Coarse geometry of groups

Workshop financed by ANR PLAGE

March 30- April 03, 2026

Monday, March 30

09.00-10.20	**Genevois 1**
10.20-10.40	Coffee
10.40-12.00	**Frances 1**
	Lunch + break
17.20-18.40	**Tointon 1**

Tuesday, March 31

09.00-10.20	**Frances 2**
10.20-10.40	Coffee
10.40-12.00	**Tointon 2**
	Lunch + break
17.20-18.40	**Genevois 2**

Wednesday, April 1

09.00-10.20	**Mackay 1**
10.20-10.40	Coffee
10.40-12.00	**Bensaid 1**
	Lunch + break
18.45-19.35	**Krutoy**

Thursday, April 2

09.00-10.20	**Pallier 1**
10.20-10.40	Coffee
10.40-12.00	**Bensaid 2**
	Lunch + break
17.20-18.40	**Mackay 2**
18.45-19.35	**Patil**

Friday, April 3

09.00-10.20	**Azuelos**
10.20-10.40	Coffee
10.40-12.00	**Pallier 2**

Titres et résumé:

- **Anthony Genevois**

Title: An introduction to stringy geometry

Abstract: In this mini-course, I will explain how to coarsify the topological notion of local cut point and how this naturally leads to (relatively) stringy spaces. Loosely speaking, a space is stringy relative to a collection of subspaces if the latters are the pieces one gets when one cuts the space at its coarse local cut points. As an application of this new geometry, I will describe some consequences to the coarse geometry of lamplighter-like groups.

- **Matt Tointon**

Title: Growth and structure in vertex-transitive graphs

Abstract: This course will centre on recent sharp versions of Gromov's theorem on groups of polynomial growth, and ensuing breakthroughs in probability on vertex-transitive graphs.

Gromov's theorem famously says that if a group has *polynomial growth* then it must be *virtually nilpotent*, i.e. contain a nilpotent subgroup of finite index. This in turn has had some striking applications to probability, from Varopoulos's classical characterisation of those vertex-transitive graphs on which the random walk is *transient*, to Duminil-Copin et al's recent characterisation of those vertex-transitive graphs that exhibit a non-trivial *percolation phase transition*.

Recent breakthroughs have yielded a *finitary*, *quantitative* sharpening of Gromov's theorem. Here, q*uantitative* means that certain parameters of the conclusion (say the index or nilpotency class of a nilpotent subgroup) are bounded in terms of the polynomial growth rate. *Finitary* means that, rather than requiring asymptotic polynomial growth of balls as the radius tends to infinity, it is enough to assume a polynomial upper bound on the size of some finite set of balls. Analogously to the classical setting, this sharpening in turn leads to finitary and quantitative results in probability, such as non-triviality of a suitably defined percolation phase transition on *finite* vertex-transitive graphs, or a quantitative lower bound on the escape probability of a transient random walk on a vertex-transitive graph.

I will spend the first hour or so of this course giving an overview of both the classical and the finitary theory, and present more details on some of the probabilistic applications. I will then close the first lecture by explaining how to reduce the setting of vertex-transitive graphs to that of finitely generated groups. In the second lecture, I will paint a picture of the fine structure of groups of polynomial growth, and explain how this can be used to make the conclusion of Gromov's theorem quantitatively completely sharp.

- **Gabriel Pallier**

Title: Nilpotent Lie groups and sublinear bilipschitz equivalence

Abstract: As it turns out, the quasiisometric classification of compactly generated locally compact groups of polynomial growth reduces to that of simply connected nilpotent Lie groups. The large-scale geometry of these groups exhibits some sublinear features that we will explore in these lectures. In particular, we will see how sublinear bilipschitz equivalences (generalized quasiisometries) arise between certain pairs of simply connected nilpotent Lie groups, and reframe the quasiisometric classification problem into a quantitative perspective.

- **John Mackay**

Title: Asymptotic dimension using a variety of covers

Abstract: The asymptotic dimension of a space was introduced by Gromov in the early 1990s as a natural large-scale analogue of topological dimension. It is monotone under coarse embeddings -- in particular, it is a well-defined invariant of a finitely generated group -- and has proved very useful in a variety of applications. However, there are spaces which are not distinguished by asymptotic dimension but are nonetheless very different, such as n-dimensional Euclidean and hyperbolic spaces. In the 2000s Buyalo and Schroeder introduced a number of variations on asymptotic dimension in order to rule out certain quasi-isometric embeddings between products of hyperbolic and Euclidean spaces not obstructed by the usual asymptotic dimension. In the usual definition of asymptotic dimension, one considers covers by sets of uniformly bounded diameters; Buyalo and Schroeder instead allow different families of sets in the cover: for example, set satisfying a doubling property; in this case, Euclidean spaces would be zero dimensional, while n-dimensional hyperbolic spaces remains n dimensional. In recent work with Hume and Tessera we use control on growth of sets in covers to rule out not only quasi-isometric embeddings but also coarse embeddings in many situations. I'll discuss this story and the questions which remain.

- **Charles Frances**

Title: A coarse perspective on automorphisms of rigid geometric structures

Abstract: There have been several approaches to defining what it means for a class of geometric structures to be “rigid” (Cartan, Kobayashi, Gromov, …). Regardless of the approach chosen, a key feature of such structures is that their automorphism group is a finite-dimensional Lie group. This raises the natural question of which Lie groups may occur as (sub)groups of automorphisms. The aim of these two talks is first to review important results, mainly due to R. Zimmer, concerning the connected component of the automorphism group. In a second step, we will emphasize how tools from coarse geometry make it possible to better understand the discrete part of this group, with concrete applications to Lorentzian geometry.

- **Harsh Patil**

Title: Coarse separation of metric spaces

Abstract: A subspace S of a topological space X is said to separate it if X-S contains more than one path-component. The classical Alexander duality theorem implies that if a subset A of the n-dimensional sphere separates it, then A must be of dimension n-1. 'Coarse separation' is an analogue of topological separation in the world of metric spaces. Coarse separation arises naturally in geometric group theory. Suppose a finitely generated group G (equipped with a word metric) splits over a subgroup C, then C coarsely separates G. I will start by introducing the definition of coarse separation. Given a metric space X, the task is to find a necessary condition that every separating subset of X must satisfy. I will discuss two tools: Growth and Asymptotic dimension, which allow us to obtain such results. I will define everything and give many examples.

- **Pénélope Azuelos**

Title: Median metric groups

Abstract: Median spaces form a broad and increasingly important class of metric spaces, encompassing both CAT(0) cube complexes and real trees. Finitely generated groups which admit free transitive (or proper cocompact) actions on discrete median spaces — equivalently, on the 0-skeletons of CAT(0) cube complexes — are reasonably well understood. In contrast, much less is known about their continuous analogue: groups acting freely and transitively on connected median spaces. I will present several methods for constructing such actions, focusing on actions on real trees and their products, and discuss some of the surprising behaviours that show up. Even when considering real trees, the class of groups acting on such spaces is vastly more diverse than in the discrete setting: while any simplicial tree admits at most one free vertex transitive action, we will see that there are 2^{2^{\aleph_0}} pairwise non-isomorphic groups which admit a free transitive action on the universal real tree with continuous valence.

- **Kostyantyn Krutoy**

Title: Uniform Roe algebras: rigidity and K-theory

Abstract: Uniform Roe algebras are C*-algebras associated to uniformly locally finite (ULF) metric spaces that capture their coarse geometry. Their K-theory groups are of particular importance for variants of the Baum-Connes and Novikov conjectures. In this talk, we will introduce two homological invariants of ULF metric spaces: the 0-th uniformly finite homology group, and the K0-group of the uniform Roe algebra. We will show that these invariants are intrinsically connected to the rigidity phenomenon of uniform Roe algebras.

- **Oussama Bensaid**

Title: Right-angled Artin groups and their large-scale geometry

Abstract: Right-angled Artin groups form a rich class of groups, ranging from free abelian groups to free groups, and their large-scale geometry exhibits both rigidity and flexibility phenomena. In this minicourse, I will discuss some aspects of their large-scale geometry, with a particular focus on quasi-isometries and quasi-isometric embeddings. I will begin with higher-rank analogues of the Morse lemma for quasi-geodesics, which provide useful tools for studying quasi-isometries in non-hyperbolic settings. I will then turn to right-angled Artin groups and give a brief overview of some results and techniques concerning their quasi-isometries. In the second part, I will focus on quasi-isometric embeddings between right-angled Artin groups. This part is based on joint work with Shaked Bader and Harry Petyt.