Renormalization of Tensorial (Group) Field Theories

Sylvain Carrozza

Centre de Physique Théorique, Marseille

Montpellier, 27/08/2015

Workshop on "Renormalization in statistical physics and lattice field theories "

Table of Contents

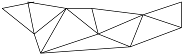
Matrix Models

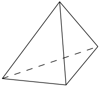
Statistical model for M_{ij}

Discretized 2d quantum gravity

Continuum limit

[David '85, Ginsparg '91...]





[Ambjorn et al., Gross '91, Sasakura '92...]

[Gurau '09...] $\rightarrow$ **large N expansion**

Tensor Models

Statistical model for $T_{i_1 \ldots i_d}$

Quantum gravity $d \geq 3$?

Same combinatorics

Group Field Theories

QFTs of $\varphi(g_1, \ldots, g_d)$

Formally define amplitudes $\mathcal{A}_s$

Well-behaved QFTs?

Renormalizability?

Phase structure?

'Many-body' sector

$\hat{\varphi}^\dagger(g_1, \ldots, g_d)$ [Oriti '13]

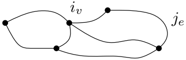
Loop Quantum Gravity

Kinematical $\mathcal{H} = L^2(\overline{\mathcal{G}})$

Dynamics?

[Ashtekar, Rovelli, Smolin... '90s]

spin networks s



'Histories' of spin network labeled by **2-complexes** C

[Reisenberger, Rovelli... '00s]

Sum over C

[De Pietri, Rovelli, Freidel, Oriti '00s...]

Feynman expansion

Spin Foam Models

Define amplitudes $\mathcal{A}_{s,C}$

How to deduce $\mathcal{A}_s$?

What is a Group Field Theory?

It is an approach to quantum gravity at the crossroads of **loop quantum gravity** (LQG) and **matrix/tensor models**.

A simple definition:

A **Group Field Theory (GFT)** is a **non-local quantum field theory** defined on a **group manifold**.

- The group manifold is **auxiliary**: should not be interpreted as space-time!
- Rather, **the Feynman amplitudes** are thought of as describing **space-time processes** $\rightarrow$ QFT *of* space-time rather than *on* space-time.
- Specific non-locality: determines the **combinatorial structure** of space-time processes (graphs, 2-complexes, triangulations...).

General structure of a GFT, and objectives

Typical form of a GFT: field $\varphi(g_1, \ldots, g_d)$, $g_\ell \in G$, with partition function

$$Z = \int [\mathcal{D}\varphi]_\Lambda \, \exp\left(-\varphi \cdot \mathcal{K} \cdot \varphi + \sum_{\{\mathcal{V}\}} t_{\mathcal{V}} \, \mathcal{V} \cdot \varphi^{n_{\mathcal{V}}}\right) = \sum_{k_{\mathcal{V}_1}, \ldots, k_{\mathcal{V}_i}} \prod_i (t_{\mathcal{V}_i})^{k_{\mathcal{V}_i}} \, \{\mathrm{SF\ amplitudes}\}$$

Main objectives of the GFT research programme:

1. Model building: define the **theory space**.
   e.g. spin foam models + combinatorial considerations (tensor models) $\rightarrow$ *d, G, $\mathcal{K}$ and $\{\mathcal{V}\}$.*
2. Perturbative definition: prove that the spin foam expansion is **consistent** in some range of Λ.
   e.g. perturbative multi-scale renormalization.
3. Systematically explore the theory space: **effective continuum regime reproducing GR** in some limit?
   e.g. functional RG, constructive methods, condensate states...

Purpose of this talk

- Illustrate the three steps with **toy models**:

1. Model building: **Tensorial** GFTs, in particular with **gauge invariance** condition.
   (in dimension 3 $\sim$ Euclidean quantum gravity)
2. Consistency check: **perturbative renormalizability** well–understood in this context
   $\rightarrow$ full classification of consistent models.
3. Systematically explore the theory space: on-going efforts aiming at making
   non-perturbative methods available.

- Show that these new QFTs have interesting mathematical properties: in particular, **asymptotic freedom** is realizable.

- Partition function for $N \times N$ **symmetric matrix**:

$$\mathcal{Z}(N,\lambda) = \int [\mathrm{d}M] \, \exp\left(-\frac{1}{2}\mathrm{Tr} M^2 + \frac{\lambda}{N^{1/2}} \mathrm{Tr} M^3 \right)$$

- **Large N expansion**:

$$\mathcal{Z}(N,\lambda) = \sum_{\text{triangulation } \Delta} \frac{\lambda^{n_\Delta}}{s(\Delta)} \mathcal{A}_\Delta(N) = \sum_{g \in \mathbb{N}} N^{2-2g} \, \mathcal{Z}_g(\lambda)$$

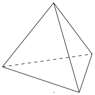
- **Continuum limit** of $\mathcal{Z}_0$: tune $\lambda \to \lambda_c \Rightarrow$ very refined triangulations dominate.
$(\mathcal{Z}_0(\lambda) \sim |\lambda - \lambda_c|^{2-\gamma})$

- Naive relation to Euclidean **2d quantum gravity**:

$$S_{EH} = \frac{1}{G} \int_S \mathrm{d}^2 x \sqrt{-g} \, (-R + \Lambda) = -\frac{4\pi}{G} \chi(S) + \frac{\Lambda}{G} A(S)$$

$$\Rightarrow \exp(-S_{EH}) \underset{\Delta}{\sim} \lambda^{n_\Delta} N^{\chi(\Delta)} \qquad \text{with } \lambda = \exp(-\Lambda/G) \; ; \; N = \exp(4\pi/G)$$

- Old idea [Ambjorn et al., Gross 91, Sasakura 92...]: generalize matrix models in the obvious way *e.g. in d=3*



$$\mathcal{Z} = \int [\mathcal{D}T]\, \mathrm{e}^{-\frac{1}{2} T_{i_1 i_2 i_3} T_{i_1 i_2 i_3} - \lambda T_{i_1 i_2 i_3} T_{i_3 i_5 i_4} T_{i_5 i_2 i_6} T_{i_4 i_6 i_1}} = \sum_{\text{triangulation } \Delta} \lambda^{n_\Delta} \mathcal{A}_\Delta$$

$\rightarrow$ a rank-d model generates **simplicial complexes** of dimension d.

- Various issues:
  - no control over the **topology** of the simplicial complexes;
  - no adapted analytical tools, in particular no $1/N$ **expansion**.

- Important improvements thanks to a modified combinatorial structure of the interactions $\rightarrow$ **colored** [Gurau '09] and **uncolored** [Bonzom, Gurau, Rivasseau '12] models.

$\Rightarrow$ action specified by a **tensorial invariance** under $\mathrm{U}(N)^{\otimes d}$:

$$T_{i_1 \dots i_d} \rightarrow U^{(1)}_{i_1 j_1} \dots U^{(d)}_{i_d j_d} T_{j_1 \dots j_d}\,, \qquad \overline{T}_{i_1 \dots i_d} \rightarrow \overline{U}^{(1)}_{i_1 j_1} \dots \overline{U}^{(d)}_{i_d j_d} \overline{T}_{j_1 \dots j_d}\,.$$

A wealth of recent results in TM, an opportunity for GFTs

- Long list of recent results in the framework of these new tensor models:
  - $1/N$ **expansion** dominated by spheres [Gurau '11...];
  - **continuum limit** of the leading order [Bonzom, Gurau, Riello, Rivasseau '11] $\rightarrow$ 'branched polymer' [Gurau, Ryan '13];
  - **double-scaling** limit [Dartois, Gurau, Rivasseau '13; Gurau, Schaeffer '13; Bonzom, Gurau, Ryan, Tanasa '14];
  - **Schwinger-Dyson** equations [Gurau '11 '12; Bonzom '12];
  - **non-perturbative** results [Gurau '11 '13; Delepouve, Gurau, Rivasseau '14];
  - **'multi-orientable'** models [Tanasa '11, Dartois, Rivasseau, Tanasa '13; Raasaakka, Tanasa '13; Fusy, Tanasa '14], $O(N)^{\otimes d}$**-invariant** models [SC, Tanasa wip], and **new scalings** [Bonzom '12; Bonzom, Delepouve, Rivasseau '15];
  - **symmetry breaking** to matrix phase [Benedetti, Gurau '15];
  - ...

- **Same techniques available in GFTs** provided that the same combinatorial restrictions are implemented.

- A tensor model can be viewed as a GFT of the simplest type
  e.g. a theory on $U(1)^d$ with sharp cut-off on the **Fourier modes** $(p_1, \ldots, p_d) \in \mathbb{Z}^d$.

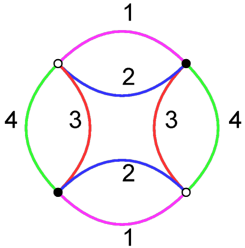
$\Rightarrow$ naturally leads to the definition of more general **Tensorial GFTs**, with **more general groups** and **more general kinetic terms**.

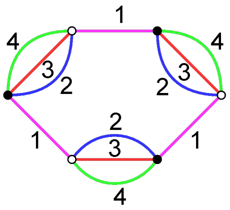
Definition: colored graph

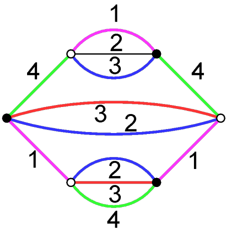
A *n*-**colored graph** is a **bipartite regular** graph of valency n, **edge-colored** by labels $\ell \in \{1, \dots, n\}$, and such that at each vertex meet n edges with distinct colors.

- Two types of nodes: black or white dots.
- n types of edges, with color label $\ell \in \{1, \dots, n\}$.

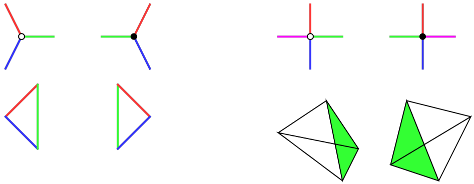
Examples: 4-colored graphs.



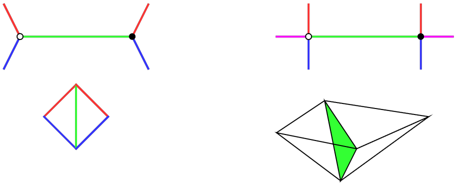




- Each node in a $(d+1)$-colored graph is dual to a d-**simplex**



- Each line represents the **gluing** of two d-simplices along their boundary $(d-1)$-**simplices**



$\Rightarrow$ **A $(d+1)$-colored graph represents a triangulation in dimension d.**

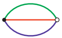
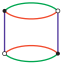
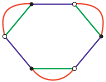
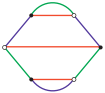
Crystallisation theory [Cagliardi, Ferri et al. '80s]

Locality as tensorial invariance

New notion of locality for **Tensor Models** and GFTs [Bonzom, Gurau, Rivasseau '12]:

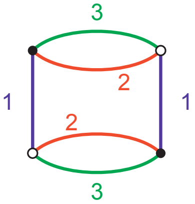
- $S^{\text{int}}(\varphi, \overline{\varphi})$ is the interaction part of the action, and should be a sum of **connected tensor invariants**

$$S^{\text{int}}(\varphi, \overline{\varphi}) \quad = \quad \sum_{b \in \mathcal{B}} t_b I_b(\varphi, \overline{\varphi})$$

$\underset{d=3}{=}$ t_2  $+\, t_4$  $+\, t_{6,1}$  $+\, t_{6,2}$  $+ \dots$

which play the role of **local** terms.

- Correspondence between colored graphs b and tensor invariants $I_b(\varphi, \overline{\varphi})$:
  - white (resp. black) **node** $\leftrightarrow$ **field** (resp. complex conjugate field);
  - **edge** of color ℓ $\leftrightarrow$ **convolution** of ℓ-th indices of φ and $\overline{\varphi}$.



$$I_b(\varphi, \overline{\varphi}) = \int [\mathrm{d}g_i]^6 \, \overline{\varphi}(g_6, g_2, g_3)\varphi(g_1, g_2, g_3)$$
$$\varphi(g_6, g_4, g_5)\overline{\varphi}(g_1, g_4, g_5)$$

Abstract notion of scale

- Scales in space-time based QFTs: **energy**. Not available in a background independent context.

- In Matrix/Tensor Models ($T_{i_1,\dots,i_d} | i_k \in \{1,\dots,N\}$): **size** N of the tensors viewed as a **cut-off**.
  - '**UV**' scales $\equiv$ large i_k;
  - '**IR**' scales $\equiv$ small i_k.

- One possible generalization to GFT: eigenvalues of $\sum_\ell \Delta_\ell$ [Ben Geloun, Bonzom '11; Ben Geloun, Rivasseau '11]. For instance, for a field $\varphi(g_1,\dots,g_d)$, with $g_k \in \mathrm{U}(1)$ or $\mathrm{SU}(2)$:

$$\text{scale} = \sum_{\ell=1}^{d} p_\ell^2 \lesssim \Lambda^2 \qquad \text{or} \qquad \sum_{\ell=1}^{d} j_\ell(j_\ell+1) \lesssim \Lambda^2$$

  - '**UV**' $\equiv$ large momenta $|p_\ell|$ or spins j_ℓ;
  - '**IR**' $\equiv$ small momenta $|p_\ell|$ spins j_ℓ.

- Natural flow from a large cut-off on the spins to a smaller one: **consistent with continuum limit** in LQG, since large spins means large building blocks (area).

'Local potential approximation'

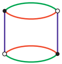
- Ansatz akin to a **'local potential approximation'**:

$$S_\Lambda(\varphi,\overline{\varphi}) = \overline{\varphi} \cdot \left(- \sum_\ell \Delta_\ell \right) \cdot \varphi + S_\Lambda^{\mathrm{int}}(\varphi,\overline{\varphi})$$

- Subtlety: invariance properties on φ imposed by **spin foam constraints**.
- **Partition function**:

$$\mathcal{Z}_\Lambda = \int \mathrm{d}\mu_{C_\Lambda}(\varphi,\overline{\varphi})\, \mathrm{e}^{-S_\Lambda^{\mathrm{int}}(\varphi,\overline{\varphi})} \,.$$

  - $S_\Lambda^{\mathrm{int}}(\varphi,\overline{\varphi})$ is **local**:

$$S_\Lambda^{\mathrm{int}}(\varphi,\overline{\varphi}) = \sum_{b \in \mathcal{B}} t_b^\Lambda I_b(\varphi,\overline{\varphi}) \underset{d=3}{=} t_2^\Lambda$$  $$+ \, t_4^\Lambda$$  $$+ \dots$$

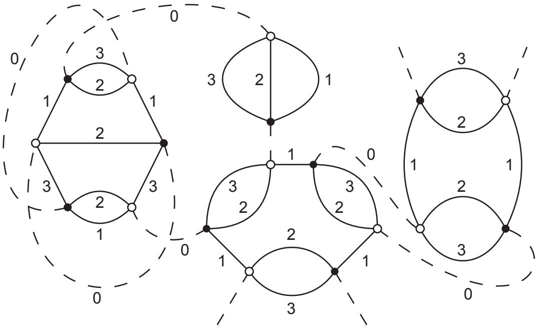
  - Gaussian measure $\mathrm{d}\mu_C$ with possibly degenerate covariance:

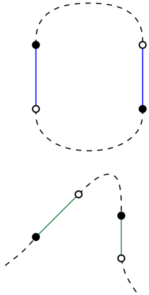
$$C = \mathcal{P} \left(- \sum_\ell \Delta_\ell \right)^{-1} \mathcal{P}$$

where $\mathcal{P}$ is a **projector** implementing the relevant constraints on the fields.

Feynman graphs:

- Elementary building blocks = **colored graphs** = **GFT vertices** = **3d cells with colored triangulated boundaries**...





- ...glued together along their boundary triangles.
- **Covariances** associated to the dashed, color-0 lines.
- **Face of color** ℓ = connected set of (alternating) color-0 and color-ℓ lines.

Overview

Goal: check that the perturbative expansion - and henceforth **the connection to spin foam models** - is consistent.

- Types of models considered so far:
  - **'combinatorial'** models on $\mathrm{U}(1)^D \to$ non-trivial propagators, but no use of the group structure;
    [Ben Geloun, Rivasseau '11; Ben Geloun, Ousmane Samary '12; Ben Geloun, Livine '12...]
  - models with **'gauge invariance'** on $\mathrm{U}(1)^D$ and $\mathrm{SU}(2) \to$ non-trivial propagators + one key dynamical ingredient of spin foam models.
    [SC, Oriti, Rivasseau '12 '13; Ousmane Samary, Vignes-Tourneret '12; SC '14 '14; Lahoche, Oriti, Rivasseau '14]
- Methods:
  - **multiscale analysis**: allow to rigorously prove renormalizability at **all orders in perturbation theory**;
  - **Connes–Kreimer** algebraic methods [Raasakka, Tanasa '13];
  - **loop-vertex expansion**: non-perturbative method allowing to resum the perturbative series [Gurau, Rivasseau,... '13].

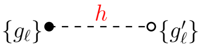
- **Gauge invariance condition**

$$\forall h \in G\,, \qquad \varphi(g_1, \dots, g_d) = \varphi(g_1 h, \dots, g_d h)$$

Common to **all** Spin Foam models: introduces a dynamical discrete connection at the level of the amplitudes.

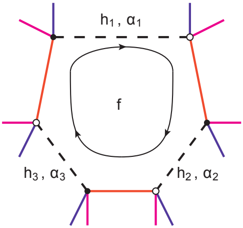
- Resulting propagator, including a regulator Λ ($\sim \sum_\ell j_\ell(j_\ell + 1) \leq \Lambda^2$):

$$C_\Lambda(g_\ell; g'_\ell) = \int_{\Lambda^{-2}}^{+\infty} \mathrm{d}\alpha \int \mathrm{d}h \prod_{\ell=1}^{d} K_\alpha(g_\ell h g'^{-1}_\ell)\,,$$



where K_α is the **heat kernel** on G at time α.

- The amplitudes are best expressed in terms of the **faces** of the Feynman graphs:



$$\longleftrightarrow \qquad K_{\alpha_1+\alpha_2+\alpha_3}(h_1 h_2 h_3)$$

Power-counting analysis $\Rightarrow$ **classification** of allowed just-renormalizable models:
[Oriti, Rivasseau, SC '13]

$d = \mathrm{rank}$	$D = \dim(G)$	order	explicit examples
3	3	6	$G = \mathrm{SU}(2)$ [Oriti, Rivasseau, SC '13]
3	4	4	$G = \mathrm{SU}(2) \times \mathrm{U}(1)$ [SC '14]
4	2	4	
5	1	6	$G = \mathrm{U}(1)$ [Ousmane Samary, Vignes-Tourneret '12]
6	1	4	$G = \mathrm{U}(1)$ [Ousmane Samary, Vignes-Tourneret '12]

- $d = D = 3$ is the only case for which the combinatorial dimension can match the dimension of space-time inferred from the symmetry group G.
- Analogy with ordinary scalar field theory: at fixed $d = 3$
  - φ^6 model in $D = 3$;
  - φ^4 model in $D = 4$.

Multiscale proof of renormalizabiliy: strategy

1) Decompose amplitudes according to slices of "momenta" (Schwinger parameter);
2) Replace **high divergent subgraphs** by effective local vertices;
3) Iterate.

Advantages of the **multiscale expansion**:

- Results at **all orders** in perturbation theory;
- sheds light on the finiteness of renormalized amplitudes (and on why they can be large).

- The Schwinger parameter α determines a momentum scale, which can be sliced in a geometric way. One fixes $M > 1$ and decomposes the propagators as

$$
\begin{aligned}
C &= \sum_i C_i \,, \\
C_0(g_\ell; g'_\ell) &= \int_1^{+\infty} \mathrm{d}\alpha \, \mathrm{e}^{-\alpha m^2} \int \mathrm{d}h \prod_{\ell=1}^{d} K_\alpha(g_\ell h g_\ell'^{-1}) \\
C_i(g_\ell; g'_\ell) &= \int_{M^{-2i}}^{M^{-2(i-1)}} \mathrm{d}\alpha \, \mathrm{e}^{-\alpha m^2} \int \mathrm{d}h \prod_{\ell=1}^{d} K_\alpha(g_\ell h g_\ell'^{-1}) \,.
\end{aligned}
$$

- A natural regularization is provided by a **cut-off on i**: $i \leq \rho$. To be removed by renormalization.

- The amplitude of a connected graph $\mathcal{G}$ is decomposed over **scale attributions** $\mu = \{i_e\}$ where i_e runs over all integers (smaller than ρ) for every line e:

$$
\mathcal{A}_{\mathcal{G}} = \sum_{\mu} \mathcal{A}_{\mathcal{G},\mu} \,.
$$

Multiscale proof of renormalizabiliy: power-counting theorem

High subgraphs

A **high subgraph** $\mathcal{H} \subset \mathcal{G}$ is a **connected** subgraph with:

$$\{\text{external scales}\} > \{\text{internal scales}\}$$

Theorem

If G has dimension D, there exists a constant K such that the following bound holds:

$$|\mathcal{A}_{\mathcal{G},\mu}| \leq K^{L(\mathcal{G})} \prod_{\text{high } \mathcal{H} \subset \mathcal{G}} M^{\omega[\mathcal{H}]} \,,$$

where the **degree of divergence** ω is given by

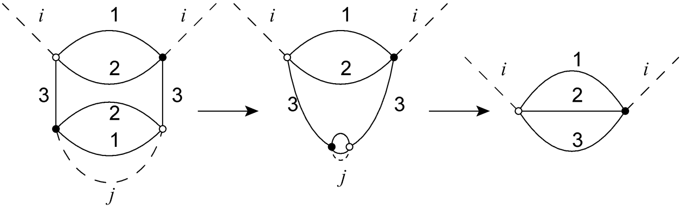
$$\omega(\mathcal{H}) = -2L(\mathcal{H}) + D(F(\mathcal{H}) - R(\mathcal{H}))$$

and $R(\mathcal{H})$ is the rank of the ϵ_{ef} incidence matrix of $\mathcal{H}$.

$\Rightarrow$ set of graphs to be renormalized, classification of potentially renormalizable theories.

Necessary condition: **divergent subgraphs must be quasi-local, i.e. look like (connected) tensor invariants.**

Example: when internal scales $j \gg$ external scales i



This property is not generic in TGFTs → **"traciality" criterion**:

- flatness condition: the parallel transports must peak around $\mathbf{1}$ (up to gauge);
- combinatorial condition: connected boundary graph.

Models studied so far dominated by **melonic graphs** → always tracial.

Overview

Short-term goal: define tools to analyse the flow of coupling constants, both in the perturbative and non-perturbative regions.

Physical question: **continuum limit** in some region of parameters? general relativity effectively recovered there?

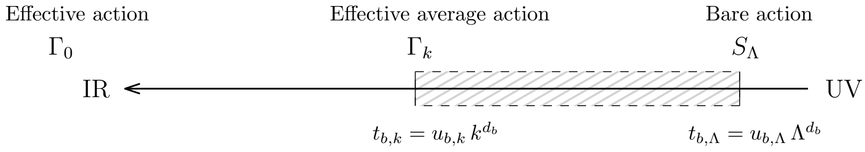
- Perturbative methods:
  - multiscale 'effective expansion' $\rightarrow$ **discrete RG** flow; [SC '14]
  - more traditional analysis (e.g. Callan–Symanzik eq.) $\rightarrow$ **continuous RG** flow. [Ben Geloun '12; Ben Geloun, Ousmane Samary '12; Ousmane Samary '13; SC '14; Lahoche, Oriti, Rivasseau '15]

  $\Rightarrow$ **Asymptotic freedom** is rather common in TGFTs! [Ben Geloun] For φ^4 theories, general explanation base on the intermediate field formalism [Rivasseau '15].

- Non-perturbative methods:
  - **functional renormalization group** (FRG): Wetterich [Eichhorn, Koslowski '13 '14; Benedetti, Ben Geloun, Oriti '14] or Polchinski equation [Krajewski, Toriumi, to appear];
  - ε**-expansion** [SC '14].

From now on: $d = 3$ i.e. $\varphi(g_1, g_2, g_3)$, and gauge invariance assumed.

Effective average action [Wetterich '93, Morris '93]



Assume also $1 \ll k \ll \Lambda \to$ approximately autonomous flow (not true in general).

Definition. **Canonical dimension** of a coupling constant t_b (N_b = valency of b):

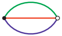
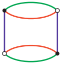
$$d_b = [t_b] = D - \frac{(D-2)N_b}{2}$$

Classification of coupling constants in the vicinity of the Gaussian fixed point:

- $[t_b] \geq 0 \Rightarrow t_b$ **relevant** or renormalizable. Marginal when $[t_b] = 0$.
- $[t_b] < 0 \Rightarrow t_b$ **irrelevant** or non-renormalizable.

Gauge invariant TGFTs in $d = 3$: $\dim G = 4$ [SC '14]

- Assume $G = \mathrm{SU}(2) \times \mathrm{U}(1)$, the effective average action at cut-off scale k reads

$\Gamma_k(\varphi, \overline{\varphi}) = u_{2,k} k^2$  $+ \, u_{4,k}$  $+ \cdots$

- A perturbative computation of the flow yields:

$$\begin{aligned} k \frac{\partial u_{2,k}}{\partial k} &= -2 u_{2,k} - 3\pi u_{4,k} + \mathcal{O}(u^2) \\ k \frac{\partial u_{4,k}}{\partial k} &= -2\pi {u_{4,k}}^2 + \mathcal{O}(u^3) \end{aligned}$$

- This model is **asymptotically free**, due to a **strong wave-function renormalization**:

$$\forall k \gg \Lambda_0 \,, \qquad u_{4,k} \approx \frac{1}{2\pi \ln\left(\frac{k}{\Lambda_0}\right)} \,.$$

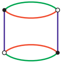
NB: this is a direct consequence of the new tensorial notion of locality.

Gauge invariant TGFTs in $d = 3$: $\dim G = 4 - \varepsilon$ [SC '14]

- TGFT in $\dim G = 4 - \varepsilon$ defined through **analytic continuation** of the numbers of $U(1)$ copies:

$$G = \mathrm{SU}(2) \times \mathrm{U}(1)^{D-3} \quad \rightarrow \quad G = \mathrm{SU}(2) \times \mathrm{U}(1)^{1-\varepsilon}$$

- The effective average action at cut-off scale k reads

$$\Gamma_k(\varphi, \overline{\varphi}) = u_{2,k} k^2$$  $$+ u_{4,k} k^{\varepsilon}$$  $$+ \cdots$$

- A perturbative computation of the flow yields:

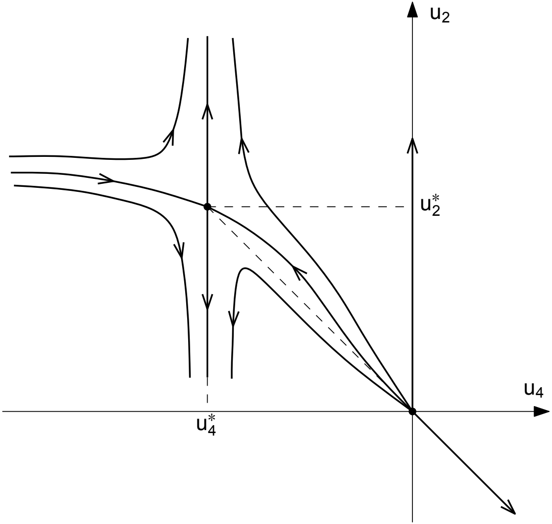
$$\begin{aligned} k \frac{\partial u_{2,k}}{\partial k} &\approx -2 u_{2,k} - 3\pi u_{4,k} + \mathcal{O}(u^2) \\ k \frac{\partial u_{4,k}}{\partial k} &\approx -\varepsilon u_{4,k} - 2\pi {u_{4,k}}^2 + \mathcal{O}(u^3) \end{aligned}$$

- New **non-trivial fixed point**:

$$u_2^* \approx \frac{3}{4} \varepsilon + \mathcal{O}(\varepsilon^2) \,, \qquad u_4^* \approx -\frac{1}{2\pi} \varepsilon + \mathcal{O}(\varepsilon^2) \,.$$

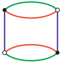
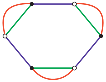
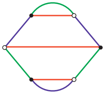
- Analogous to the **Wilson-Fischer** fixed point in ordinary scalar field theories, but with opposite signs.

Phase portrait (qualitative):



Back to $G = \mathrm{SU}(2)$.

- The effective average action at cut-off scale k reads

$\Gamma_k(\varphi, \overline{\varphi}) = u_{2,k} k^2$  $+ u_{4,k} k$  $+ u_{6,1,k}$  $+ u_{6,2,k}$  $+ \cdots$

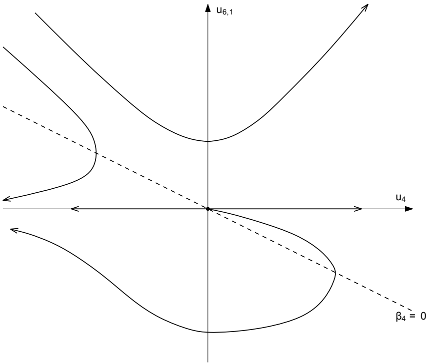
- One-loop β-functions:

$$\begin{cases} \beta_2(u) \approx -2\, u_2 - 7.5\, u_4 \\ \beta_4(u) \approx -u_4 - 5.0\, u_{6,1} - 10.0\, u_{6,2} \\ \beta_{6,1}(u) \approx -1.4\, u_4 u_{6,1} \\ \beta_{6,2}(u) \approx -3.1\, u_4 u_{6,2} \end{cases}$$

$\rightarrow$ a 3d non-linear flow determines whether the theory is asymptotically free or not.

Gauge invariant TGFTs in $d = 3$: $\dim G = 3$ [SC '14]

$\{u_{6,1}\}$ and $\{u_{6,2}\}$ are invariant subspaces $\rightarrow$ reduced 2d phase portrait:



- More generally: trajectories with $u_{6,1} > 0$ and $u_{6,2} > 0$ **cannot be asymptotically free**.

- However, if the non-trivial fixed point found in dimension $4 - \varepsilon$ survives in dimension 3, these trajectories might be **asymptotically safe**. Should be investigated further by means of non-perturbative methods such as the **FRG**.

Non-perturbative methods

- The **Wetterich equation**:

$$k\partial_k \Gamma_k = \mathrm{Tr}\left(k\partial_k \mathcal{R}_k \cdot [\Gamma_k^{(2)} + \mathcal{R}_k]^{-1} \right)$$

  - applied to matrix and tensor models; [Eichhorn, Koslowski '13 '14]
  - applied to a φ^4 TGFT without gauge-invariance; [Benedetti, Ben Geloun, Oriti '14]
  - what about gauge invariant models? The φ^6 $d = D = 3$ model is an interesting playground!

- The **Polchinski equation** ($t = \ln \Lambda$):

$$\frac{\partial S}{\partial t} = \int [\mathrm{d}g_i \mathrm{d}\tilde{g}_i] K_t(g_i \tilde{g}_i^{-1}) \left(\frac{\delta^2 S}{\delta\varphi(g_i)\delta\overline{\varphi}(\tilde{g}_i)} - \frac{\delta S}{\delta\varphi(g_i)} \frac{\delta S}{\delta\overline{\varphi}(\tilde{g}_i)} \right)$$

  General framework currently being investigated. [Krajewski, Toriumi, to appear]

- **Constructive methods** such as the **loop-vertex expansion** (intermediate field):
  - applied to tensor models; [Gurau '11 '13; Delepouve, Gurau, Rivasseau '14]
  - applied to TGFTs without gauge invariance; [Delepouve, Rivasseau '14]
  - gauge invariance makes the intermediate field construction easier! Construction of a just-renormalizable TGFT tractable? [Lahoche, Oriti, Rivasseau '15]

Background Independent lattice renormalization

Alternative to GFT approach to spin foam models: **lattice interpretation** of a given foam $\Rightarrow$ **refining** strategy.

Mainly developed by Bianca Dittrich (Perimeter Institute) and collaborators.

Outstanding challenge: **diffeomorphism invariance** and **background-independence** $\Rightarrow$ **no scale parameter** a, and no regular lattices.

Instead:

- **The lattice itself is the scale** $\rightarrow$ complicated **directed set**, not completely ordered.
- **projective methods** used to construct the renormalization group flow i.e. the (consistent) collection of all effective descriptions.

  [Dittrich, Bahr '10s...]
- practical avenue to find theories as fixed points of a truncated RG flow: **non-perturbative numerical methods** generalizing **tensor networks**.

  [Dittrich, Steinhaus, Martin-Benito, Mizera, Delcamp...]

Summary

- **Tensor models** and **tensorial field theories** are within the scope of renormalization methods.
- Interaction between **LQG** ideas and **tensor models** $\Rightarrow$ **Tensorial Group Field Theories**, which are interesting completions of spin foam models.
- **Perturbatively renormalizable** TGFTs exist, despite the complications introduced by the new notion of locality and non-commutative group structures.
- **Asymptotic freedom** can be realized in such models, especially when only quartic interactions are renormalizable $\rightarrow$ UV complete GFTs.
- **Non-perturbative features** of TGFTs are explored: new fixed points.

Outlook

- **Non-perturbative** renormalization group methods:
  - **Functional Renormalization Group** for gauge invariant models: e.g. fixed point in 3d $\mathrm{SU}(2)$ model?
  - relation to **lattice** gauge theory methods [Dittrich and collaborators '10s]?
- Towards **4d quantum gravity** GFT models:
  - imposition of (some version of) the remaining **spin foam constraints**;
  - lorentzian signature $\rightarrow$ non–compact group;
  - neat formulation of 4d **theory space** in terms of **symmetries** of the GFT action.
- Physical applications of the GFT formalism:
  - effective smooth space–time from **GFT coherent states** [Gielen, Oriti, Sindoni '12...];
  - GFT description of black holes in LQG? what is the role of coarse–graining and renormalization? [Perez, Pranzetti,... '10s]

Outlook

- **Non-perturbative** renormalization group methods:
  - **Functional Renormalization Group** for gauge invariant models: e.g. fixed point in 3d $\mathrm{SU}(2)$ model?
  - relation to **lattice** gauge theory methods [Dittrich and collaborators '10s]?

- Towards **4d quantum gravity** GFT models:
  - imposition of (some version of) the remaining **spin foam constraints**;
  - lorentzian signature $\to$ non–compact group;
  - neat formulation of 4d **theory space** in terms of **symmetries** of the GFT action.

- Physical applications of the GFT formalism:
  - effective smooth space–time from **GFT coherent states** [Gielen, Oriti, Sindoni '12...];
  - GFT description of black holes in LQG? what is the role of coarse–graining and renormalization? [Perez, Pranzetti,... '10s]

Thank you for your attention