Semi-analytical solutions for transport equations in heterogeneous media

This talk considers two PDE problems encountered when modelling groundwater flow and contaminant transport in aquifers. Such applications are particularly challenging since one must deal with a heterogeneous geological structure where physical properties vary spatially. Firstly, I will describe a semi-analytical solution to the advection-dispersion-reaction equation in a layered medium, which arises when modelling solute transport through soil layers in an aquifer. The proposed solution approach involves reformulating the problem by introducing unknown functions representing the dispersive flux at the interfaces between adjacent layers. Secondly, I will consider the issue of small-scale heterogeneity in hydraulic conductivity when modelling groundwater flow. For this problem, directly solving the governing PDE is computationally prohibitive since one has to discretise the computational domain with a very fine mesh to accurately capture the heterogeneity. A common approach to overcome this issue is to coarse-grain the model by decomposing the domain into several sub-domains and homogenizing the heterogeneous medium within each sub-domain. Here, I will show how the semi-analytical solution approach for one-dimensional layered media can be extended to two-dimensional block media and how this allows for semi-analytical homogenization of complex heterogeneous media. For both the solute and groundwater flow problems, numerical experiments will be presented to demonstrate application of our approach and verify it produces the correct results.