Convergence analysis of a numerical scheme for a tumour growth model

We consider a one-spatial dimensional tumour growth model that consists of three variables; volume fraction and velocity of tumour cells, and nutrient concentration. The model variables form a coupled system of semi–linear advection equation (hyperbolic), simplified stationary Stokes equation (elliptic), and linear diffusion equation (parabolic) with appropriate conditions on the time–dependent boundary which is governed by an ordinary differential equation. An equivalent formulation in a fixed domain is used to overcome the difficulty associated with the time–dependent boundary in the original model. Though this reduces complexity of the model, the tight coupling between the component equations and their non–linear nature offer challenges in proving suitable \emph{a priori} estimates. A numerical scheme that employs a finite volume method for the hyperbolic equation, a finite element method for the elliptic equation and a mass–lumped finite element method for the parabolic equation is developed. We establish the existence of a time interval $(0,T_{\ast})$ over which the convergence of the scheme using compactness techniques is proved, thus proving the existence of a solution. Numerical tests and justifications confirm the theoretical findings.