Scalability analysis of the distributed-memory implementation of the Aggregated unfitted Finite Element Method (AgFEM)

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Unfitted (a.k.a. immersed or embedded) finite element (FE) discretization methods discretize the PDE problem at hand by embedding the computational domain in a background mesh which does not conform to its geometrical boundary. Thus, there might be cells intersecting the domain boundary, referred to as cut cells. The background mesh can be thus as simple as a uniform (or, more generally, octree-based adaptive) Cartesian-like mesh. This sort of meshes can be very efficiently manipulated in peta-scale computing environments. Besides, unfitted FE methods are also very helpful in those scenarios in which the generation of body-conforming meshes is impractical/cumbersome, such as, e.g., PDE problems posed on complex, evolving-in-time geometries, shape or topology optimization, or uncertainty quantification under stochastic geometries.

Unfortunately, unfitted FE methods have also well-known drawbacks. The most notorious one is the so-called small cut cell problem: the condition number of the linear system also depends on the characteristic size of the cut cells. The intersection of a background cell with the physical domain can be arbitrarily small and with arbitrarily high aspect ratios, typically resulting in severely ill-conditioned linear systems. This flaw, if ignored at the discretization level, has to be dealt with by the solver itself. While iterative solvers are much better suited (compared to direct solvers) for the efficient exploitation of distributed-memory computers, it is cornerstone that they are equipped with an efficient preconditioner for robustness and high parallel scalability.

Instead of designing an ad-hoc, suitably adapted preconditioner, we consider the recently introduced AgFEM [1], an enhanced FE formulation that leads to system matrices that are not affected by small cuts. AgFEM is based on removal of basis functions associated with badly cut cells by introducing carefully designed constraints and its formulation shares the remarkable properties of standard FEs such as stability, condition number bounds, optimal convergence, and continuity with respect to data. This opens the door to considering well-known scalable preconditioners such as Algebraic MultiGrid, widespread in state-of-the-art parallel scientific computing packages (e.g., PETSc). In this talk we will first present the AgFEM method and briefly overview its main numerical properties [1]. Then, we will discuss parallelization aspects of the method [2]. Finally, numerical results will evaluate its parallel scalability in the solution of the Poisson PDE on complex 3D domains. A weak scaling test up to 16K cores and up to nearly 300M DOFs in the MN-IV supercomputer confirms the high suitability of AgFEM for large-scale simulations. Recent results corresponding to the combination of AgFEM with octree-based adaptive meshes in the solution of classical hp-FEM benchmark problems with solution singularities or multiple shocks will also be presented.

[1] S. Badia, F. Verdugo, A.F. Martin. The aggregated unfitted finite element method for elliptic problems. Computer Methods in Applied Mechanics and Engineering 336, 533-553, 2018.

[2] F. Verdugo, A. F. Martin, S. Badia. Distributed-memory parallelization of the aggregated unfitted finite element method. Computer Methods in Applied Mechanics and Engineering 357, 112583, 2019.