Transition in subcritical turbulent shear flows – invariant solutions and optimally growing states on the edge of chaos

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The rate of flow through a pipe is measured by the Reynolds number, Re, and the flow typically undergoes a transition from (smooth) laminar flow to (chaotic) turbulence for $Re > 2000$. This result dates back to Osborne Reynolds' observations in 1883 [1]. For a long time, the only theoretical bound for transition, based on energy stability, predicted that $Re > 81$ [2], a long way off the observed $Re > 2000$. The difficulty stemmed from the absence of nonlinear solutions to the Navier-Stokes equations that could be found for pipe flow. (The situation was simiar for many other simple shear-flows.) Reynolds had observed that the transition occurs in the absence of a linear instability, so that the transition is inherently nonlinear. Without the linear instability, no new branch of solutions can be tracked off the laminar solution. A new approach was needed.

The development of methods to numerically compute solutions for high-dimensional systems has revolutionised our approach to understanding the transition to turbulence. This has enabled us to extract invariant solutions (equilibria and periodic orbits) directly from simulations. Taken directly from the natural measure of turbulence, we have good reason to believe that these solutions provide a 'skeleton' or 'backbone' for the dynamics, which can now be viewed as a 1-dimensional trajectory that shadows these solutions in a high-dimensional phase space [3].

In this talk I will outline some of the methods that have enabled us to find solutions within turbulence (Newton-Krylov [4]; slicing [5]), methods to probe the laminar-turbulent boundary (edge-tracking [6]; adjoint optimisation [7]), and I will point out some of the challenges that still remain in their application to a high-dimensional system.

References

[1] O. Reynolds. An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and the law of resistance in parallel channels. *Proc. Roy. Soc. Lond. Ser A*, 174:935–982, 1883.

[2] DD Joseph and S Carmi. Stability of poiseuille flow in pipes, annuli, and channels. *Quarterly of applied Mathematics*, 26(4):575–599, 1969.

[3] N.B. Budanur, K.Y. Short, M. Farazmand, A.P. Willis, and P. Cvitanović. Relative periodic orbits form the backbone of turbulent pipe flow. *J. Fluid Mech.*, 833:274–301, 2017.

[4] Dana A Knoll and David E Keyes. Jacobian-free newton–krylov methods: a survey of approaches and applications. *Journal of Computational Physics*, 193(2):357–397, 2004.

[5] A. P. Willis, K. Y. Short, and P. Cvitanović. Symmetry reduction in high dimensions, illustrated in a turbulent pipe. *Phys. Rev. E*, 93:022204, 2016.

[6] J. D. Skufca, J. A. Yorke, and B. Eckhardt. Edge of Chaos in a parallel shear flow. *Phys. Rev. Lett.*, 96:174101, 2006.

[7] C. C. T. Pringle, A. P. Willis, and R. R. Kerswell. Minimal seeds for shear flow turbulence: using nonlinear transient growth to touch the edge of chaos. *J. Fluid Mech.*, 702:415–443, 2012.