lusztig cells

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- **Involution:** $\overline{v^{\gamma}} = v^{-\gamma}$, $\overline{T}_w = T_{w^{-1}}^{-1}$

Theorem (Kazhdan-Lusztig 1979, Lusztig 1983)

If $w \in W$, there exists a unique $C_w \in \mathcal{H}$ such that

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$$C_s = \begin{cases} T_s + v^{-\varphi(s)} & \text{if } \varphi(s) > 0 \\ T_s & \text{if } \varphi(s) = 0 \\ T_s - v^{\varphi(s)} & \text{if } \varphi(s) < 0 \end{cases}$$

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- By construction, $I_{\leqslant_L \mathcal{C}}$ and $I_{<_L \mathcal{C}}$ are left ideals of $\mathcal{H}$ and $V_{\mathcal{C}}$ is a left $\mathcal{H}$-module: $V_{\mathcal{C}}$ is called the **left cell representation** associated to $\mathcal{C}$.

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However, the preorder $\leqslant_L$ or $\leqslant_R$ is in general unknown (even in the symmetric group). The preorder $\leqslant_{LR}$ seems to be easier (for instance, it is given by the dominance order on partitions through the Robinson-Schensted correspondence).

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$\mathcal{H}(W, S, \varphi) = W_\varphi \ltimes \mathcal{H}(\tilde{W}, \tilde{I}, \tilde{\varphi})$, where $\tilde{\varphi}(wtw^{-1}) = \varphi(t)$ ($w \in W_\varphi$, $t \in I$).

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Corollary

Since $C_s = T_s$ and $C_{sw} = C_s C_w$ for all $s \in S_\varphi$ and $w \in W$, the left cells of (W, S, φ) are of the form $W_\varphi \cdot \mathcal{C}$, where $\mathcal{C}$ is a left cell of $(\tilde{W}, \tilde{I}, \tilde{\varphi})$.

Examples

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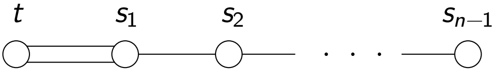

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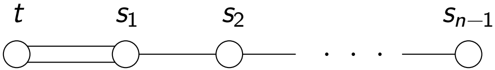

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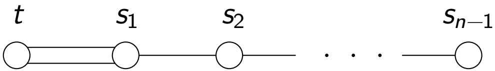

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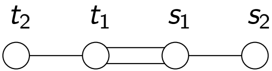

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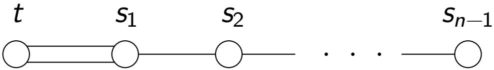

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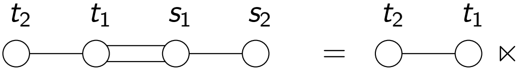

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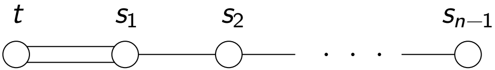
- Type B



$$W(B_n) = \mathfrak{S}_n \times (\mathbb{Z}/2\mathbb{Z})^n = \langle t \rangle \ltimes W(D_n)$$

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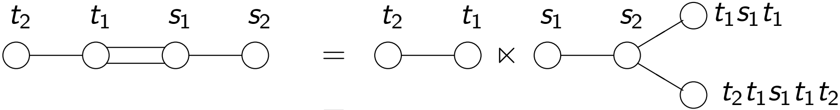

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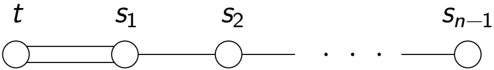

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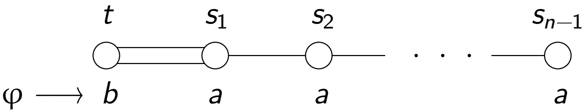

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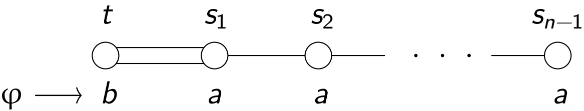

We identify W_n with the group of permutations w of $I_n = \{\pm 1, \pm 2, \ldots, \pm n\}$ such that $w(-i) = -w(i)$ through

$$t \mapsto (-1, 1) \qquad \text{and} \qquad s_i \mapsto (i, i+1)(-i, -i-1)$$

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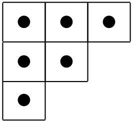

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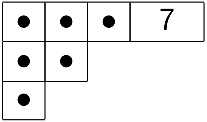

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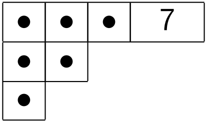

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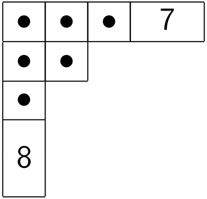

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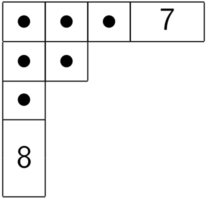

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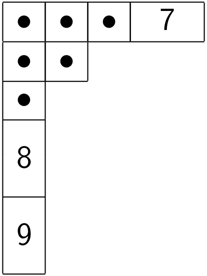

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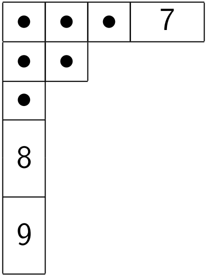

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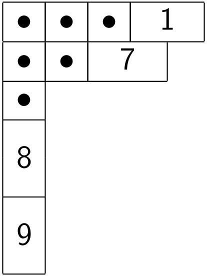

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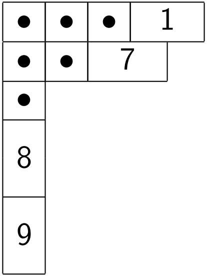

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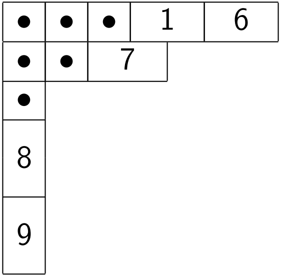

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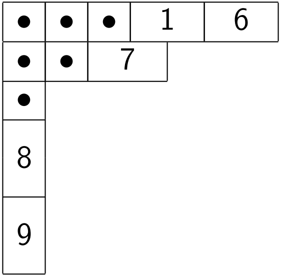

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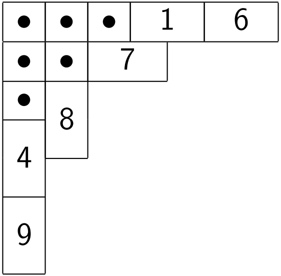

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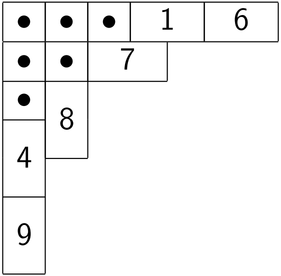

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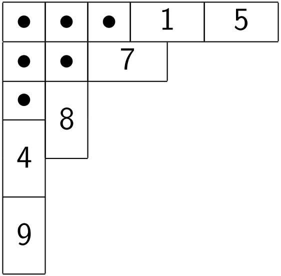

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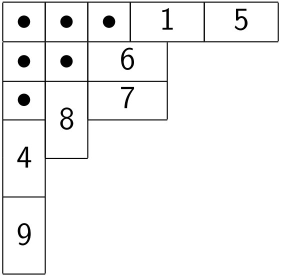

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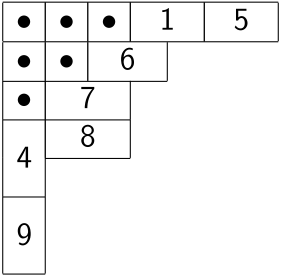

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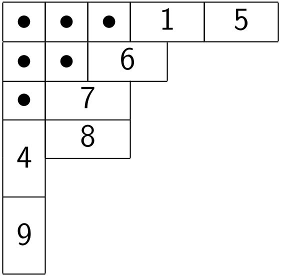

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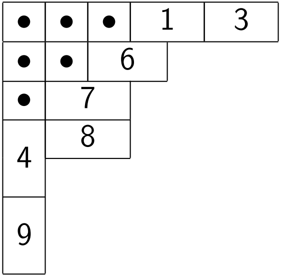

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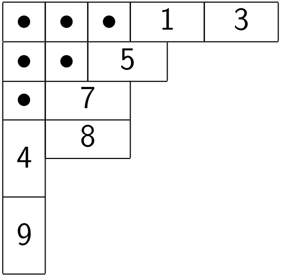

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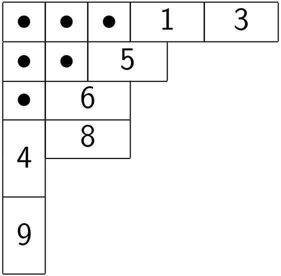

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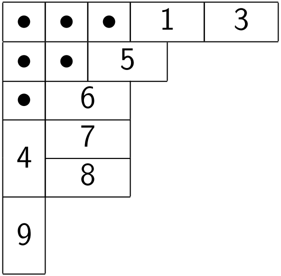

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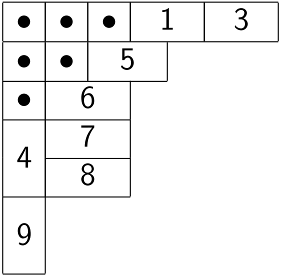

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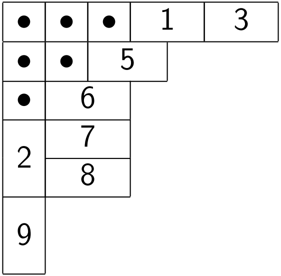

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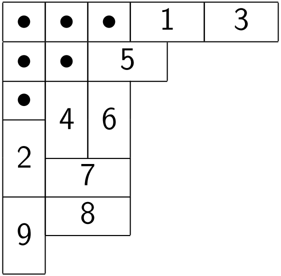

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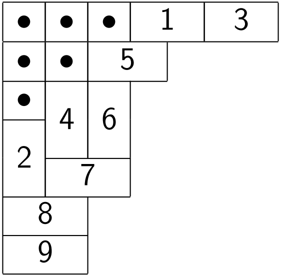

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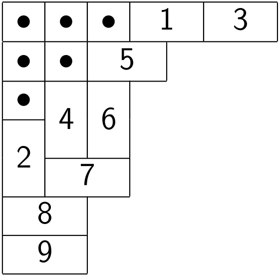

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Theorem (Garfinkle, van Leeuwen)

$$\begin{array}{ccc} W_n & \xrightarrow{\sim} & SDT_r^{(2)}(n) \\ w & \longmapsto & (D_r(w), D_r(w^{-1})) \end{array}$$

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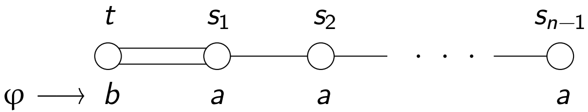

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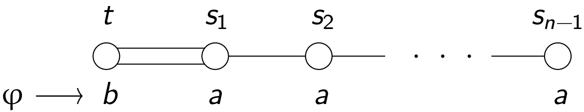

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Assume a, $b > 0$ and assume that $0 \leqslant r < b/a < r+1$. Then:

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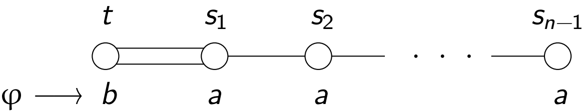

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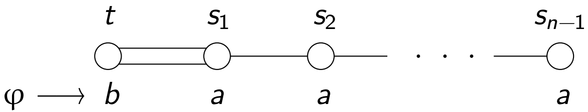

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Conjecture B (Geck-Iancu-Lam-B. 2003)

Let $r \geqslant 1$ and assume that $b = ra$.

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Conjecture C (maybe only for finite or affine Weyl groups)

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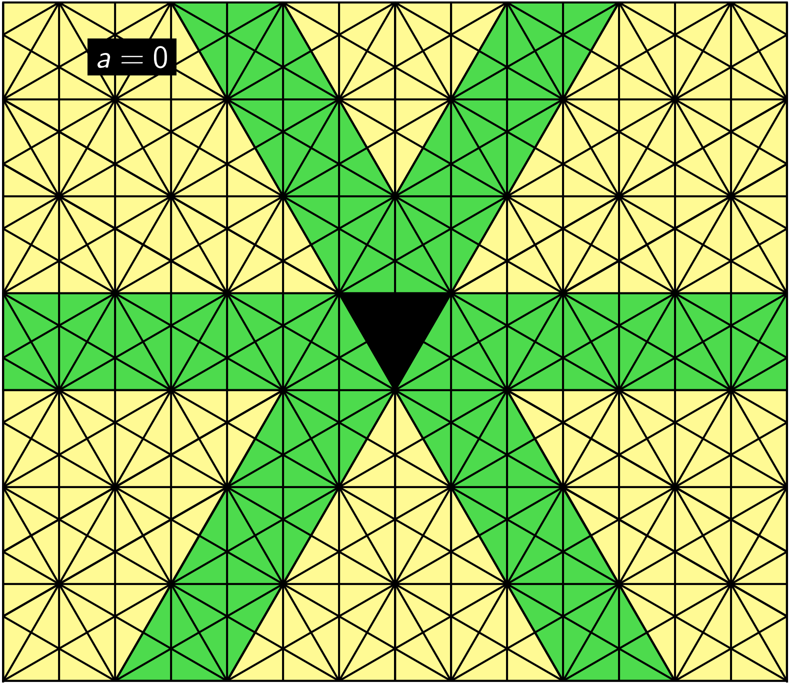
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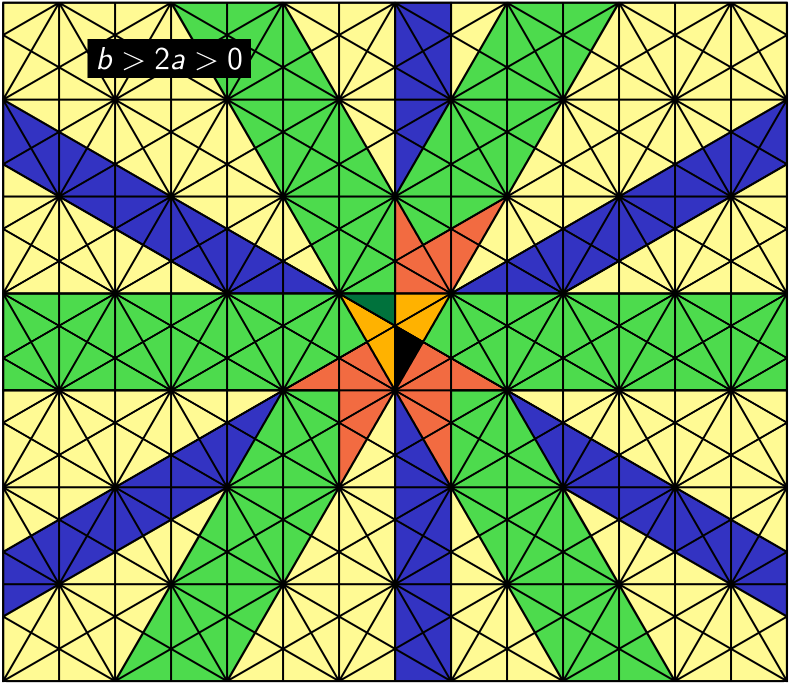
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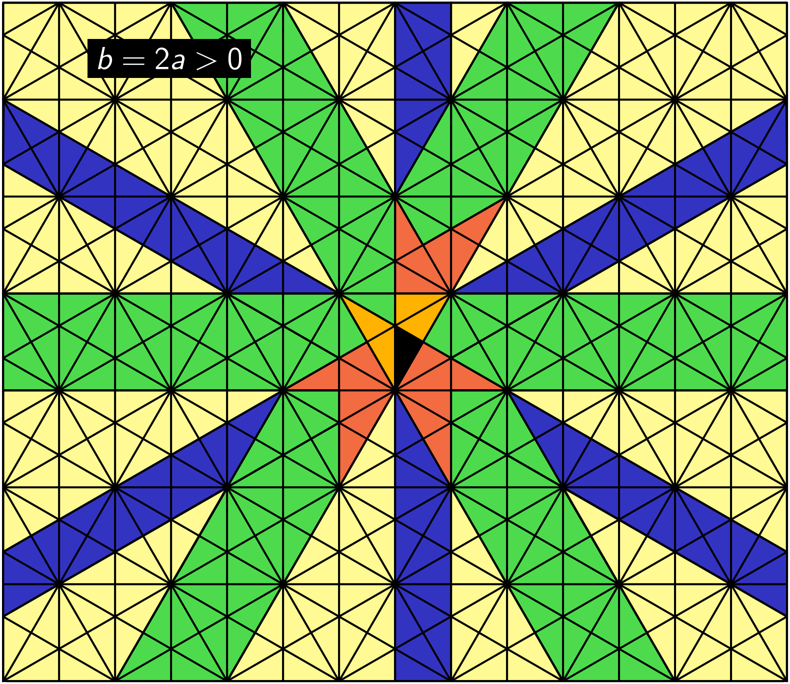
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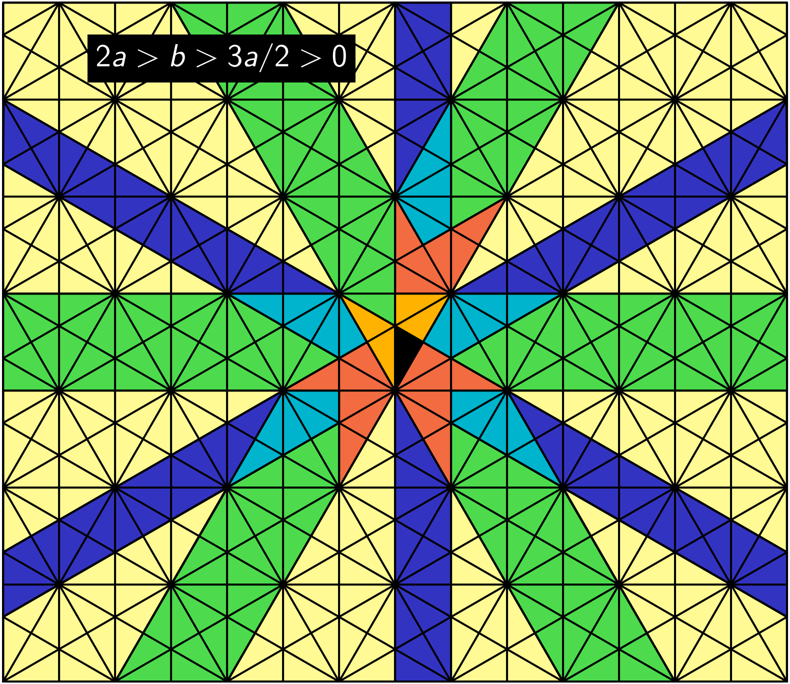
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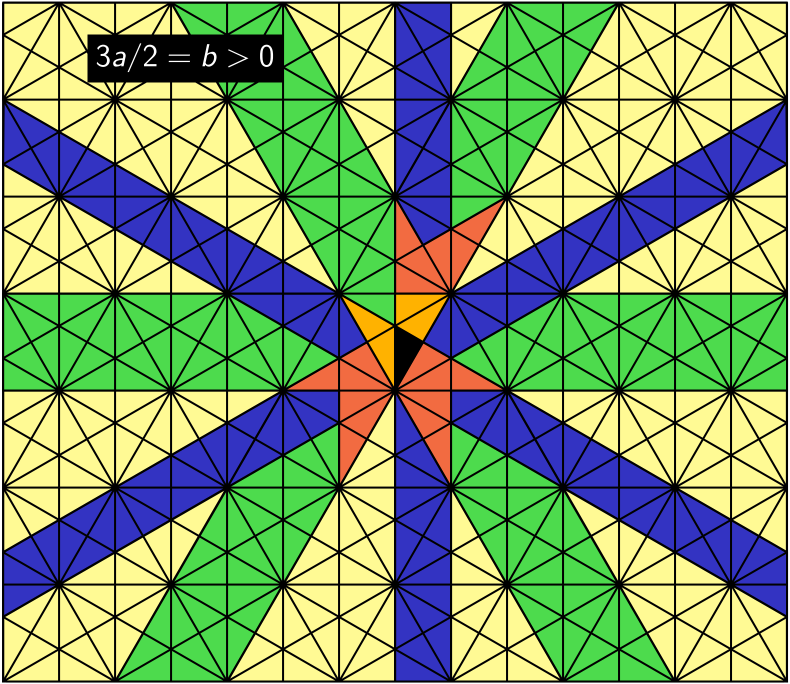

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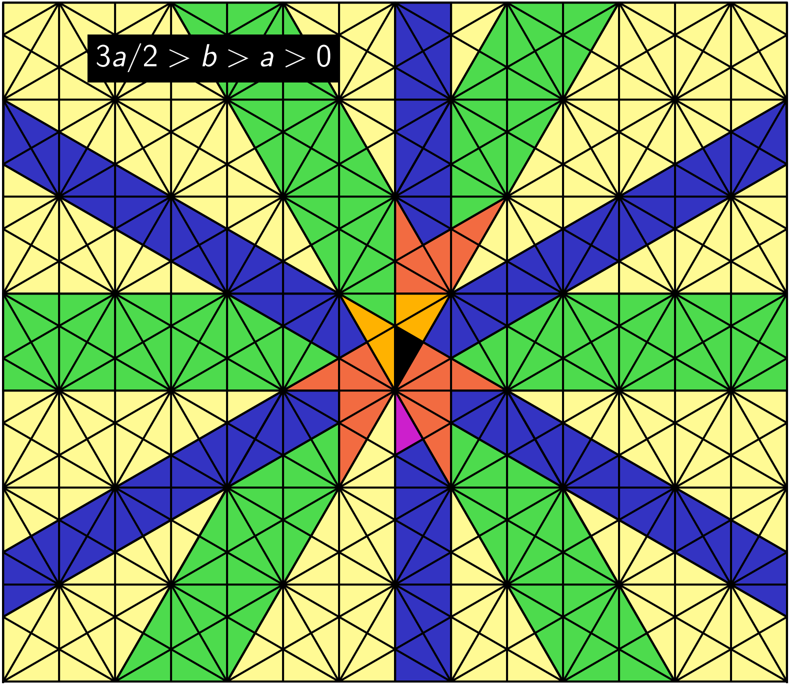

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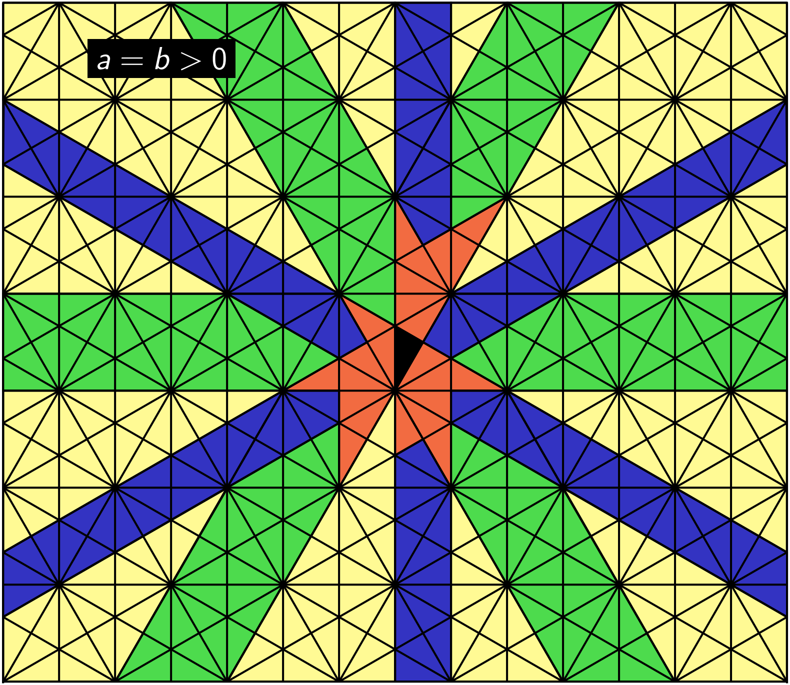
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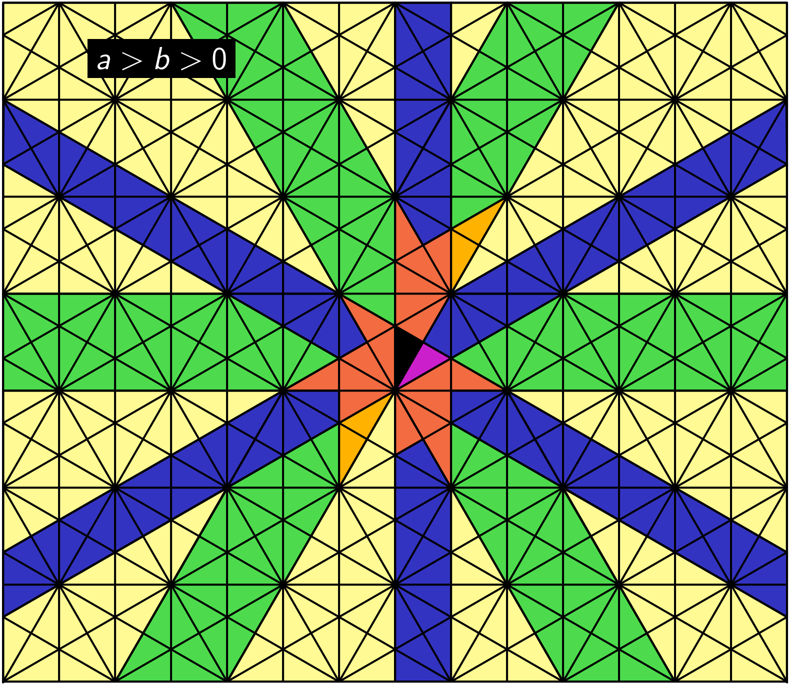
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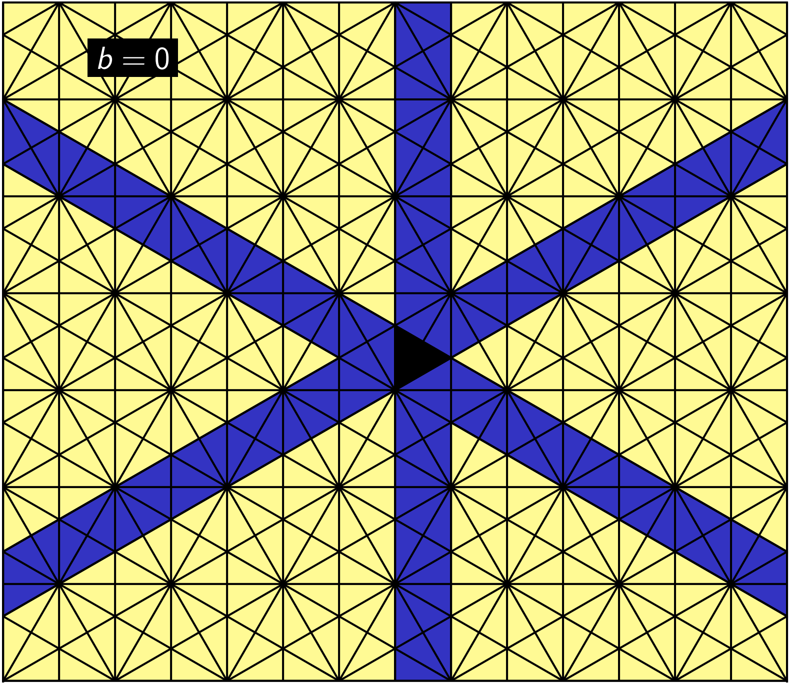
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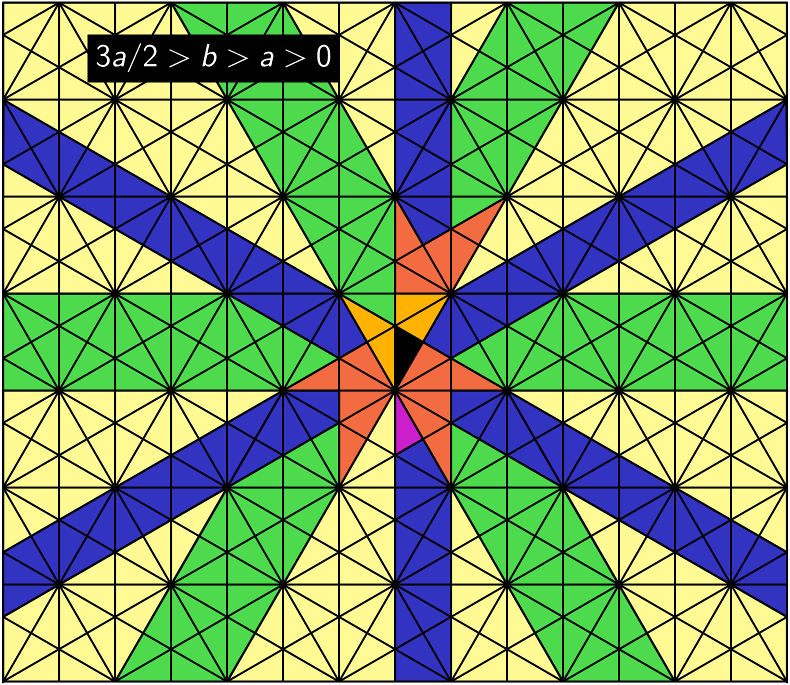
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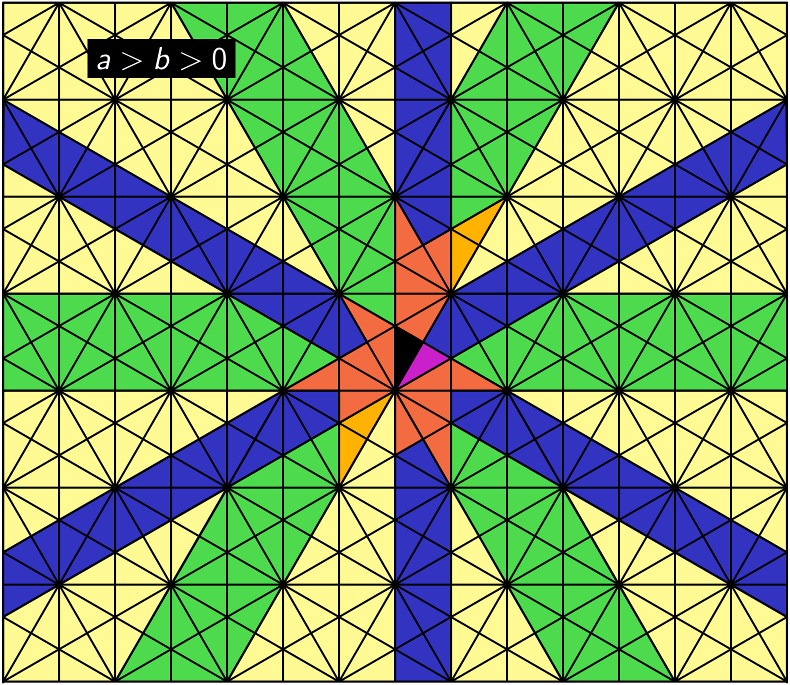
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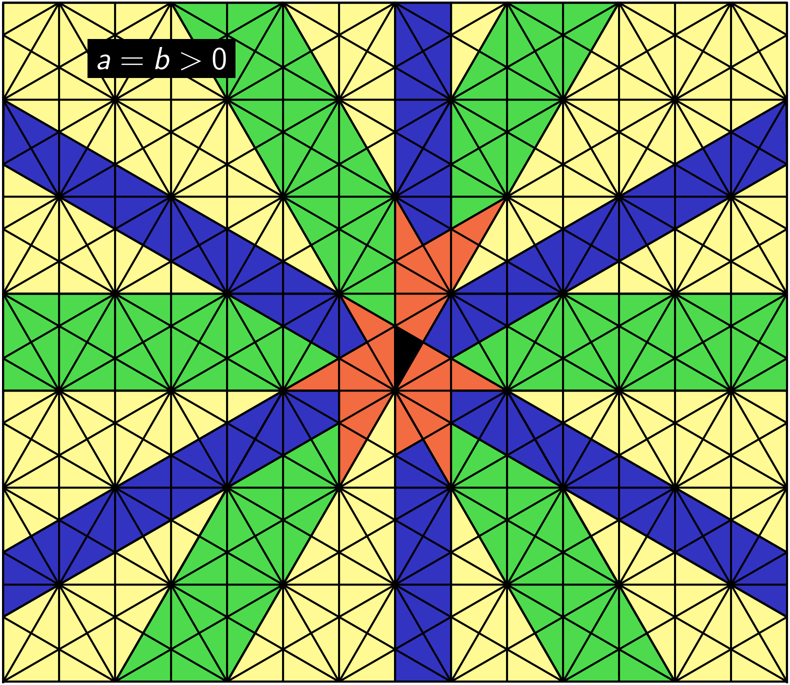
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