Tristan

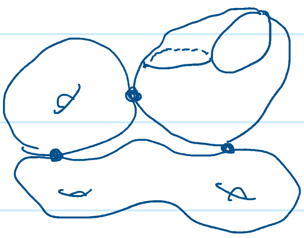
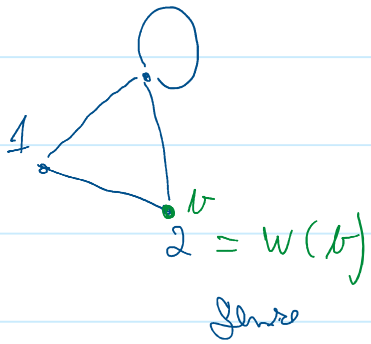
Thursday 21 April 2022 15:18

Stratification

$$\overline{M}_g = \bigsqcup_{G \text{ connexe}} M_G$$

Espace de modules de courbes stables de graphe (dual) G

Déf / Exemple :

C :  $\rightsquigarrow$ 

$$M_G = M_{1,2} \times M_{2,2} \times M_{0,4} \Big/ \mathrm{Aut}\, G$$

$$\operatorname{codim} M_G = |E|$$

C stable $\rightsquigarrow$ $\chi(v) = 2w(v) - 2 + \mathrm{val}(v) > 0 \quad \forall v \in V$

$\forall v \in V$

g_C $\qquad g_{G,w} = |E| - |V| + 1 + \sum_{v \in V} w(v)$

$M_G \subseteq \overline{M_{G'}}$ si $G' = G/e_1/e_2 \ldots$

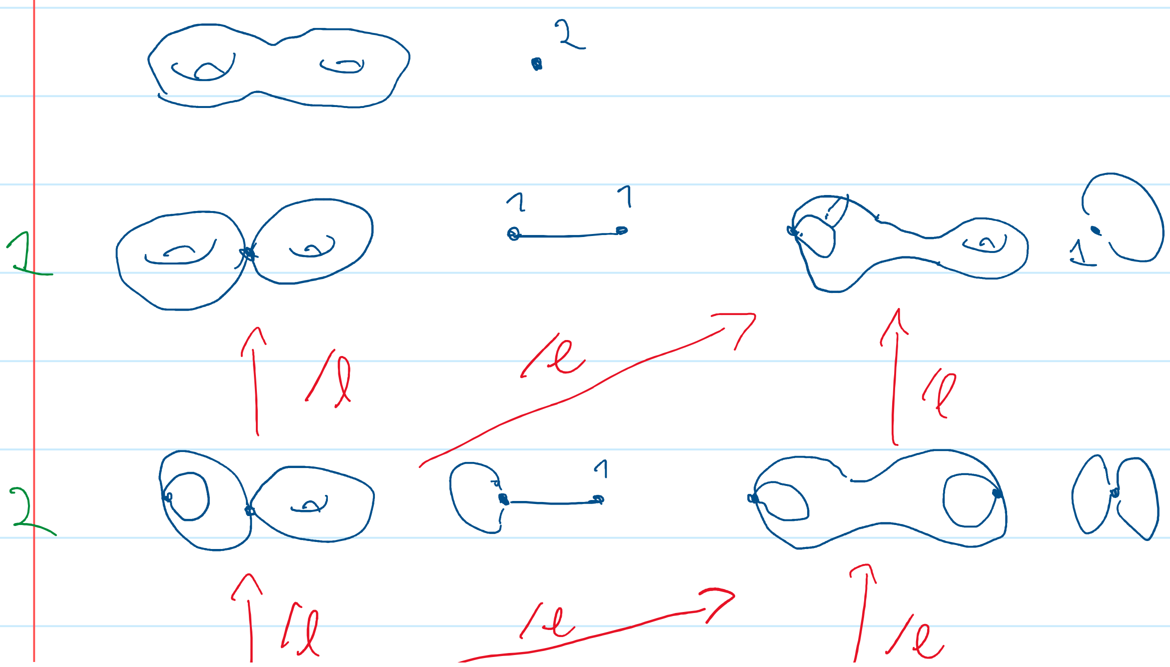
contractions successives

où $\begin{cases} w'(v) = w(v)+1 & \text{si on contracte une boucle} \\ \text{sinon additif} \end{cases}$

Ex : $g = 2$

Contrainte sur G : $\sum_v \mathrm{st}(v) = 2g-2 = 2$

codim



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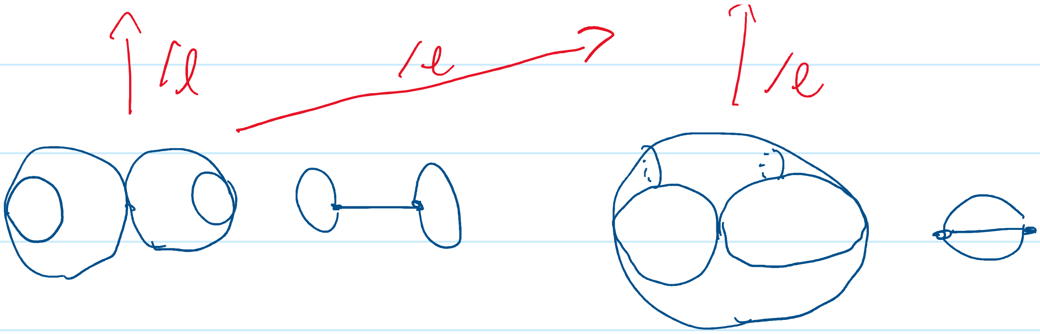
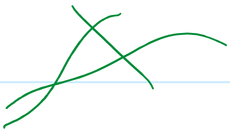


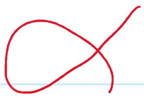
Ensuite on s'intéresse à

$$D_g = \overline{M_g}/M_g$$

diviseur à croisements normaux mais pas simple.



int transverse

pas de 

Quand un diviseur D à croisements normaux est simple, on lui associe un Δ-complexe

$\Delta(D)$: sommets = composantes irréductibles

1-faces = intersections doubles

2-faces = $\cap$ triples

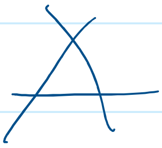
⋮

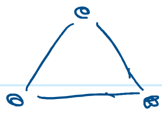
Dans ce cas, en utilisant la filtration par le poids de $H^{\bullet}$, on a une surjection

$$\left(\text{si } X \subset \overline{X}_{\dim d},\ D = \overline{X} \setminus X\right)$$

$$H^{2d-\bullet}(\underbrace{\overline{X} \setminus D}_{X}) \twoheadrightarrow \tilde{H}_{\bullet-1}(\Delta(D))$$

<u>Exemple</u> : $\overline{X} = \mathbb{P}^2_{\mathbb{C}}$, $X = (\mathbb{C}^{\times})^2$

$D =$ 

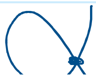
$\Delta(D) =$ 

$$\begin{array}{ccc} H^2(X) & \twoheadrightarrow & H_1(\Delta(D)) \\ \wr\| & & \wr\| \\ \mathbb{Q} & & \mathbb{Q} \end{array}$$

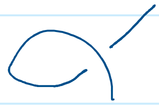
Dans notre cas, D_g pas simple :

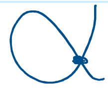
$$\{(z, b),\ z \in b\} = \tilde{D}_g \longrightarrow D_g$$

$\nearrow$ branche

branche





$$(\tilde{D}_g)_p = \{(z, b_0, \dots, b_p) \mid z \in D_g,\ z \in b_i\}$$

$$\Delta(D_g)_p := \mathrm{IrrComp}((\tilde{D}_g)_p)$$

$\Delta(D_g)$ Δ-complexe symmetrique

Def: Soit I la catégorie $[p] = \{0, .., p\}$ et toutes les injections.
Un Δ-complexe symétrique est

$$X : I^{op} \longrightarrow Sets$$

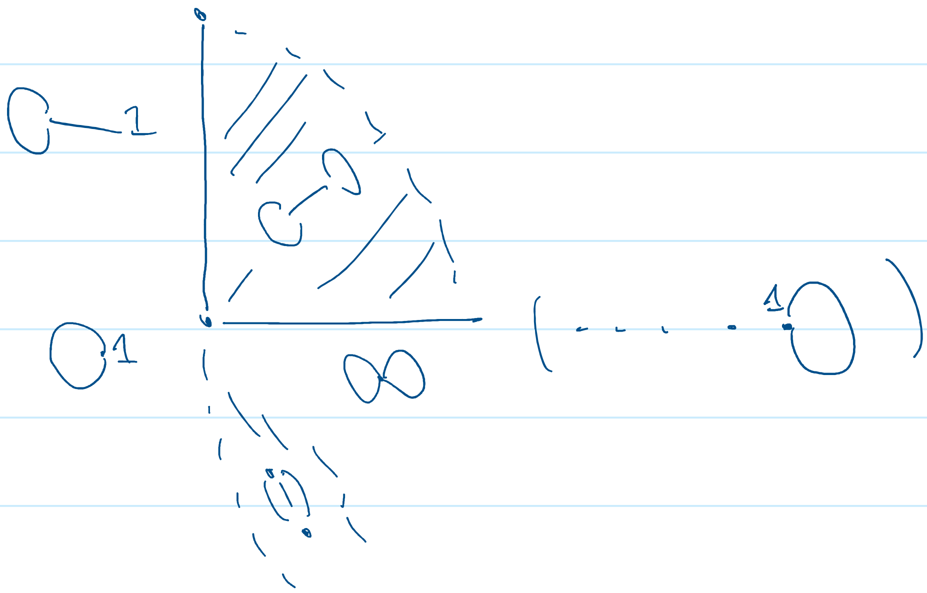
Dans notre cas,

$$(\tilde{D}_g)_p = \{X \text{ courbe stable},\ z_0, \dots, z_p \text{ noeuds sur } X\}$$

$$= \bigsqcup_{G : |E| = p+1} M_G / \ker(\mathrm{Aut}\,G \longrightarrow \mathrm{Aut}\,E)$$

$\Rightarrow \Delta(D_g)_p = \{$ graphes stables de genre g, avec $p+1$ arêtes ordonnées $\} / \sim$

<u>Ex:</u> $|\Delta(D_2)|$



contractile [Hodge ...]

$gr_6^W H^\bullet(M_2) = 0$

On a la généralisation

$H^{2d-k}(M_g) \twoheadrightarrow \tilde{H}_{k-1}(\Delta(D_g))$

avec $d = 3g-3$

$$\Delta(D_g)$$
$$\text{On a } C_\bullet(\Delta_g) \overset{!!}{=} \ldots \text{ engendré par}$$

$$[G, \omega]$$

graphe st. de genre g

numérotation $\omega : E \longrightarrow [p]$

soumis à $[G, \omega] = \operatorname{sgn} \sigma \, [G, \sigma \circ \omega]$

et $G \cong G' \Rightarrow [G, \omega] = [G', \omega']$

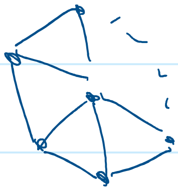
avec $\partial [G, \omega] = \sum_{\substack{i=0 \\ \omega(e_i) = i}}^{p} (-1)^i \left[G/e_i \, , \, \omega|_{E(G/e_i)} \right]$

Déf : Cx de Kontsevich GC est (un shift de)

$$C_\bullet(\Delta_g) / \text{poids total} > 0$$

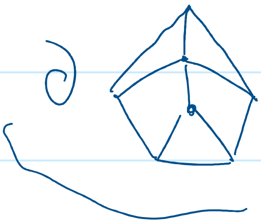
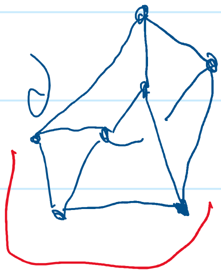
i.e. comme à gauche $\operatorname{val}(v) \geq 3$, $\forall v \in V$

Ex : $W_0 =$

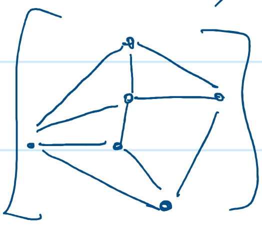
Ex : $W_g =$  $|E| = 2g$

$\Rightarrow \partial W_g = 0$ car $[\cdot / /] = 0$

Ainsi $[W_g] = 0$ si g pair.

Sous-ex : W_5 "G" ∂  et ∂ 

$= 0$ par symétrie

$W_5 + 2$ 

N'apparait pas dans d'autres ∂G

$$\Rightarrow [W_5] \neq 0 \in \tilde{H}_0 (GC^{(g)})$$

$$\cong$$

$$gr^W_{24} H^{14}(\mathcal{M}_5)$$

Exo : $\{w_3\} \neq 0$

Résumé :

$$\mathrm{gr}^W_{6g-6} H^{6g-6-k}(\mathcal{M}_g)$$

$$\cong$$

$$\tilde{H}_{k-1}(\underbrace{\Delta(\overline{\mathcal{M}}_g \setminus \mathcal{M}_g)}_{\Delta_g})$$

$$= \tilde{H}_{k-1}(C_\bullet(\Delta_g))$$

$$= \tilde{H}_{k-1}(GC^{(g)})$$

acyclicité de $w>0$
$$= \tilde{H}_{k-2g}(GC^{(g)})$$

On a (avec le shift)

$$H^{4g-6}(\mathcal{M}_g) \twoheadrightarrow H_0(GC^{(g)})$$

$$\cong \text{ Willwacher}$$

$$\mathfrak{grt}$$

$$\begin{array}{ll} & \sigma \\ & \big\uparrow \ \text{Brown} \\ \text{où } \langle \sigma_g , w_g \rangle \neq 0 \qquad & \text{Free Lie}(\sigma_3, \sigma_5, \dots) \end{array}$$