Multilevel quasi-Monte Carlo methods for random elliptic eigenvalue problems

Motivated by uncertainty quantification for the neutron diffusion criticality problem, we will study an elliptic eigenvalue problem with coefficients that depend on infinitely many stochastic parameters. The stochasticity in the coefficients causes the eigenvalues and eigenfunctions to also be stochastic, and so our goal is to compute the expectation of the minimal eigenvalue. In practice, to approximate this expectation one must:

1) truncate the stochastic dimension;
2) discretise the eigenvalue problem in space (e.g., by finite elements); and
3) apply a quadrature rule to estimate the expected value.

In this talk, we will present a multilevel Monte Carlo method for approximating the expectation of the minimal eigenvalue, which is based on a hierarchy of finite element meshes and truncation dimensions. To improve the sampling efficiency over Monte Carlo we will use a quasi-Monte Carlo rule to generate the sampling points. Quasi-Monte Carlo rules are deterministic (or quasi-random) quadrature rules that are well-suited to high-dimensional integration and can converge at a rate of 1/N, which is faster than the rate for Monte Carlo. Also, to make each eigenproblem solve on a given level more efficient, we utilise the two-grid scheme from [Xu & Zhou 1999] to obtain the eigenvalue on the fine mesh from the coarse (i.e., coarser FE mesh and lower truncation dimension eigenvalue (and eigenfunction) with a single linear solve.