The numerical solution of time-fractional initial-boundary value problems

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An introduction to fractional derivatives and some of their properties will be presented. The regularity of solutions to Caputo fractional initial-value problems is then discussed; it is shown that typical solutions have a weak singularity at the initial time $t = 0$. This singularity has to be taken into account when designing and analysing numerical methods for the solution of such problems. To address this difficulty we use graded meshes, which cluster mesh points near $t = 0$, and answer the question: how exactly should the mesh grading be chosen? Finally, initial-boundary value problems are considered, where the time derivative is a Caputo fractional derivative. (This is a fractional-derivative generalisation of the classical parabolic heat equation.) Once again a weak singularity appears at $t = 0$, and the mesh in the time coordinate should be graded to compute satisfactory numerical solutions.