An h-adaptive unfitted finite element method for interface elliptic boundary value problems

A frequent hurdle in numerical simulation pipelines is the generation of meshes conforming to the geometry of the domain of analysis (a.k.a. body–fitted meshes). Current body–fitted mesh generators cannot scale up to a large number of CPUs and, often, require manual work to tune parameters. As a result, mesh generation is estimated to make 80% of the total time spent in the design of simulations in practical industrial applications [1]. This bottleneck is especially severe when modelling transient multi–physics problems, such as fluid–structure interaction or additive manufacturing (3D printing) processes. In those cases, body–fitted meshes tracking evolving interfaces or geometries in time are impractical to produce. An alternative approach is to resort to unfitted (a.k.a. embedded) methods. The main idea is to embed the complex physical domain into an artificial background domain with simple geometry. The background domain can then be efficiently meshed with, e.g., Cartesian grids. Adding to this setting a strategy to deal with the cells cut by the physical domain allows one to perform the analysis on the physical domain using a simple background mesh.

Even though unfitted methods circumvent the mesh generation bottleneck, they are prone to serious ill conditioning problems. The most prominent cause is the so–called small cut cell problem, arising when the condition number depends on the characteristic size of the cut cells. Small cuts can lead to arbitrarily high condition numbers and preclude efficient use of iterative Krylov methods and, thus, practical usage of unfitted methods for realistic large scale applications. To avoid this issue in the context of finite elements, we consider the aggregated unfitted finite element method, referred to as AgFEM. The technique is based on constructing enhanced finite element spaces by cell aggregation; shape functions on small cut cells are removed by enforcing suitable algebraic constraints. AgFEM has been introduced for elliptic [2] and Stokes [3] PDEs. For these problems, it retains good properties of standard finite element methods, independent on cut location, and exhibits good parallel performance and scalability on both uniform [4] and tree–based meshes.

This work presents the extension of AgFEM to interface elliptic boundary value problems. The Nitsche method is used to approximate the problem, whereas independent cell aggregation schemes are carried out at each subdomain to remove ill–posed degrees of freedom. Mathematical analysis proves well–posedness, approximation properties and a priori error estimates analogous to standard finite elements. Numerical experiments assess optimality and robustness of the method on tree–based meshes, regardless of cut location and material contrast. Good scaling properties are also retained.

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