

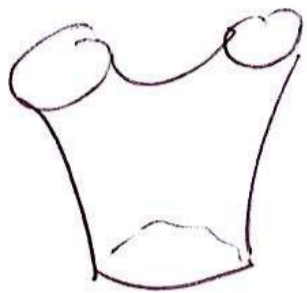
25.11.21
Sylvain

①
Compactification de $S_{g,m}$

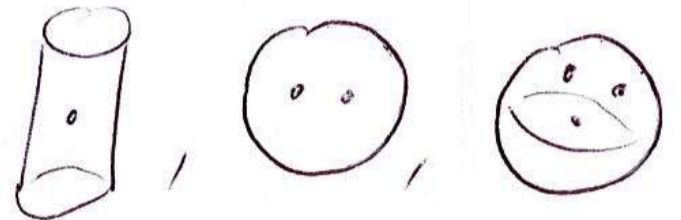
$g, m \mid \chi(S_{g,m}) < 0$

Décomposition en pentagons = multicouche menue.

$(\delta_{11}, \dots, \delta_{3g-3+m})$



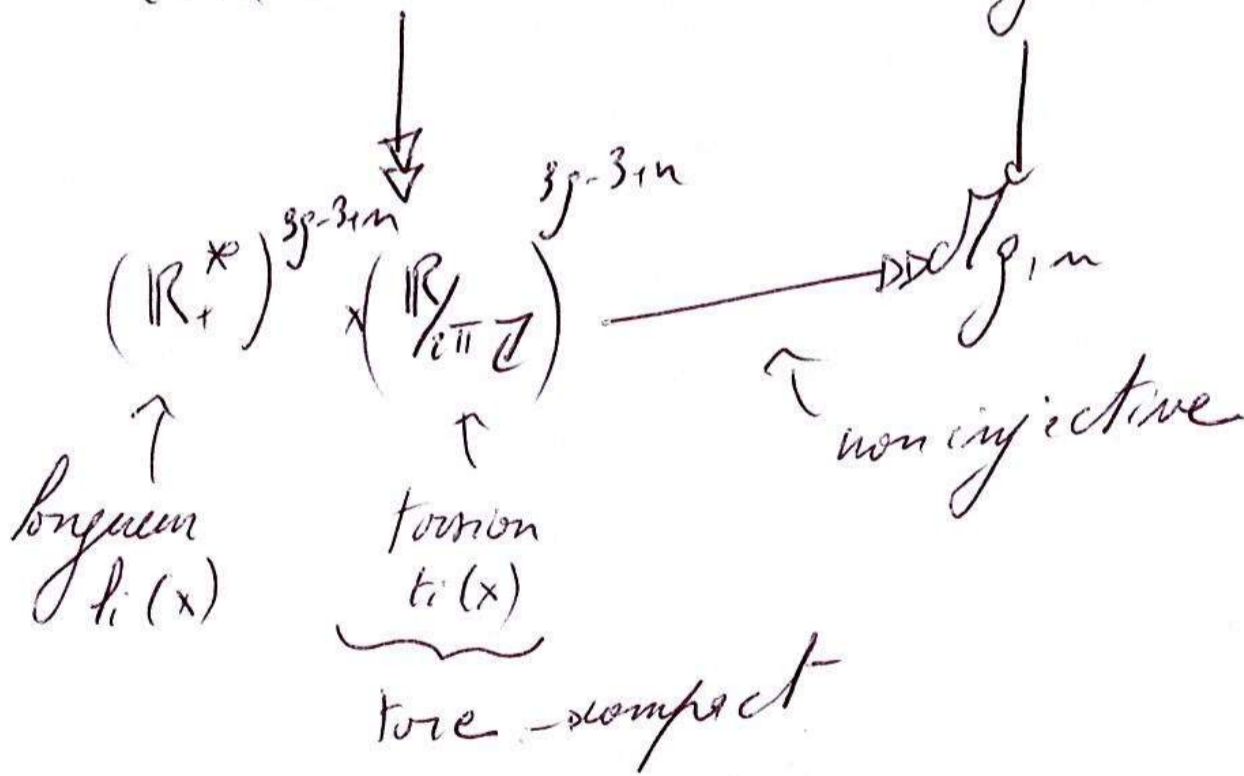
, éventuellement dépeçure :



Fenchel-Nielsen:

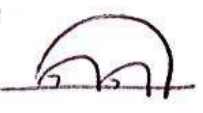
$\mathcal{L}_{g,m}$:

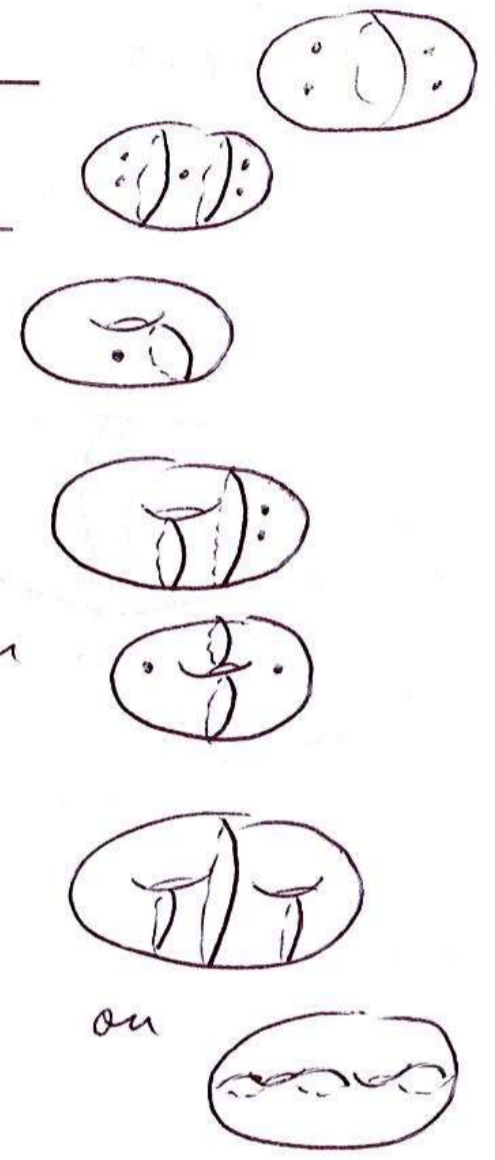
$(\mathbb{R}_+^*)^{3g-3+m} \times \mathbb{R}^{3g-3+m} \xrightarrow{\sim} \mathcal{L}_{g,m}$



Exemples:

g	n	$3g-3-n$ # courbes	$2g-2+n$ # composantes
0	3	0	1
0	4	1	2
0	n	$n-3$	$n-2$
1	1	1	1
1	2	2	2
2	0	3	2

g & H^2
 if $n=1$: 
 note: unique structure up to \mathbb{R}^2 !



Definition:

Une surface de Riemann avec n points marqués de type (g, n) est:

- X espace topologique compact avec n points marqués,
- atlas: carte $\rightarrow \varphi: D \xrightarrow{\sim} X$

$$\uparrow \varphi: \{(z_1, z_2) \in \mathbb{C}^2 \mid |z_1| < 1, |z_2| < 1, z_1 z_2 = 0\} \rightarrow X$$

nombre fini de fois \rightarrow morceaux

avec: $X \setminus \{\text{marqués}\}$: \forall composante connexe est hyperbolique

$$g = \sum g_i + n - p + 1.$$

ou: g_i = genre des composantes connexes de $X \setminus \{\text{marqués}\}$

n : # marqués.

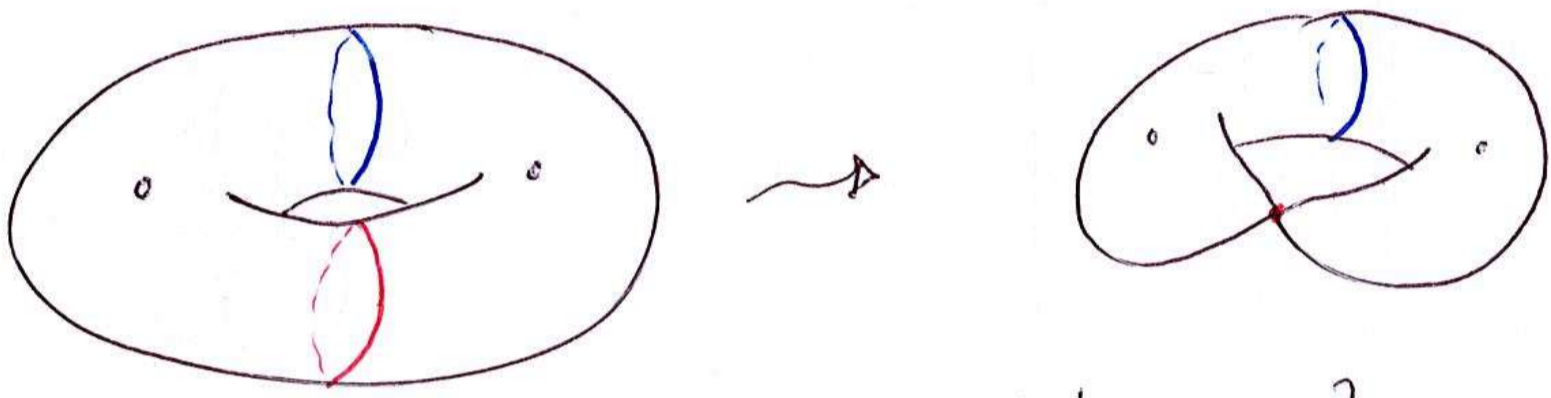
p : # comp. connexes de $X \setminus \{\text{marqués}\}$.

$$\overline{\mathcal{M}}_{g,m} = \{ \text{surfaces avec nœuds} \} / \text{isom.}$$

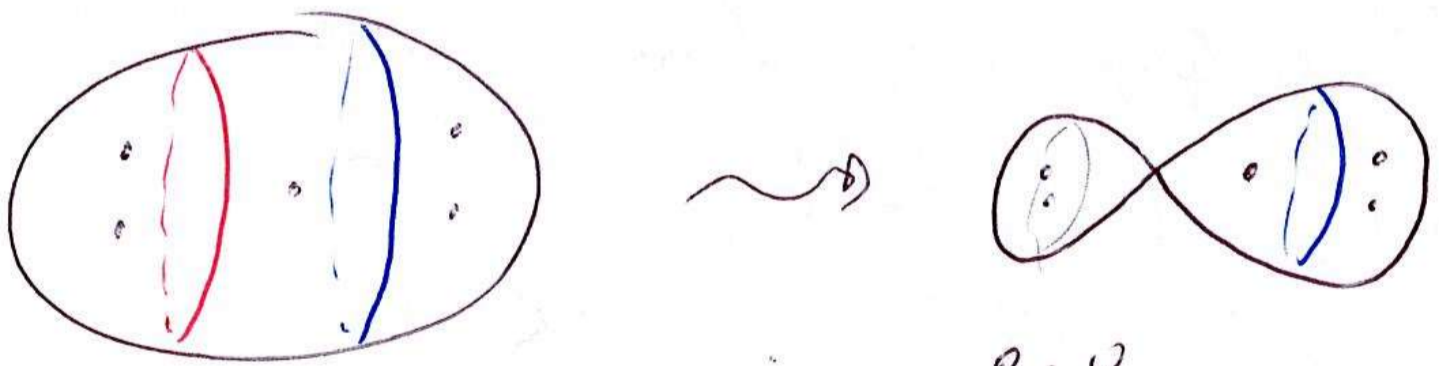
$$\cup$$

$$\mathcal{M}_{g,m}$$

But: munir $\overline{\mathcal{M}}_{g,m}$ d'une topologie qui étende celle de $\mathcal{M}_{g,m}$ /
 $\mathcal{M}_{g,m}$ est dense dans $\overline{\mathcal{M}}_{g,m}$ et $\overline{\mathcal{M}}_{g,m}$ est compact.

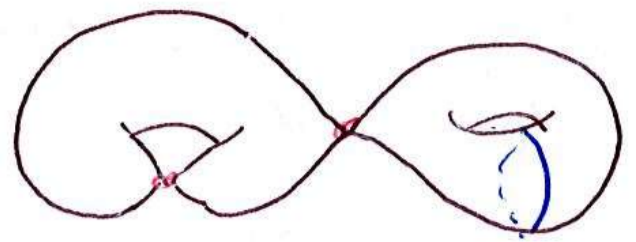
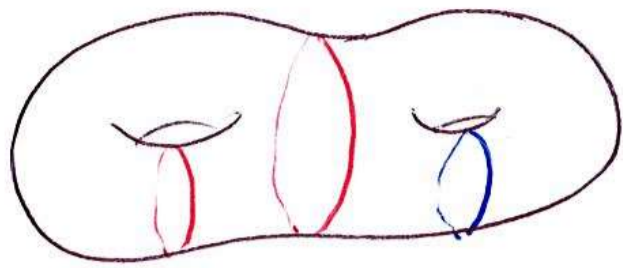


$$\left. \begin{array}{l} g=1 \\ n=2 \\ m=1 \\ h=1 \\ g_1=0 \end{array} \right\} 1 = 0 + 1 - 1 + 1$$



$$\left. \begin{array}{l} g=0 \\ n=4 \\ m=1 \\ h=2 \\ g_1=0 \\ g_2=0 \end{array} \right\} 1 - 2 + 1 = 0$$

(9)



$$\begin{aligned}
 g &= 2 \\
 m &= 0 \\
 m &= 2 \\
 p &= 2 \\
 g_1 &= 0 \\
 g_2 &= 1
 \end{aligned}$$

$$1 + 2 - 2 + 1 = 2$$

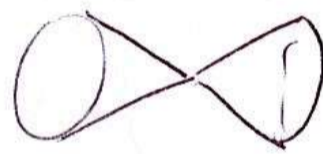
Extension de Fenichel-Kiehn:

$$X_0 \in \overline{\mathcal{M}_{g,m}} \setminus \mathcal{M}_{g,m}$$

\rightarrow déingulariser $\xrightarrow{\varepsilon > 0}$ X_ε

$$\left\{ \begin{array}{l} |z_1| < 1 \\ |z_2| < 1 \\ z_1 z_2 = 0 \end{array} \right\} \hookrightarrow z_1 z_2 = \varepsilon$$

\rightsquigarrow



Base de voisinages: multicourbe minimale $(\delta_1, \dots, \delta_{3g-3+m})$

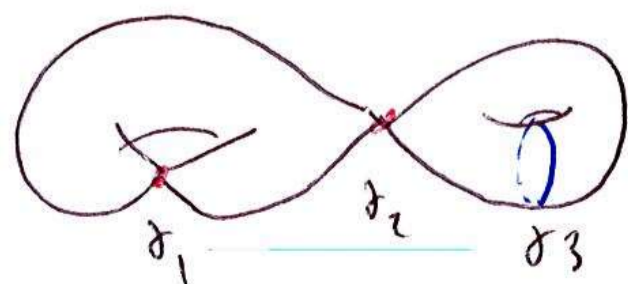
premières \rightarrow necks

$$(\mathbb{R}_+^*)^{3g-3+m} \times \mathbb{R}^{3g-3+m} \rightarrow \text{on étend à:}$$

$$\mathbb{R}_+^{3g-3+m} \times \mathbb{R}^{3g-3+m}$$

: neck $\rightarrow \rho_i(a) = 0$ (et on peut prendre $t_i = 0$?)

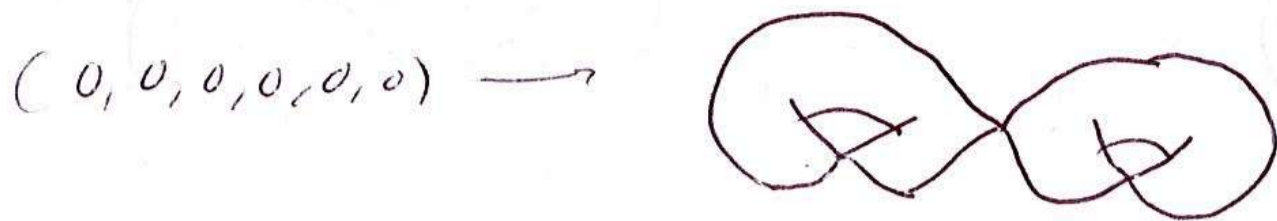
Ex:



$$X_0: (0, 0, \rho(\delta_3), 0, 0, t(\delta_3))$$

(10)

On peut faire dériver en outre:



Alors les composantes connexes sont des sphères à 3 points \rightarrow elles portent une unique structure complexe multivoisinnée.

Lemme: il n'y a qu'un nombre fini de P à l'action de $\text{Diff}(S_{g,m})$

$$K_P = \prod_P ([0, c], \dots, [0, c], [0, 2\pi], \dots, [0, 2\pi])$$

Topologie: X_0 , lieu des drainages: $\mu, \tau > 0$:

$$U_{\mu, \tau} = \left\{ X \in \mathcal{D}_{g,m} \mid \forall i, |p_i(X) - p_i(X_0)| < \mu, \forall i > m, |\tau_i(X) - \tau_i(X_0)| < \tau \right\}$$

$$\cup K_P \supset \mathcal{D}_{g,m}$$

$$\text{compact} \supset \overline{\mathcal{D}_{g,m}}$$

Proposition: $\forall g, m, \chi_{g,m} < 0, \exists c = c(g, m) /$

$\forall X$ surface de Riemann lisse de type (g, m) ,

\exists multicourbe minimale $P = (\delta_1, \dots, \delta_{3g-3+m}) /$

$$l(\delta_i) < c(g, m) \forall i$$

②

$\Rightarrow \forall X \in \sigma\tau_{g,m}, \exists P / X \in K_P$

$K = \cup K_P$ compact

$\Rightarrow \sigma\tau_{g,m} \subset K$

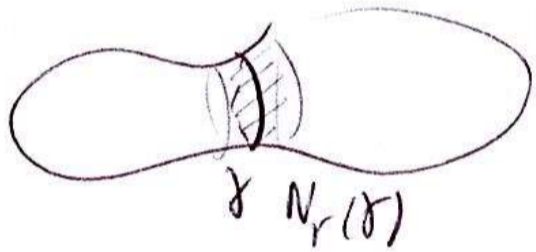
et $\overline{\sigma\tau_{g,m}} = \bar{\sigma\tau_{g,m}} \subset K$

Idees de preuve pour la proposition:

Lemme de Nagata / décomposition mine le pisme:

$X =$  surface compacte (points entiers).

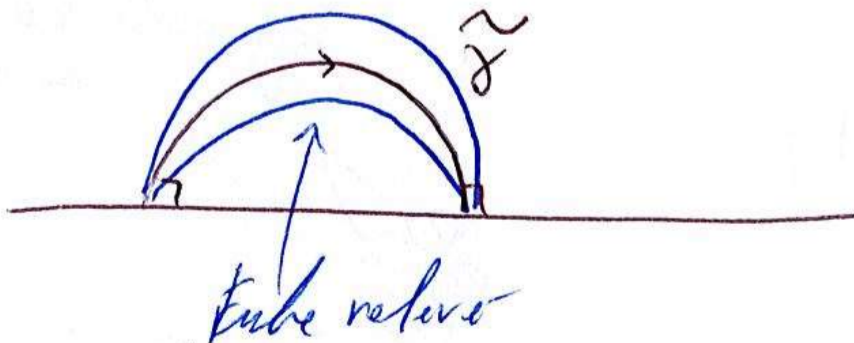
Definition: tube de Nagata:



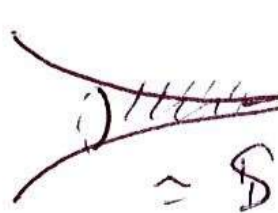
δ petite épaisseur fermée
 r suffisamment petit pour que r
soit topologiquement un collier.

Revêtement universel: $\tilde{X} = \mathbb{H}^2$

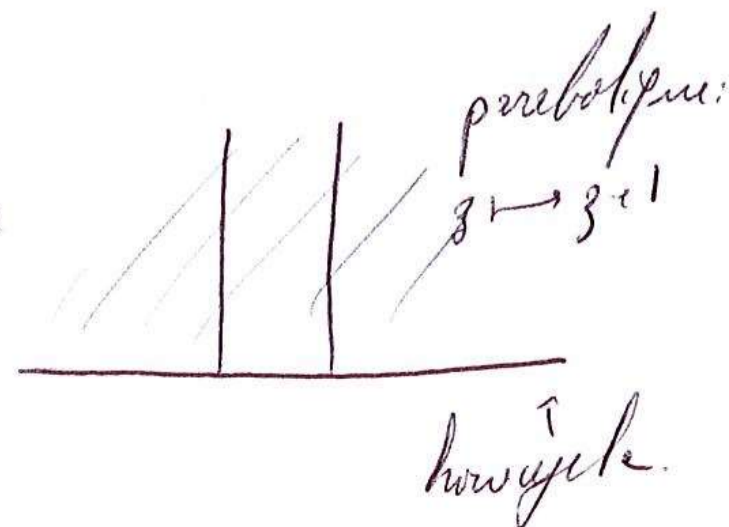
$\mathbb{Z} \langle \pi_1(X) \rangle$ engendré par un élément hyperbolique



Definition: voisinage de pointe (cusp):

 voisinage $\approx S^1 \times]0, +\infty[$

$\tilde{X} = \mathbb{H}^2$



Lemme de Targoulis:

$\exists \mu > 0, \forall X, \exists$ collection finie de tubes de Targoulis T_1, \dots, T_n et de courbes de pointes C_1, \dots, C_s disjointes

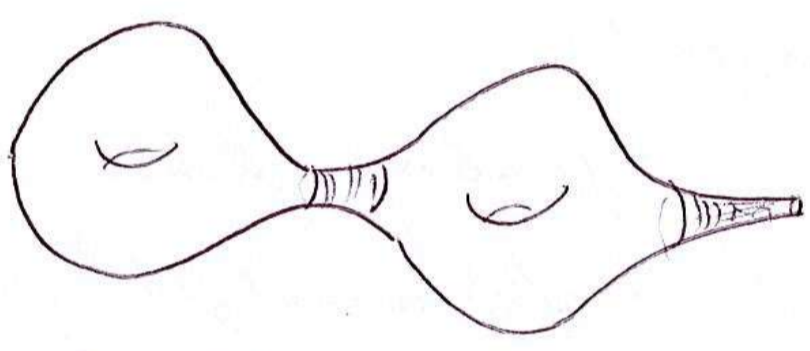
dont le complémentaire:

$\forall n \in \mathbb{N} \setminus \{ \underbrace{\cup T_i \cup C_i}_{\text{partie } \mu\text{-mince}} \} \quad \text{inj}^n \geq \mu.$

pour μ rayon / $B(n, r)$ isotopie dans tout \mathbb{H}^2

$X \setminus X_{\mu\text{-mince}} = X_{\mu\text{-épaisse}}.$

Exemple:



$\exists C' = C'(g, n) / \forall X$ et

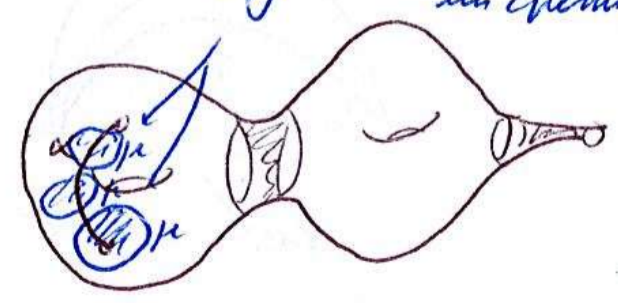
Lemme: \exists c constante minimisant $c X_{\mu\text{-épaisse}}.$

|| Alors $P(L) \leq c'$

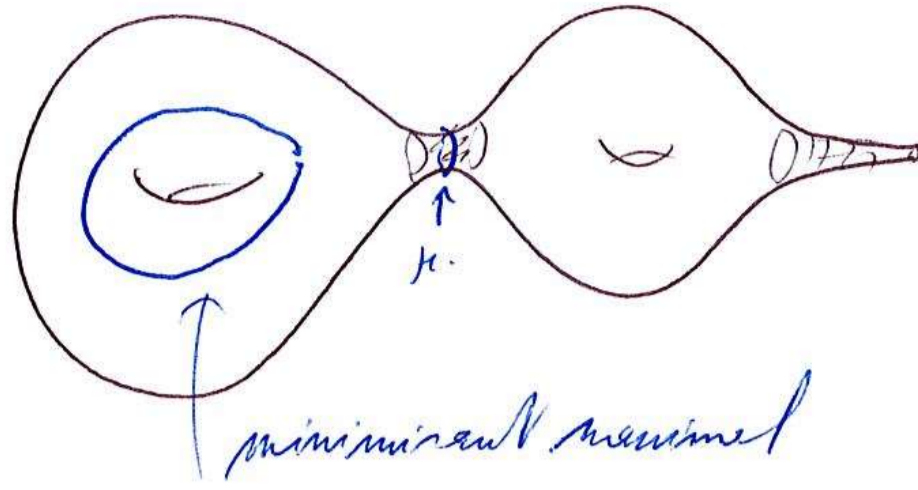
Preuve:

aire $\leq 2\pi |X(S_{g,n})|$

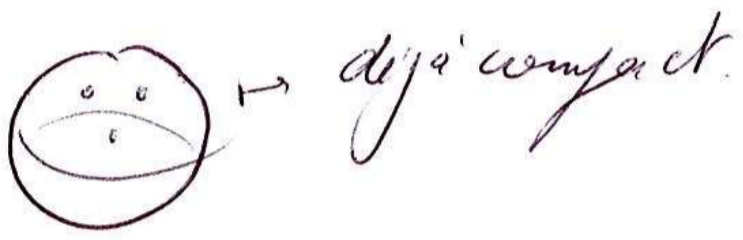
disjointes sinon on trouve un chemin plus court.



points: *points:*



Exemples:



$M_{0,4} \cong \mathbb{P}^1 \setminus 3pt$ et 3 manières de décomposer les 4 points.