Finite Element Methods for Modeling Multiphase Flow and Fluid-Structure Interaction in Coastal and Hydraulic Engineering Applications

We ultimately seek to improve predictions of water levels and hydrodynamic forces on coastal and hydraulic solid structures, possibly including erosion and deposition of granular materials and displacement of structures. The direct approach to such predictions involves solving the incompressible Navier-Stokes equations for water and air, possibly also with the coupled dynamics of a collection of rigid or flexible solid structures. I will present the level set finite element methods we have implemented in software at the US Army Coastal and Hydraulics Laboratory for approximating the air/water immersed boundary problem and the fluid/solid embedded interface problem and some of our recent research directions. The methods presented are based fundamentally on representing interfaces as signed distance functions in a manner that maintains mass and volume conservation [1,2]. Our initial research and development employed a well-known collection of level set methods that regularize the phase interfaces and avoid cut cell integration. This approach both reduces accuracy and limits convergence rates to first order in the characteristic mesh diameter. Such methods employ regularized or discrete Heaviside and Dirac distributions, which are the primary source of reduced accuracy. More recently, we have employed the equivalent polynomials approach [3] to recover exact integration over cut cells and cut cell boundaries. This approach does not require explicit formation of a cut cell mesh but nevertheless supports exact integration of cut cell methods. We then reformulate no-slip fluid/solid boundaries using Nitsche's method [4,5] and air/water jump conditions using immersed finite element basis functions constructed on the fly to recover jump conditions exactly [6,7]. It is well-known that coercivity and accuracy issues can arise due to arbitrarily cut cells in these approaches, but several authors have addressed these issues with various types of edge penalties or aggregate cells that expand the stencil of cut cells [5,7,8]. I will present the resulting formulations as well as numerical results on test problems involving moving solids and water waves motivated by coastal and hydraulic engineering applications.

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