Introduction to Deligne-Lusztig Theory

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(3) $\{1_G, \mathrm{St}_G\} \cup \{R'_\eta \mid \eta \in S^\wedge, \eta \neq 1\}$: principal block (defect S).

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Curiosities

- **Abhyankar's conjecture (Raynaud's Theorem):** *A finite group* Γ *is the Galois group of a Galois étale covering of* $\mathbf{A}^1(\mathbb{F})$ *if and only if it is generated by its Sylow p-subgroups.*

  Example: $\Gamma = \mathbf{SL}_2(\mathbb{F}_q)$,

$$\begin{array}{ccc} \mathbf{Y} & \longrightarrow & \mathbf{A}^1(\mathbb{F}) \\ (x, y) & \longmapsto & xy^{q^2} - yx^{q^2} \end{array}$$

- $q = 7$, $\mathbf{Y}/\{\pm 1\}$ is acted on by $\mathbf{PSL}_2(\mathbb{F}_7) \simeq \mathbf{GL}_3(\mathbb{F}_2)$: it is the reduction modulo 7 of the Klein's quartic (whose group of automorphism is exactly $\mathbf{PSL}_2(\mathbb{F}_7)$, reaching Hurwitz' bound).