



A polytopal domain-specific language with a Rust interpreter

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Paris, September 2026

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A workflow you know all too well...

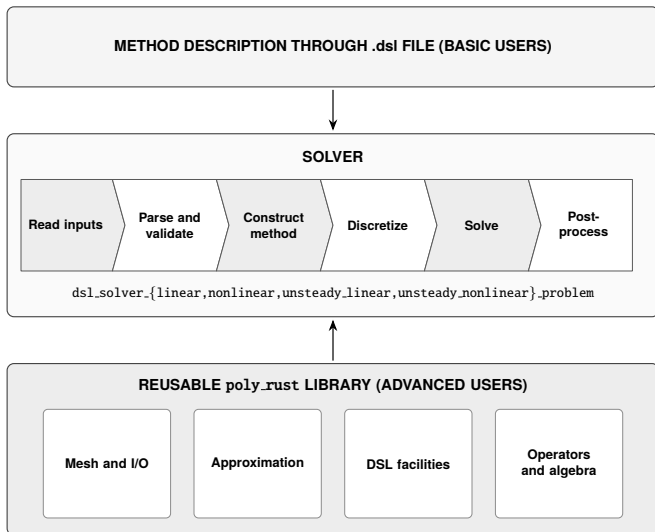


A workflow you know all too well...



Implementation: translate from human (mathematical) language to computer language

Architecture



The DSL

Driving example

- Let $\Omega \subset \mathbb{R}^2$ a polygonal domain with boundary Γ s.t.

$$\Gamma = \Gamma^D \sqcup \Gamma^N$$

- Let $f : \Omega \rightarrow \mathbb{R}$, $g : \Gamma^D \rightarrow \mathbb{R}$, and $\phi : \Gamma^N \rightarrow \mathbb{R}$ be given data
- We consider a **Hybrid High-Order (HHO)** scheme¹ for the problem:
Find $u : \Omega \rightarrow \mathbb{R}$ s.t.

$$\begin{aligned} -\Delta u &= f && \text{in } \Omega, \\ u &= g && \text{on } \Gamma^D, \\ -\nabla u \cdot n &= \phi && \text{on } \Gamma^N, \end{aligned}$$

where n denotes the outward unit normal to Γ

¹Di Pietro et al. 2014

Method

```
method hho_poisson {  
  // Parameters and functions  
  // Discrete spaces  
  // Interpolants  
  // Operators  
  // Forms  
  // Norm functionals  
  // Boundary conditions  
  // Problem description  
}
```

Parameters and functions

```
parameter pi = 3.14159265358979

function exact_solution(vector X) -> scalar = sin(pi * X[0])
    * sin(pi * X[1])
function exact_solution_gradient(vector X) -> vector
    = vector(
        pi * cos(pi * X[0]) * sin(pi * X[1]),
        pi * sin(pi * X[0]) * cos(pi * X[1])
    )
function exact_normal_flux(vector X) -> scalar on edge E of
    element T
    = - exact_solution_gradient(X) dot normal

function source(vector X) -> scalar = 2.0 * pow(pi, 2.0) *
    sin(pi * X[0]) * sin(pi * X[1])
```

Discrete spaces

The standard HHO space is defined by:

$$\underline{U}_h^k := \left\{ \underline{v}_h = ((v_T)_{T \in \mathcal{T}_h}, (v_E)_{E \in \mathcal{E}_h}) : \right. \\ \left. v_T \in \mathcal{P}^k(T) \text{ for all } T \in \mathcal{T}_h, v_E \in \mathcal{P}^k(E) \text{ for all } E \in \mathcal{E}_h \right\}$$

```
space Uh {  
  element Poly(k, scalar)  
  edge Poly(k, scalar)  
}
```

Discrete spaces

The standard HHO space is defined by:

$$\underline{U}_h^k := \left\{ \underline{v}_h = ((v_T)_{T \in \mathcal{T}_h}, (v_E)_{E \in \mathcal{E}_h}) : \right. \\ \left. v_T \in \mathcal{P}^k(T) \text{ for all } T \in \mathcal{T}_h, v_E \in \mathcal{P}^k(E) \text{ for all } E \in \mathcal{E}_h \right\}$$

```
space Uh {  
  element Poly(k, scalar) called element_dof  
  edge Poly(k, scalar) called edge_dof  
}
```

Interpolants

Components are interpreted through the interpolator $\underline{I}_h^k : H^1(\Omega) \rightarrow \underline{U}_h^k$:

$$H^1(\Omega) \ni v \mapsto \underline{I}_h^k v := ((\pi_T^k v)_{T \in \mathcal{T}_h}, (\pi_E^k v)_{E \in \mathcal{E}_h}) \in \underline{U}_h^k$$

```
interpolant interpolant_exact_solution on Uh {  
  on element T:  
    dof(T) = l2_project(exact_solution, Poly(k, scalar))  
  on edge E:  
    dof(E) = l2_project(exact_solution, Poly(k, scalar))  
}
```

Operators

The discrete gradient $G_T^k : \underline{U}_T^k \rightarrow \mathcal{P}^k(T; \mathbb{R}^2)$ is s.t., for all $\underline{v}_T \in \underline{U}_T^k$,

$$\int_T G_T^k \underline{v}_T \cdot \tau = - \int_T \underline{v}_T \operatorname{div} \tau + \sum_{E \in \mathcal{E}_T} \int_E \underline{v}_E (\tau \cdot n_{TE}) \quad \forall \tau \in \mathcal{P}^k(T; \mathbb{R}^2)$$

```
operator gradient_reconstruction : Uh(v) -> Poly(k, vector)
  on element T {
    forall tau in Poly(k, vector):
      int(T) gradient_reconstruction(v) dot tau =
        - int(T) dof(v, T) * div(tau)
        + sum_edges(int(E) dof(v, E) * (tau dot normal))
  }
```

Using interpolants to test polynomial exactness I

- A crucial design property is **polynomial exactness**:

$$G_T^k I_T^k v = v \quad \forall v \in \mathcal{P}^{k+1}(T)$$

- When coding a new operator, checking polynomial consistency is a common **debugging step**
- `poly_rust` provides language facilities that makes this easy!

Using interpolants to test polynomial exactness II

```
function test_linear(vector X) -> scalar = X[0] - 4.0 * X[1]
    + 7.0
function test_quadratic(vector X) -> scalar = pow(X[0], 2.0)
    - 3.0 * X[0] * X[1] + pow(X[1], 2.0) - 5.0
```

```
operator gradient_reconstruction : Uh(v) -> Poly(k, vector)
    on element T {
    ...
    test exactness for k = 0 against test_linear_gradient using
        interpolant_test_linear
    test exactness for k = 1 against test_quadratic_gradient
        using interpolant_test_quadratic
}
```

Potential and difference operators I

$$\int_T \nabla p_T^{k+1} \underline{u}_T \cdot \nabla w = \int_T G_T^k \underline{u}_T \cdot \nabla w \quad \forall w \in \mathcal{P}^{k+1}(T),$$
$$\int_T \nabla p_T^{k+1} \underline{u}_T = \int_T u_T$$

```
operator potential_reconstruction : Uh(u) -> Poly(k+1, scalar
) on element T {
  forall w in Poly(k+1, scalar):
    int(T) grad(potential_reconstruction(u)) dot grad(w) =
    int(T) gradient_reconstruction(u) dot grad(w)
  constraint int(T) potential_reconstruction(u) = int(T) dof(
    u, T)
}
```

Potential and difference operators II

$$\delta_T^k \underline{v}_T := \pi_T^k(p_T^{k+1} \underline{v}_T - v_T), \quad \delta_{TE}^k \underline{v}_T := \pi_E^k(p_T^{k+1} \underline{v}_T - v_E) \quad \forall E \in \mathcal{E}_T$$

```
operator element_difference : Uh(v) -> Poly(k, scalar) on
  element T {
  forall q in Poly(k, scalar):
    int(T) element_difference(v) * q =
    int(T) (potential_reconstruction(v) - dof(v, T)) * q
}

operator edge_difference : Uh(v) -> Poly(k, scalar) on edge E
  of element T {
  forall q in Poly(k, scalar):
    int(E) edge_difference(v) * q =
    int(E) (potential_reconstruction(v) - dof(v, E)) * q
}
```

Forms I

- Forms define the LHS and RHS of the discrete variational problem
- The bilinear form $a_h : \underline{U}_h^k \times \underline{U}_h^k \rightarrow \mathbb{R}$ s.t., for all $(\underline{u}_h, \underline{v}_h) \in \underline{U}_h^k \times \underline{U}_h^k$,

$$a_h(\underline{u}_h, \underline{v}_h) := \sum_{T \in \mathcal{T}_h} \left[\int_T G_T^k \underline{u}_T \cdot G_T^k \underline{v}_T + h_T^{-2} \int_T \delta_T^k \underline{u}_T \delta_T^k \underline{v}_T + h_T^{-1} \sum_{E \in \mathcal{E}_T} \int_E \delta_{TE}^k \underline{u}_T \delta_{TE}^k \underline{v}_T \right]$$

- The source linear form $\ell_h : \underline{U}_h^k \rightarrow \mathbb{R}$ s.t., for all $\underline{v}_h \in \underline{U}_h^k$,

$$\ell_h(\underline{v}_h) := \sum_{T \in \mathcal{T}_h} \int_T f v_T$$

- We also need to account for the Neumann contribution $\int_{\Gamma^N} \phi v_E$

Forms II

```

bilinear form poisson : Uh(trial u) times Uh(test v) {
  sum_elements(
    int(T) gradient_reconstruction(u) dot
    gradient_reconstruction(v)
    + pow(diameter(T), -2) * int(T) element_difference(u) *
    element_difference(v)
    + pow(diameter(T), -1) * int(dT) edge_difference(u) *
    edge_difference(v)
  )
}

```

Forms III

```
linear form load : Uh(test v) {  
    sum_elements(int(T) source * dof(v, T))  
}  
  
boundary labels { left = 1, right = 2, bottom = 3, top = 4 }  
  
linear form neumann_contribution : Uh(test v) {  
    sum_boundary_edges(right)(int(E) exact_normal_flux * dof(v,  
        E))  
}
```

Strongly-enforcing boundary conditions

- Boundary conditions can be enforced strongly considering

$$\underline{U}_{h,g}^k := \{ \underline{v}_h \in \underline{U}_h^k : v_E = \pi_E^k g \text{ for all } E \in \mathcal{E}_h^D \},$$

where $\mathcal{E}_h^D := \{ E \in \mathcal{E}_h : E \subset \Gamma^D \}$

- This is done by enforcing DOF values much like in interpolants

```
boundary conditions mixed_boundary_conditions on Uh {  
  on edge E in left, bottom, top:  
    dof(E) = l2_project(exact_solution, Poly(k, scalar))  
}
```

Linear problem

The HHO problem reads: Find $\underline{u}_h \in \underline{U}_{h,g}^k$ such that

$$a_h(\underline{u}_h, \underline{v}_h) = \ell_h(\underline{v}_h) - \int_{\Gamma^N} \phi v_E \quad \forall \underline{v}_h \in \underline{U}_{h,0}^k$$

```
linear problem hho_poisson_mixed_boundary_conditions_problem
  on Uh {
    lhs { poisson }
    rhs { load - neumann_contribution }
    boundary_conditions mixed_boundary_conditions
  }
```

Export I

- A problem block can be completed with a list of fields to export
- Exportable fields are operators defined on elements

```
operator potential_field : Uh(u) -> Poly(k, scalar) on
  element T {
    potential_field(u) = dof(u, T)
  }

linear problem hho_poisson_mixed_boundary_conditions_problem
  on Uh {
    ...
    export { potential_reconstruction, potential_field }
  }
```

Export II

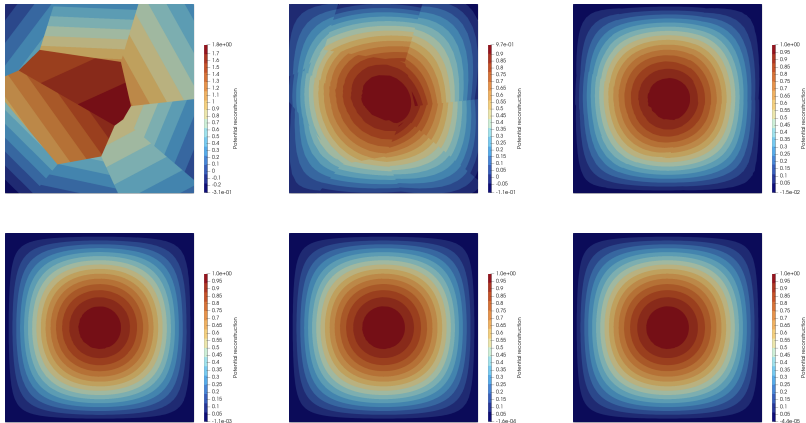


Figure: Evolution of the HHO potential field on a coarse polygonal mesh upon degree-refinement from $k = 0$ to $k = 5$. Notice the high-order representation.

Functionals and error computation I

- When prototyping, one usually need to perform error computations
- In the fully discrete setting, the error is measured by

$$\underline{e}_h := \underline{u}_h - \underline{I}_h^k u$$

- Appropriate error measures in our context are, e.g.,

$$\|\underline{v}_h\|_{a,h} := a_h(\underline{v}_h, \underline{v}_h)^{\frac{1}{2}},$$

$$\|\underline{v}_h\|_{1,h} := \sum_{T \in \mathcal{T}_h} \left[\|\nabla v_T\|_{L^2(T)}^2 + h_T^{-1} \sum_{E \in \mathcal{E}_T} \|v_E - v_T\|_{L^2(E)}^2 \right]$$

- These norms can be implemented in the DSL through **functionals**

Functionals and error computation II

- Functionals are similar to functions, but act on discrete variables

```
function h1_operator_norm : Uh(v) -> scalar {  
  sqrt(poisson(v, v))  
}  
  
function h1_component_norm : Uh(v) -> scalar {  
  sqrt(  
    sum_elements(  
      int(T) squared_norm(grad(dof(v, T)))  
      + pow(diameter(T), -1) * int(dT) pow(dof  
(v, E) - dof(v, T), 2)  
    )  
  )  
}
```

Functionals and error computation III

- We can now tell the code to compute errors as follows:

```
linear problem
  hho_poisson_mixed_boundary_conditions_problem on Uh {
  ...
  compute errors using interpolant_exact_solution {
    h1_component_norm, h1_operator_norm
  }
}
```

- The field `report` can be used to export functionals evaluated at $\underline{u}_h$

Functionals and error computation IV

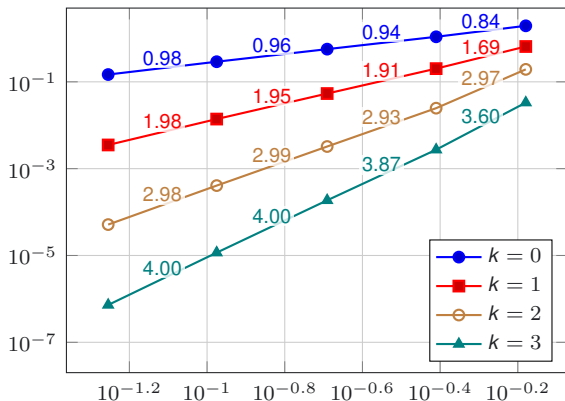


Figure: Convergence plot generated by the script `convergence_test.sh`.

Functionals and error computation V

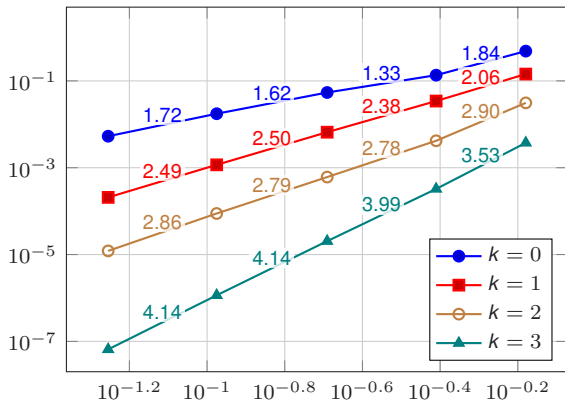


Figure: And DDR^2 for comparison, since we were all waiting for this 🤖.

The solvers

- Basic solvers **without static condensation**
- Optimized for both singleton and Cartesian product spaces
- For non-linear problems, **line search** and **Newton** are implemented
 - ▶ Line search requires **fixed_point** to be defined
 - ▶ Newton requires **jacobian** to be defined
- If both are defined, line search is used for warming-up and as a fallback
- Initialization can be done solving a linear problem or interpolating

- Based on Singly Diagonally Implicit Runge-Kutta (SDIRK):
 - ▶ **Diagonally Implicit:** each stage only depends on the current and previous stages
 - ▶ **Singly Implicit:** all the diagonal elements of the method's internal stage matrix are identical
- Available schemes: `implicit-euler`, `sdirk-4-3`, `sdirk-5-4`
- General formulation possibly including **non-linear unsteady terms**

Highlights

- Thread-based parallelism using rayon
- Automatic **fast polynomial integration**³ through polynomial algebra
- **Centralized repositories** for polynomial families and operators
- Integration of leading **linear solver libraries**:
 - ▶ UMFPACK for direct solvers
 - ▶ PETSc for Krylov solvers

³Chin et al. 2015

Advanced features

Cartesian product spaces

```
space Uh {  
  element Poly(k, vector)  
  edge Poly(k, vector)  
}  
  
space Ph {  
  element Poly(k, scalar)  
}  
  
product space Xh = Uh times Ph  
  
bilinear form velocity_pressure : Uh(trial u) times Ph(test q  
  ) {  
  sum_elements(-int(T) dof(u, T) dot grad(q) + int(dT) (dof(u  
    , E) dot normal) * q)  
}
```

Operators between polynomial spaces

```
operator raviart_thomas_interpolator_polykpo
: Poly(k+1, vector)(u) -> RaviartThomasPoly(k+1) on element
  T {
forall v in Poly(k-1, vector):
  int(T) raviart_thomas_interpolator_polykpo(u) dot v = int
  (T) u dot v
forall edge E, forall q in Poly(k, scalar):
  int(E) (raviart_thomas_interpolator_polykpo(u) dot normal
  ) * q
  = int(E) (u dot normal) * q
}
```

Comparison between HHO and HYPRE for the Kovasznay problem I

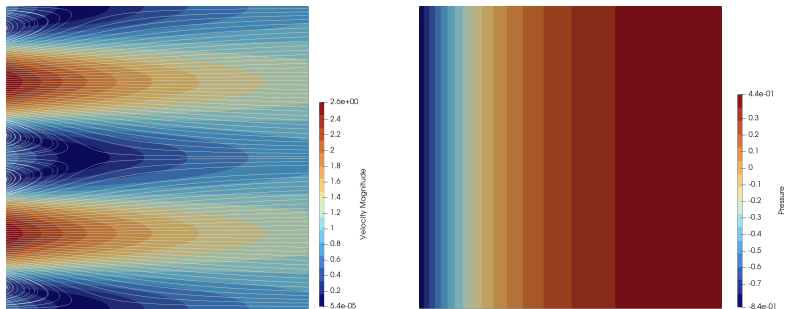


Figure: Velocity streamlines and magnitude (left) and pressure magnitude (right) for the HHO solution of the Kovasznay problem with the scheme of Botti et al. 2019 on a Cartesian mesh.

Comparison between HHO and HYPRE for the Kovasznay problem II

i	h	size	velocity h1 norm		velocity l2 norm		pressure l2 norm	
			error	rate	error	rate	error	rate
$k = 0$								
1	3.5355e-01	736	4.3351e+00	—	1.6481e+00	—	1.5461e-01	—
2	1.7678e-01	3008	2.9067e+00	0.58	5.5621e-01	1.57	7.1725e-02	1.11
3	8.8388e-02	12160	1.5853e+00	0.87	1.5297e-01	1.86	3.0772e-02	1.22
4	4.4194e-02	48896	7.9540e-01	1.00	3.8816e-02	1.98	1.3839e-02	1.15
$k = 1$								
1	3.5355e-01	1856	1.7135e+00	—	3.1462e-01	—	2.7420e-02	—
2	1.7678e-01	7552	4.7569e-01	1.85	4.6311e-02	2.76	5.9505e-03	2.20
3	8.8388e-02	30464	1.1923e-01	2.00	5.8691e-03	2.98	1.4300e-03	2.06
4	4.4194e-02	122368	2.8157e-02	2.08	6.8695e-04	3.09	3.7818e-04	1.92

Table: Convergence of the HHO method using the velocity-pressure coupling bilinear form `velocity_pressure`.

Comparison between HHO and HYPRE for the Kovasznay problem III

i	h	size	velocity h1 norm		velocity l2 norm		pressure l2 norm	
			error	rate	error	rate	error	rate
$k = 0$								
1	3.5355e-01	1073	2.7186e+00	—	9.7862e-01	—	1.8236e-01	—
2	1.7678e-01	4321	2.1961e+00	0.31	4.0189e-01	1.28	9.1591e-02	0.99
3	8.8388e-02	17345	1.4356e+00	0.61	1.3301e-01	1.60	4.1867e-02	1.13
4	4.4194e-02	69505	8.5016e-01	0.76	4.0344e-02	1.72	1.9450e-02	1.11
$k = 1$								
1	3.5355e-01	2529	9.8806e-01	—	1.6405e-01	—	3.0343e-02	—
2	1.7678e-01	10177	2.3755e-01	2.06	2.2946e-02	2.84	5.7044e-03	2.41
3	8.8388e-02	40833	6.6929e-02	1.83	3.2898e-03	2.80	1.4608e-03	1.97
4	4.4194e-02	163585	1.8953e-02	1.82	4.6687e-04	2.82	3.6503e-04	2.00

Table: Convergence of the HYPRE method of Beirão da Veiga et al. 2025 using the `raviart_thomas_interpolator_polykpo` operator.

Edge operators with vertex terms I

- Let

$$\underline{u}_E = (u_E, g_{n,E}, (u_V, g_V)_{V \in \mathcal{V}_E}) \in \mathcal{P}^{k-1}(E) \times \mathcal{P}^k(E) \times \bigtimes_{V \in \mathcal{V}_E} (\mathbb{R} \times \mathbb{R}^2)$$

- Consider the **edge Hessian reconstruction** s.t., for all $w \in \mathcal{P}^{k+1}(E)^2$,

$$\begin{aligned} \int_E \mathbf{H}_E^{k+1} \underline{u}_E \cdot w &= \int_E u_E \partial_{t_E}^2 w \cdot t_E - \int_E g_{n,E} \partial_{t_E} w \cdot n_E \\ &\quad + \sum_{V \in \mathcal{V}_E} \omega_{EV} (g_V \cdot w(V) - u_V \partial_{t_E} w(V) \cdot t_E) \end{aligned}$$

- This operator approximates $\nabla^2 u t_E$

Edge operators with vertex terms II

```
operator edge_hessian : Uh(u) -> Poly(k+1, vector) on edge E
{
  forall w in Poly(k+1, vector):
    int(E) edge_hessian(u) dot w =
      int(E) dof(u, E, edge_value)
        * (tangential_derivative(tangential_derivative(w))
dot tangent(E))
    - int(E) dof(u, E, edge_normal_derivative)
      * (tangential_derivative(w) dot normal(E))
    + sum_vertices(orientation(V, E) * (
      dof(u, V, vertex_gradient) dot w(V)
      - dof(u, V, vertex_value) * (tangential_derivative(
w) dot tangent(E))
    )
  )
}
```

Orthogonal complement polynomial spaces I

■ Let

$$\underline{u}_T = (u_T, (u_E, G_E)_{E \in \mathcal{E}_T}) \in \mathcal{P}^k(T) \times \bigtimes_{E \in \mathcal{E}_T} (\mathcal{P}^k(E)^2 \times \mathcal{P}^k(E))$$

■ Consider the **deflection reconstruction** of Bonaldi et al. 2018 s.t.

► For all $z \in \mathcal{P}^{k+2}(T)$ orthogonal to $\mathcal{P}^1(T)$,

$$\begin{aligned} \int_T \nabla^2 p_T^{k+2} \underline{u}_T : \nabla^2 w &= \int_T u_T \operatorname{div} \operatorname{div} \nabla^2 w \\ &+ \sum_{E \in \mathcal{E}_T} \left[\int_E G_E \nabla^2 w n_{TE} - \int_E u_E (\operatorname{div} \nabla^2 w \cdot n_{TE}) \right], \end{aligned}$$

► For all $z \in \mathcal{P}^1(T)$,

$$\int_T p_T^{k+2} \underline{u}_T z = \int_T u_T z$$

Orthogonal complement polynomial spaces II

```
operator deflection_reconstruction : Uh(u) -> Poly(k+2,
  scalar) on element T {
  forall w in orthogonal complement of Poly(1, scalar)
    relative to Poly(k+2, scalar):
    int(T) grad(grad(deflection_reconstruction(u))) dot grad(
      grad(w)) =
    int(T) dof(u, T) * div(div(grad(grad(w))))
    + int(dT) dof(u, E, edge_gradient) dot (grad(grad(w)) dot
      normal)
    - int(dT) dof(u, E, edge_value) * (div(grad(grad(w))) dot
      normal)
  forall w in Poly(1, scalar):
    int(T) deflection_reconstruction(u) * w = int(T) dof(u, T
    ) * w
}
```

Another comparison, this time on plates

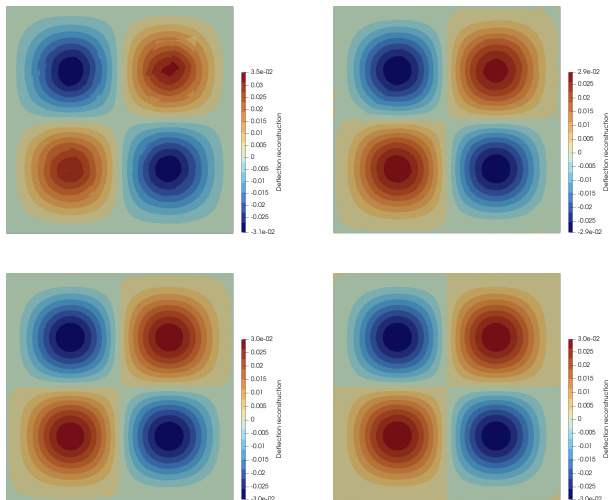


Figure: Comparison of HHO (left) and DDR (right) for the four-point load Kirchhoff–Love problem on a coarse (top) and fine (bottom) mesh.

Subdomains labels I

```
subdomain labels { navier_stokes = 1, darcy = 2 }

residual form convective_residual
: Uh(state w) times Uh(test v) on element T {
  sum_elements(navier_stokes)
  (
    int(T) (grad(dof(w, T)) dot dof(w, T)) dot dof(v, T)
    + 0.5 * int(dT) (dof(w, T) dot normal) * (dof(w, E) -
dof(w, T)) dot (dof(v, E) + dof(v, T))
    + 0.5 * int(dT) abs(dof(w, E) dot normal) * (dof(w, E)
- dof(w, T)) dot (dof(v, E) - dof(v, T))
  )
}
```

Subdomains labels II

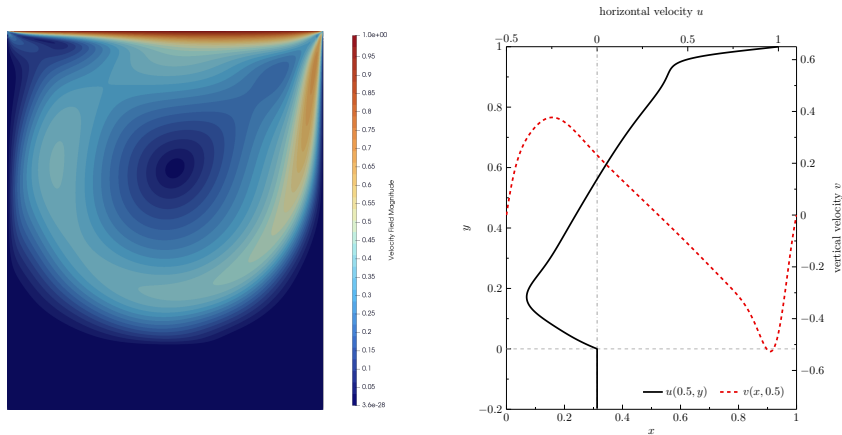


Figure: Lid-driven cavity flow over an impermeable bedding.

An unsteady non-linear problem

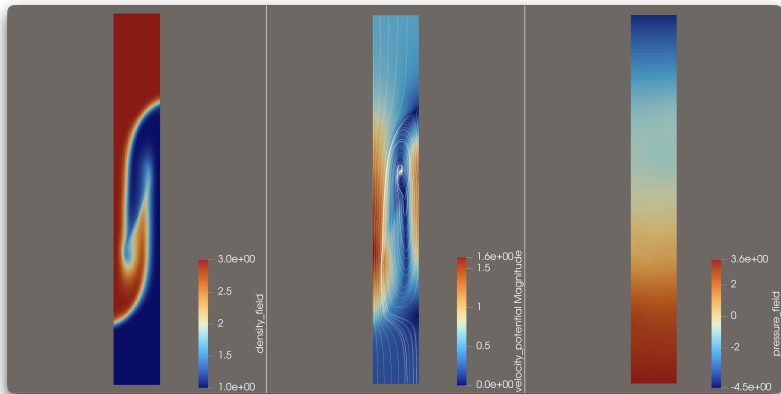


Figure: HHO simulation of the Rayleigh–Taylor instability

3D available since v0.11.0

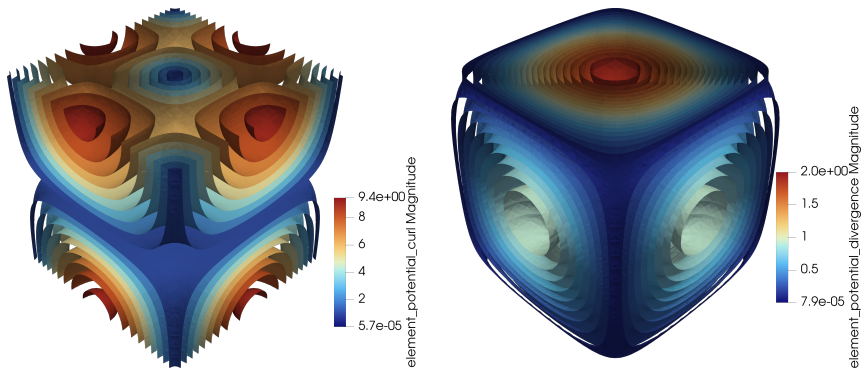


Figure: DDR simulation of the magnetostatic problem⁴

⁴Di Pietro and Droniou 2021



Funded by the European Union (ERC Synergy, NEMESIS, project number 101115663). Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.

Thank you for your attention!

https://imag.umontpellier.fr/~di-pietro/poly_rust/

References



Beirão da Veiga, L., D. A. Di Pietro, and J. Droniou (May 2026). *Key challenges and bridges among convergence analysis techniques for polytopal methods*. arXiv: [2605.23405](https://arxiv.org/abs/2605.23405).



Beirão da Veiga, L., D. A. Di Pietro, J. Droniou, K. B. Haile, and T. J. Radley (2025). “A Reynolds-semi-robust method with hybrid velocity and pressure for the unsteady incompressible Navier–Stokes equations”. In: *SIAM J. Numer. Anal.* 63.6, pp. 2317–2342. DOI: [10.1137/25M1736104](https://doi.org/10.1137/25M1736104).



Bonaldi, F., D. A. Di Pietro, G. Geymonat, and F. Krasucki (2018). “A Hybrid High-Order method for Kirchhoff–Love plate bending problems”. In: *ESAIM: Math. Model Numer. Anal.* 52.2, pp. 393–421. DOI: [10.1051/m2an/2017065](https://doi.org/10.1051/m2an/2017065).



Botti, L., D. A. Di Pietro, and J. Droniou (2019). “A Hybrid High-Order method for the incompressible Navier–Stokes equations based on Temam’s device”. In: *J. Comput. Phys.* 376, pp. 786–816. DOI: [10.1016/j.jcp.2018.10.014](https://doi.org/10.1016/j.jcp.2018.10.014).



Chin, E. B., J. B. Lasserre, and N. Sukumar (2015). “Numerical integration of homogeneous functions on convex and nonconvex polygons and polyhedra”. In: *Comput. Mech.* 56.6, pp. 967–981. DOI: [10.1007/s00466-015-1213-7](https://doi.org/10.1007/s00466-015-1213-7).



Di Pietro, D. A. and J. Droniou (2021). “An arbitrary-order method for magnetostatics on polyhedral meshes based on a discrete de Rham sequence”. In: *J. Comput. Phys.* 429.109991. DOI: [10.1016/j.jcp.2020.109991](https://doi.org/10.1016/j.jcp.2020.109991).



Di Pietro, D. A., A. Ern, and S. Lemaire (2014). “An arbitrary-order and compact-stencil discretization of diffusion on general meshes based on local reconstruction operators”. In: *Comput. Meth. Appl. Math.* 14.4, pp. 461–472. DOI: [10.1515/cmam-2014-0018](https://doi.org/10.1515/cmam-2014-0018).