High-order generalized-α methods for dynamic phase-field problems

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ABSTRACT

The generalized-α method was introduced by Chung and Hulbert in [4] for solving structural dynamics problems. Later, the mothod was modified to deal with the computational fluid dynamics governed by the parabolic differential equations [6]. Then, the analysis corresponding to applying the method to non-linear problems was introduced in [5]. This time-marching time integrator provides second-order accuracy in time and has a feature of user-control on the numerical dissipation in the higher frequencies of the discrete spectrum. This method includes a wide range of time integrators such as the Newmark method, the HHT-α method by Hilber, Hughes, and Taylor, and the WBZ-α method by Wood, Bossak, and Zienkiewicz; see [4].

We propose a new class of high-order generalized-α methods following [1, 2, 3] for general types of non-linear phase-field models featuring high-order spatial differential equations that maintain all the attractive features of the original generalized-α method. In particular, we extend the time-marching technique to any arbitrary order of accuracy in time that is unconditionally stable to address a wide range of dynamic problems such as Swift-Hohenberg, Cahn-Hilliard, and Phase-field crystal. Our method also provides the user-control on the numerical dissipation in the higher frequencies of the discrete spectrum as well as delivering an optimal behavior for low-frequencies domains. Finally, we present numerical results to verify the stability and its high-order accuracy in time. For this aim, we consider a NURBS-based variational formulation, and the spatial discretization relies on isogeometric analysis.

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