Multilevel solution strategies for discontinuous Galerkin methods in continuum mechanics.

I will consider agglomeration based multilevel solution strategies as an effective means of solving discontinuous Galerkin (dG) discretizations of linear elasticity problems in continuum mechanics. The capabilities of the solution strategy will be demonstrated tackling thin-walled tree-dimensional geometries with the final ambitious goal of simulating injection-blow molding of plastic containers.

The dG method's ability of dealing with polyhedral elements meshes [1] dramatically simplifies the generation of nested elements meshes, a fundamental ingredient of geometric multigrid implementations. A coarse mesh can be obtained starting from the fine computational grid by agglomeration coarsening, that is clustering together a number of neighboring elements of the fine grid to produce coarse agglomerated elements of very general shape. The agglomeration process can be repeated recursively to generate a sequence of nested grids whose coarsening steepness can be controlled at will by tuning the cardinality of clusters. Since arbitrarily unstructured, possibly hybrid, polyhedral grids are admissible as input for the agglomeration process, the solution strategy does not require to alter the mesh generation workflow, even in the case of complex computational domains.

Based on the tools provided in [2,3] the performance of the resulting multilevel solution strategy can be analysed to obtain convergence rates estimates that are uniform with respect to the number of levels. Numerical evidence will supplement the theoretical findings. Moreover, I will provide implementation details regarding the construction of coarse grid operators that are crucial for the sake of computational efficiency [4].

[1] F. Bassi, L. Botti, A. Colombo, D.A. Di Pietro, P. Tesini; On the flexibility of agglomeration based physical space discontinuous Galerkin discretizations; Journal of Computational Physics, Volume 231, Issue 1, 2012, Pages 45-65.

[2] S. Brenner; Convergence of nonconforming *V*-cycle and *F*-cycle multigrid algorithms for second order elliptic boundary value problems; Mathematics of Computation, Volume 73, Number 247, 2004, Pages 1041–1066.

[3] P. F. Antonietti, M. Sarti, and M. Verani; Multigrid Algorithms for *hp*-Discontinuous Galerkin Discretizations of Elliptic Problems; SIAM Journal on Numerical Analysis, Volume 53, Issue 1, 2015, Pages 598-618.

[4] L. Botti, A. Colombo, F. Bassi; h-multigrid agglomeration based solution strategies for discontinuous Galerkin discretizations of incompressible flow problems; Journal of Computational Physics, Volume 347, 2017, Pages 382-415.