B-char: an efficient (and feasible!) approach for mass-conserving characteristic schemes in 2D and 3D

Models in underground reservoir often mix diffusion and advection process, the latter being dominant most of the time. For linear advection, very classical in transport of solutes, characteristic-based methods are an interesting approach compared to classical upwinding (which initroduces numerical diffusion). These methods must be patched with the underlying mesh, to respect the data coming from the discretisation (e.g. via finite-volume methods) of the diffusive and reactive processes.

Combining characteristic-based methods with finite-volume methods requires to transport, via the flow, each cell and compute its intersection with other (non-transported) cells. This leads to a problem of computing (in 2D) the area of the intersection of two generic polygons, which is computationaly expensive. The situation is even more problematic in 3D, where the question is to compute the volume of the intersection of two polyhedras – actually, very few algorithms exist, and they are expensive and require convexity of at least one of the two polyhedra.

We will present here a novel approach to implement characteristic-based methods, using the Eulerian Lagrangian Localised Adjoint Method (ELLAM) as an illustration. The idea is to approximate polyhedra by balls, to track these balls, and to compute only intersections of balls – which is extremely cheap. The standard ELLAM, tracking and intersecting polygons, has local and global mass-conservation properties, which are broken when approximating the cells by balls. An adjustment of the intersection, combining algebraic balancing and optimisation, is used to recover local and global mass balance. Numerical tests demonstrate the accuracy of the method, despite the rough approximation of polyhedras by balls, and its efficiency – the computational time in 2D is reduced by several orders of magnitudes compared to the standard ELLAM, and 3D implementations are for the first time accessible.