A Weak Galerkin Finite Element Method for Elliptic and Parabolic Problems on Polygonal Meshes.

Weak Galerkin (WG) method is based on weak functions and weak gradient and it allows the use of totally discontinuous function in the finite element algorithm. We will discuss how the numerical schemes are designed and why they provide reliable numerical approximation for the underlying partial differential equations. In the implementation, the weak function and weak gradient are approximated by polynomial with various degree of freedom. The accuracy and the computational complexity of the corresponding WG scheme is significantly impacted by the selection of such polynomials. Our main task is that how to choose an optimal combination for the polynomial spaces that minimize the number of unknowns in the numerical scheme without compromising the accuracy of the numerical approximation. For illustrative purpose, we use the second order elliptic and linear parabolic equations. Optimal order errors are obtained from the corresponding WG−FEM approximation in both discrete H^1−norm and standard L^2−norm. Moreover, the new WG algorithm allows the use of finite element partition consisting of general polygonal meshes and can be easily generalized to high orders. With the help of WG algorithm, we will discuss semi−discrete and fully discrete finite element error estimate for linear parabolic problem under minimal regularity with the help of L^2− projections. At the end, we will discuss numerical experiments to verified the theoretical results.