Current state of $k-R$ WL cylinder, $k-\varepsilon-\gamma$ model and Imersed boundary method

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Averaged N.S equations

■ Problem Averaged Navier-Stokes compressible equations with $k - \varepsilon$ closure model :
Find $(\rho, \rho\mathbf{u}, \rho E, \rho k, \rho\epsilon)$ solution of :

$$\left\{\begin{array}{rcl} \frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) & = & 0, \\ \frac{\partial\rho\mathbf{u}}{\partial t} + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u}) & = & -\nabla p + \frac{1}{Re}\nabla\cdot\sigma + \nabla\cdot\tau, \\ \frac{\partial\rho E}{\partial t} + \nabla\cdot(\rho E + p)\mathbf{u} & = & \frac{1}{Re}\nabla\cdot\sigma\mathbf{u} + \nabla\cdot\tau\mathbf{u} + \left(\frac{\gamma}{PrRe} + \frac{\gamma}{Pr_t Re_t}\right)\nabla h \\ \frac{\partial\rho k}{\partial t} + \nabla\cdot(\rho\mathbf{u}k) & = & \tau : \nabla\mathbf{u} + \nabla\cdot[(\mu + \mu_t\sigma_k)\nabla k] - \rho\varepsilon, \\ \frac{\partial\rho\varepsilon}{\partial t} + \nabla\cdot(\rho\mathbf{u}\varepsilon) & = & \left(c_\varepsilon^{(1)}\frac{\varepsilon}{k}\tau : \nabla\mathbf{u} - c_\varepsilon^{(2)}\rho\frac{\varepsilon^2}{k} + C^{(2)}\right)C^{(1)} + \nabla\cdot[(\mu + \mu_t\sigma_\varepsilon)\nabla\varepsilon] \end{array}\right. \tag{1}$$

with $\mu_T = c_\mu f_\mu \frac{k^2}{\epsilon}$.

- Change $k-\varepsilon$ by a simplified $k-R$ closure model :

$$\begin{aligned}\frac{\partial \rho k}{\partial t}+\nabla\cdot(\rho\mathbf{u}k)&=\mu_t\mathfrak{S}^2+\nabla\cdot[(\mu+\mu_t\sigma_k)\nabla k]+\rho\frac{k^2}{R},\\ \frac{\partial \rho R}{\partial t}+\nabla\cdot(\rho\mathbf{u}R)&=c_1T_t\mu_t\mathfrak{S}^2-\min\left(\rho c_2 k,\mu_t\frac{|\Omega|}{a_1}\right)+\nabla\cdot\left[\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\nabla R\right].\end{aligned} \tag{2}$$

With eddy viscosity

$$\mu_t=\rho c_\mu f_\mu\left[\underbrace{kT_t(1-f_c)}_{\mu_t^{(1)}}+\underbrace{Rf_c}_{\mu_t^{(2)}}\right]. \tag{3}$$

- Using [Jaeger and Dhatt, 1992] and [GOLDBERG and OTA, 1990] works :

$$\text{if } y^+ > 10 : \quad k = \frac{u_f^2}{\sqrt{C_\mu}}, \quad \varepsilon = \frac{|u_f|^3}{\kappa\delta} \tag{4}$$

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$$\text{if } y^+ > 10 : \quad k = \frac{u_f^2}{\sqrt{C_\mu}}, \quad \varepsilon = \frac{|u_f|^3}{\kappa\delta} \tag{5}$$

$$\text{if } y^+ < 10 : \quad k = \frac{u_f^2}{\sqrt{C_\mu}}\left(\frac{y^+}{\delta}\right)^2, \quad \varepsilon = Re\frac{u_f^4}{10\kappa}\left[\left(\frac{y^+}{\delta}\right)^2 + 0.2\frac{\kappa}{\sqrt{C_\mu}}\left(1 - \left(\frac{y^+}{\delta}\right)^2\right)\right] \tag{6}$$

- Using [Jaeger and Dhatt, 1992] and [GOLDBERG and OTA, 1990] works :

$$\text{if } y^+ > 10: \quad k = \frac{u_f^2}{\sqrt{C_\mu}}, \quad \varepsilon = \frac{|u_f|^3}{\kappa\delta} \tag{7}$$

$$\text{if } y^+ < 10: \quad k = \frac{u_f^2}{\sqrt{C_\mu}}\left(\frac{y^+}{\delta}\right)^2, \quad \varepsilon = Re\frac{u_f^4}{10\kappa}\left[\left(\frac{y^+}{\delta}\right)^2 + 0.2\frac{\kappa}{\sqrt{C_\mu}}\left(1-\left(\frac{y^+}{\delta}\right)^2\right)\right] \tag{8}$$

- Using equations 7, 8 and $R = \frac{k^2}{\varepsilon}$:

$$\text{if } y^+ > 10: \quad R = \frac{|u_f|}{C_\mu}\kappa\delta,$$

$$\text{if } y^+ < 10: \quad R = \frac{10\kappa}{C_\mu Re}\left[\frac{\alpha}{\alpha + 0.2\frac{\kappa}{\sqrt{C_\mu}}(1-\alpha)}\right], \quad \text{with } \alpha = \left(\frac{y^+}{\delta}\right)^2.$$

■ Mesh : $y_w^+ = 1 \Leftrightarrow \delta = 2 \times 10^{-4}$

■ Set up

- Mach $= 0.1$, Re $= 3900$
- $U_\infty = 34.025$, $\rho_\infty = 1.225$
- turbulence intensity : $I_k = 0.5\%$
- $k_\infty = \frac{3}{2}(I_k U_\infty)^2$, $\varepsilon_\infty = k_\infty/10$

■ Boundary conditions :

$$R_{\partial C} = 0, \quad \text{and } R_\infty = \frac{k_\infty^2}{\varepsilon_\infty}$$

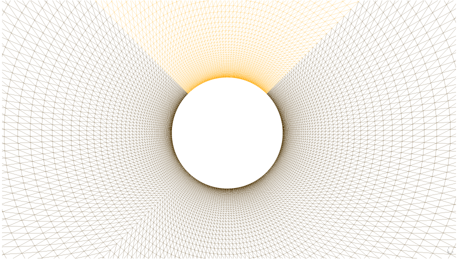
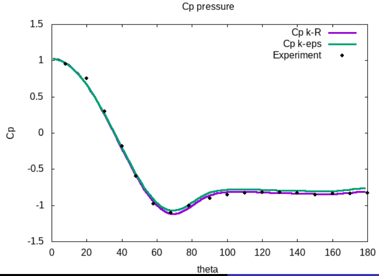


Figure – Cylinder mesh

Name	Mesh size	δ_w	$\overline{C}_d$	C'_l	$-\overline{C}_{pb}$	L_r	$\overline{\theta}$	St
Present simulation								
$k-\varepsilon$ **Goldberg 3D**	**176K**	**0.002**	**0.96**	**0.11**	**0.85**	**1.56**	**111**	**0.20**
$k-R$	**176K**	**0.002**	**1.00**	**0.11**	**0.86**	**1.53**	**93**	**0.20**
Numerical simulation								
Spalart 3D [?]	-	**0.002**	**0.97**	**0.11**	**0.83**	**1.67**	**89**	**0.21**
DVMS WALE 3D [?]	**1.46M**	**0.004**	**0.94**	-	**0.85**	**1.47**	-	**0.22**
Experiment								
[Norberg, 1994]	-	-	**0.94-1.04**	-	**0.84-0.93**	-	-	**0.20**
[Parnaudeau et al., 2008]	-	-	-	**0.1**	-	**1.41-1.58**	-	-
[Lourenço, 1993]	-	-	-	-	-	-	**86**	-

Table – Bulk coefficient of the flow around a circular cylinder at Reynolds number 3900, $\overline{C}_d$ holds for the mean drag coefficient, $\overline{C}'_l$ is the root mean square of lift time fluctuation, $\overline{C}_{p_b}$ is the pressure coefficient at cylinder basis, L_r is the mean recirculation length, $\overline{\theta}$ is the mean separation angle.



- Re = 1M using WL

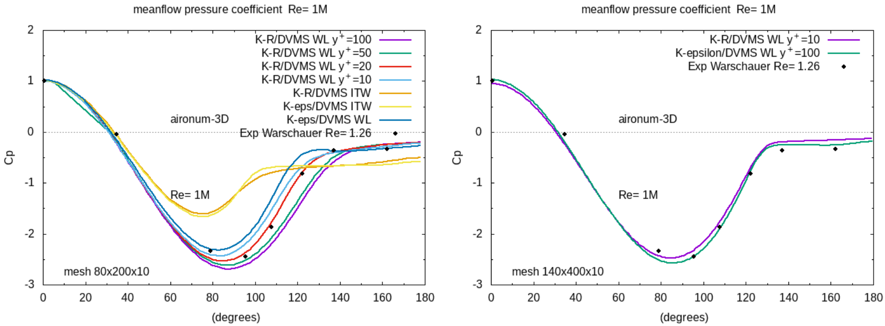


Figure – Mean pressure distribution on body

Name	Mesh size	δ_w	$\overline{C}_d$	C_l'	$-\overline{C}_{pb}$	L_r	$\overline{\theta}$	St
ITW simulations								
$k-\varepsilon$ **Goldberg / DVMS 3D**	**176K**	**0.002**	**0.65**	**0.13**	**0.63**	**1.30**	**100**	**0.28**
$k-R$ **/ DVMS**	**176K**	**0.002**	**0.60**	**0.04**	**0.50**	**1.74**	**105**	**0.30**
WL simulations								
$k-\varepsilon$ **Goldberg / DVMS 3D**	**176K**	**0.002**	**0.25**	**0.08**	**0.25**	**1.10**	**125**	**0.05**
$k-R$ **/ DVMS** $y_m^+ = $ **10**	**176K**	**0.002**	**0.29**	**0.08**	**0.21**	**0.77**	**133**	**0.08**
$k-R$ **/ DVMS** $y_m^+ = $ **20**	**176K**	**0.002**	**0.31**	**0.11**	**0.20**	**0.62**	**140**	**0.06**
$k-R$ **/ DVMS** $y_m^+ = $ **10**	**572K**	5×10^{-5}	**0.18**	**0.02**	**0.14**	**0.84**	**135**	**0.56**
Experiments								
al **[Shih et al., 1993]**			**0.24**	**-**	**0.33**			
[Schewe, 1983]			**0.22**	**-**	**-**			
[Gölling, 2006]						**-**	**130**	
[Zdravkovich, 1997]			**0.2-0.4**	**0.1-0.15**	**0.2-0.34**			

Table – Bulk coefficient of the flow around a circular cylinder at Reynolds number 3900, $\overline{C}_d$ holds for the mean drag coefficient, $\overline{C}_l'$ is the root mean square of lift time fluctuation, $\overline{C}_{p_b}$ is the pressure coefficient at cylinder basis, L_r is the mean recirculation length, $\overline{\theta}$ is the mean separation angle.

In summery :

- $k - R$ works very well for low Reynolds
- Hybrid wall law $k - R$ gives better results using coarse grid.
- ITW computation can't be established for fine mesh.

To do :

- Improved implicitation ?
- Modify the mesh ?
- given up the $k - R$

Part 2 : Current status of $k - \varepsilon - \gamma$

- Akhter 2015 transitional model :

$$\frac{\partial \rho\gamma}{\partial t} + \nabla\cdot\rho\mathbf{u}\gamma = \underbrace{c_{g1}\gamma(1-\gamma)\frac{2\mu_t S^2}{k}}_{Production} + \underbrace{\rho\frac{c_{g2}}{\beta*}\rho\frac{k}{\omega}\nabla\gamma\cdot\nabla\gamma}_{Auxiliary\ \ production} \tag{9}$$

$$+\underbrace{\nabla\cdot[\sigma_\gamma(1-\gamma)(\mu+\mu_t)\nabla\gamma]}_{Dissipation}$$

with $c_{g1} = 0.19$, $c_g2 = 1.0 = \sigma_\gamma$, $c_{\mu g} = 10^{-3}$ $\mu_t = k/\omega$ and

$$\mu_t^* = \left[1 + c_{\mu g}\frac{k}{\omega^2}\gamma^{-2}(1-\gamma)\|\nabla\gamma\|^2\right]\mu_t \tag{10}$$

- Akhter 2015 transitional model :

$$\frac{\partial \rho\gamma}{\partial t} + \nabla \cdot \rho \mathbf{u}\gamma = \underbrace{c_{g1}\gamma(1-\gamma)\frac{2\mu_t S^2}{k}}_{Production} + \underbrace{\rho\frac{c_{g2}}{\beta^*}\rho\frac{k}{\omega}\nabla\gamma\cdot\nabla\gamma}_{Auxiliary\ \ production} \tag{11}$$

$$+ \underbrace{\nabla\cdot[\sigma_\gamma(1-\gamma)(\mu+\mu_t)\nabla\gamma]}_{Dissipation}$$

with $c_{g1} = 0.19$, $c_g 2 = 1.0 = \sigma_\gamma$, $c_{\mu g} = 10^{-3}$ $\mu_t = k/\omega$ and

$$\mu_t^* = \left[1 + c_{\mu g}\frac{k}{\omega^2}\gamma^{-2}(1-\gamma)\|\nabla\gamma\|^2\right]\mu_t \tag{12}$$

- Using $\varepsilon = \beta^*\omega k$, equations can be transformed in :

$$\frac{\partial \rho\gamma}{\partial t} + \nabla \cdot \rho \mathbf{u}\gamma = \underbrace{C_{g1}\gamma(1-\gamma)\frac{P_k}{k}}_{Production} + \underbrace{\rho C_{g2}\frac{k^2}{\varepsilon}\nabla\gamma\cdot\nabla\gamma}_{Auxiliary\ \ production} \tag{13}$$

$$+ \nabla\cdot\underbrace{[\sigma_\gamma(1-\gamma)(\mu+\mu_t)\nabla\gamma]}_{\mathcal{D}_\gamma} \tag{14}$$

with $C_{\mu g} = 10^{-7} = c_{\mu g}(\beta^*)^2$ and the turbulent viscosity

$$\mu_t^* = \left[1 + C_{\mu g}\frac{k^3}{\varepsilon^2}\gamma^{-2}(1-\gamma)\|\nabla\gamma\|^2\right]c_\mu f_\mu\frac{k^2}{\epsilon} \tag{15}$$

■ Problem Averaged Navier-Stokes compressible equations with $k - \varepsilon$ closure model :
Find $(\rho, \rho\mathbf{u}, \rho E, \rho k, \rho\epsilon, \rho\gamma)$ solution of :

$$\left\{\begin{array}{rcl}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) & = & 0, \\
\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) & = & -\nabla p + \frac{1}{Re} \nabla \cdot \sigma + \nabla \cdot \tau, \\
\frac{\partial \rho E}{\partial t} + \nabla \cdot (\rho E + p)\mathbf{u} & = & \frac{1}{Re} \nabla \cdot \sigma \mathbf{u} + \nabla \cdot \tau \mathbf{u} + \left(\frac{\gamma}{PrRe} + \frac{\gamma}{Pr_t Re_t} \right) \nabla h \\
\frac{\partial \rho k}{\partial t} + \nabla \cdot (\rho \mathbf{u} k) & = & \tau : \nabla \mathbf{u} + \nabla \cdot [(\mu + \mu_t \sigma_k) \nabla k] - \rho \varepsilon, \\
\frac{\partial \rho \varepsilon}{\partial t} + \nabla \cdot (\rho \mathbf{u} \varepsilon) & = & \left(c_\varepsilon^{(1)} \frac{\varepsilon}{k} \tau : \nabla \mathbf{u} - c_\varepsilon^{(2)} \rho \frac{\varepsilon^2}{k} + C^{(2)} \right) C^{(1)} + \nabla \cdot [(\mu + \mu_t \sigma_\varepsilon) \nabla \varepsilon] \\
\frac{\partial \rho \gamma}{\partial t} + \nabla \cdot \rho \mathbf{u} \gamma & = & C_{g1} \gamma (1 - \gamma) \frac{P_k}{k} + \rho C_{g2} \frac{k^2}{\varepsilon} \nabla \gamma \cdot \nabla \gamma \\
 & + & \nabla \cdot [\sigma_\gamma (1 - \gamma)(\mu + \mu_t^*) \nabla \gamma]
\end{array}\right. \tag{16}$$

with $\mu_t^* = \left[1 + C_{\mu g} \frac{k^3}{\varepsilon^2} \gamma^{-2} (1 - \gamma) \|\nabla \gamma\|^2 \right] c_\mu f_\mu \frac{k^2}{\epsilon}$ and $P_k = \tau : \nabla \mathbf{u}$.

Discretization

- Concatenate equation

$$\frac{\partial \mathbf{W}}{\partial t} + \nabla \cdot F_c(\mathbf{W}) - \nabla \cdot F_v(\mathbf{W}) = \tau^{k-\varepsilon}(\mathbf{W}) + \tau^{\gamma}(\mathbf{W}) \qquad (17)$$

- Concatenate equation

$$\frac{\partial \mathbf{W}}{\partial t} + \nabla \cdot F_c(\mathbf{W}) - \nabla \cdot F_v(\mathbf{W}) = \tau^{k-\varepsilon}(\mathbf{W}) + \tau^{\gamma}(\mathbf{W}) \tag{18}$$

- Using a correct definition of Φ :

$$\begin{cases} \partial_t \mathbf{W}_i \, |\mathcal{C}_i| + \overbrace{\Phi_i(\mathbf{W}_i, \phi_i, \chi_i) - \left(\tau^{k-\varepsilon}(\mathbf{W}_i) + \tau^{\gamma}(\mathbf{W}_i), \phi_i\right)}^{\Phi_i^{Total}} = \mathbf{0} \\ \mathbf{W}_i(0) = \mathbf{W}_i^0 \end{cases} \tag{19}$$

- Concatenate equation

$$\frac{\partial \mathbf{W}}{\partial t} + \nabla \cdot F_c(\mathbf{W}) - \nabla \cdot F_v(\mathbf{W}) = \tau^{k-\varepsilon}(\mathbf{W}) + \tau^{\gamma}(\mathbf{W}) \tag{20}$$

- Using a correct definition of Φ :

$$\begin{cases} \partial_t \mathbf{W}_i \, |\mathcal{C}_i| + \overbrace{\Phi_i(\mathbf{W}_i, \phi_i, \chi_i) - \left(\tau^{k-\varepsilon}(\mathbf{W}_i) + \tau^{\gamma}(\mathbf{W}_i), \phi_i\right)}^{\Phi_i^{Total}} = \mathbf{0} \\ \mathbf{W}_i(0) = \mathbf{W}_i^0 \end{cases} \tag{21}$$

- Time discretization and implicit scheme

$$|\mathcal{C}_i| \left(\mathbf{W}_i^{n+1} - \mathbf{W}_i^n\right) + \Delta t \Phi_i^{total}(\mathbf{W}_i^n, , \phi_i, \chi_i) + \Delta t \frac{\partial \Phi^{Total}}{\partial \mathbf{W}}(\mathbf{W}_i^n)(\mathbf{W}_i^{n+1} - \mathbf{W}_i^n) = \mathbf{0} \tag{22}$$

$$\left(\frac{|\mathcal{C}_i|}{\Delta t} Id - \frac{\partial \Phi^{Total}}{\partial \mathbf{W}}(\mathbf{W}_i^n)\right)(\mathbf{W}_i^{n+1} - \mathbf{W}_i^n) = -\Phi_i^{total}(\mathbf{W}_i^n, , \phi_i, \chi_i) \tag{23}$$

- Approximation of the γ jacobian source term on a tetrahedron :

$$\left.\frac{\partial \mathcal{P}_{\gamma,h}}{\partial \rho\gamma}\right|_T \simeq C_{g1} \overline{\frac{1}{\rho_h k_h}(1-2\gamma_h)}^T P_k \tag{24}$$

$$\left(\left.\frac{\partial \mathcal{D}_{\gamma,h}}{\partial \rho\gamma}\right|_T\right)_i \simeq \sigma_\gamma \left(\mu + \overline{\mu_t}^T\right)\left[(1-\overline{\gamma_h}^T)\sum_{j=1}^{4}\frac{1}{\rho_j}\frac{\partial \phi_j}{\partial \mathbf{x}_i} - \overline{\left(\frac{1}{\rho}\right)_h}^T \sum_{j=1}^{4}\gamma_j \frac{\partial \phi_j}{\partial \mathbf{x}_i}\right] \tag{25}$$

- Mesh : $y_w^+ = 1 \Leftrightarrow \delta = 2 \times 10^{-5}$

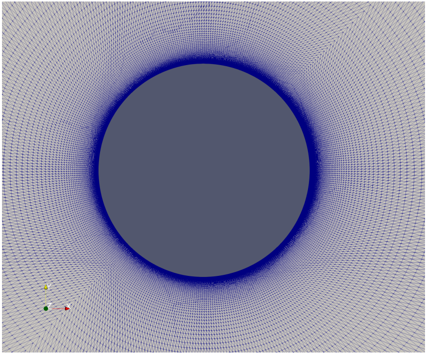


Figure – IBM mesh

- Set up
  - Mach $= 0.1$, Re $= 1$M
  - $U_\infty = 34.025$, $\rho_\infty = 1.225$
  - turbulence intensity : $I_k = 0.5\%$
  - $k_\infty = \frac{3}{2}\left(I_k U_\infty\right)^2$, $\varepsilon_\infty = k_\infty/10$

- Boundary conditions :

$$\gamma_{\partial C} = 1, \quad \text{and } \gamma_\infty = 0.01$$

Name	Mesh size	y_w^+	$\overline{C}_d$	C_l'	$-\overline{C}_{pb}$	L_r	$\overline{\theta}$
Present simulation							
URANS $k-\varepsilon$	0.6M	1	0.50	0.24	0.51	1.00	109
URANS $k-\varepsilon-\gamma$	0.6M	1	0.51	0.23	0.49	1.10	110
DDES $k-\varepsilon$ Goldberg ITW	4.8M	1	0.50	0.07	0.54	1.22	103
k - ε / cubic WALE ITW	4.8M	1	0.48	0.11	0.55	1.14	109
Experiments							
[Shih et al., 1993]			0.24	-	0.33		
[Schewe, 1983]			0.25	-	0.32		
[Gölling, 2006]						-	130
[Zdravkovich, 1997]			0.2-0.4	0.1-0.15	0.2-0.34		

Table – Bulk coefficient of the flow around a circular cylinder at Reynolds number 1M, $\overline{C}_d$ holds for the mean drag coefficient, C_l' is the root mean square of lift time fluctuation, $\overline{C}_{p_b}$ is the pressure coefficient at cylinder basis, L_r is the mean recirculation lenght, $\overline{\theta}$ is the mean separation angle.

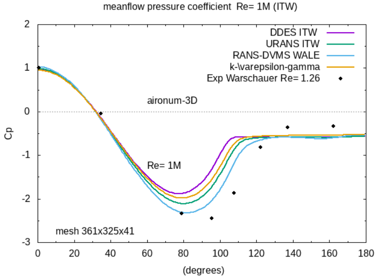
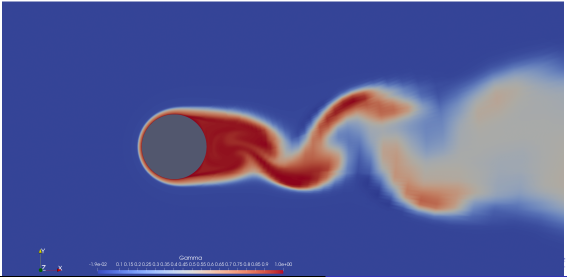


Figure – Pressure distribution



Problems occurs :

- Low CFL number $\Rightarrow$ low time advancing
- Pressure distribution not better than URANS

To do :

- Improved implicitation ?
- Improved transitional model, modify the production ?
- Compute hybrid $k - \varepsilon - \gamma$ model.

Part 3 : Immersed Boundary Method applied on $k - \varepsilon$

■ Averaged Navier-Stokes compressible equations with $k-\varepsilon$ closure model and *Brinkman Penalisation* :
Find $(\rho, \rho\mathbf{u}, \rho E, \rho k, \rho\epsilon)$ solution of :

$$\left\{\begin{array}{rcl}
\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{u}) & = & 0,\\
\frac{\partial\rho\mathbf{u}}{\partial t}+\nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u}) & = & -\nabla p+\frac{1}{Re}\nabla\cdot\sigma+\nabla\cdot\tau-\frac{\chi}{\eta}\rho\mathbf{u},\\
\frac{\partial\rho E}{\partial t}+\nabla\cdot(\rho E+p)\mathbf{u} & = & \frac{1}{Re}\nabla\cdot\sigma\mathbf{u}+\nabla\cdot\tau\mathbf{u}+\left(\frac{\gamma}{PrRe}+\frac{\gamma}{Pr_t Re_t}\right)\nabla h-\frac{\chi}{\eta}\rho\|\mathbf{u}\|^2\\
\frac{\partial\rho k}{\partial t}+\nabla\cdot(\rho\mathbf{u}k) & = & \tau:\nabla\mathbf{u}+\nabla\cdot[(\mu+\mu_t\sigma_k)\nabla k]-\rho\varepsilon-\frac{\chi}{\eta}\rho k,\\
\frac{\partial\rho\varepsilon}{\partial t}+\nabla\cdot(\rho\mathbf{u}\varepsilon) & = & \nabla\cdot[(\mu+\mu_t\sigma_\varepsilon)\nabla\varepsilon]+\left(c_\varepsilon^{(1)}\frac{\varepsilon}{k}\widetilde{\tau}:\nabla\overline{\mathbf{u}}-c_\varepsilon^{(2)}\overline{\rho}\frac{\varepsilon^2}{k}+C^{(2)}\right)C^{(1)}\\
 & & -\frac{\chi}{\eta}\left(\rho\varepsilon-\frac{2}{Re}\nabla\sqrt{k}\cdot\mathbf{n}\right)
\end{array}\right. \tag{26}$$

and $\chi=\left\{\begin{array}{ll} 1 & \textit{if } \mathbf{x}\in C,\\ 0 & \textit{otherwise.}\end{array}\right.$

References : I.V.Abalakin,A.P.Duben, N.S.Zhdanova, T.K.Kozubskaya, Simulating an unsteady turbulent flow arround a cylinder by the immersed boundary method, *Mathematical Models ans Computer Simulation*, 2019, vol 11, No 1, pp 74-85.

- Mesh : $y_w^+ = 1 \Leftrightarrow \delta = 2 \times 10^{-4}$

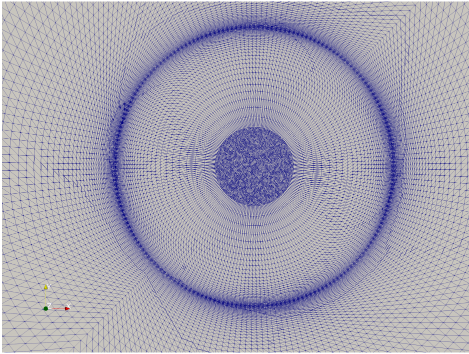


Figure – IBM mesh

- Set up
  - Mach $= 0.1$, Re $= 3900$
  - $U_\infty = 34.025$, $\rho_\infty = 1.225$
  - turbulence intensity : $I_k = 0.6\%$
  - $k_\infty = \frac{3}{2}\left(I_k U_\infty\right)^2$, $\varepsilon_\infty = k_\infty/10$
  - Immersed parameter : $\eta = 10^{-2}$

Name	Mesh size	δ_w	$\overline{C}_d$	C_l'	$-\overline{C}_{pb}$	L_r	$\overline{\theta}$
Present simulation							
$k-\varepsilon$ Goldberg 3D	176K	0.002	0.96	0.11	0.85	1.56	111
IBM $k-\varepsilon$ Goldberg 3D	176K	0.002	0.98	0.12	0.85	1.49	80
Numerical simulation							
Spalart 3D NOisette	-	0.002	0.97	0.11	0.83	1.67	89
IBM Spalart 3D NOisette	-	0.002	1.04	0.11	0.86	1.58	87
Experiment							
[Norberg, 1994]	-	-	0.94-1.04	-	0.84-0.93	-	-
[Parnaudeau et al., 2008]	-	-	-	0.1	-	1.41-1.58	-
[Lourenço, 1993]	-	-	-	-	-	-	86

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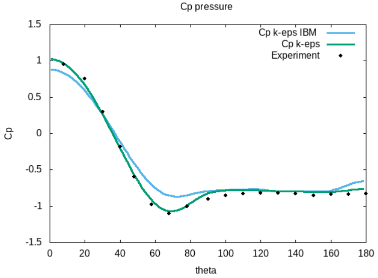

Figure – Mean pressure distribution

- Moving geometry center parametrized by $(0, \sin(4t))$

Problems occurs :

- low value of η (10^{-2} instead of 10^{-12})
- incorrect pressure distribution

To do :

- Add ghost cell method with Brinkmann penalization ?
- Implement Caradonna Thung geometry
- Run Caradonna test case

Part 4 : Current state of aeroacoustic post-treatment

- Root mean square pressure

$$p_\sim = p - \overline{p}, \tag{27}$$

$$p_{rms}^2 = \frac{1}{T}\int_T (p - \overline{p})^2 dt, \tag{28}$$

$$= \frac{1}{T}\int_T p_\sim^2 dt, \tag{29}$$

- We can proved :

$$p_{rms}^2 = \overline{p_\sim^2} = \overline{p}^2 - \overline{p^2}, \tag{30}$$

Then

$$d_B = 10\log\left(\frac{p_{eff}^2}{p_\infty^2}\right) = 20\log\left(\frac{\overline{p_\sim}}{p_\infty}\right) \quad [dB] \tag{31}$$

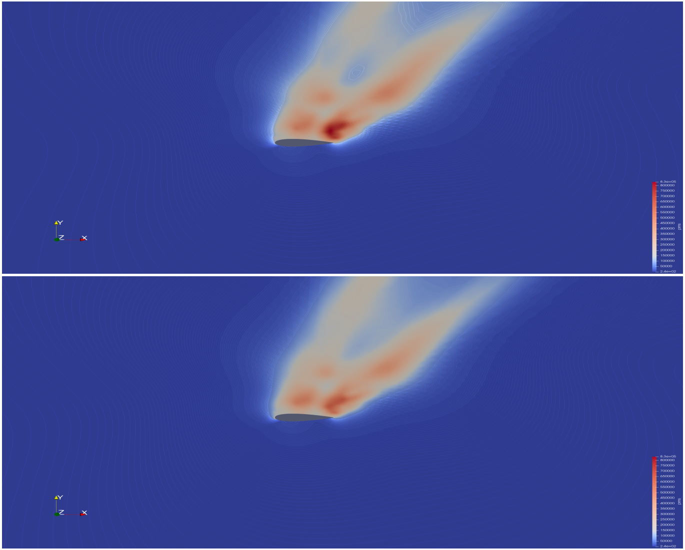

Figure – Root mean square of the pressure field

Results

- Radar representation

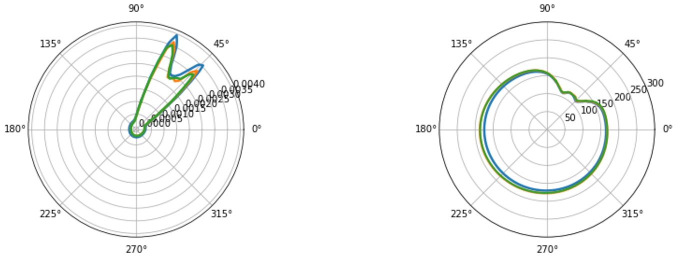

Figure – On left $\frac{p_{rms}^2}{p_\infty^2}$ is shown at $r = 5$, and on right side we show the acoustic level of the pressure in [dB]

Problems occurs :

- $p_{rms}/p_{\infty} < 1$.
- The mesh density in the wake is not adapted.

To do :

- Adapted the Lemma's mesh to Aironum
- Run the Lemma's adapted mesh
- Compute instantaneous aeroacoustic field using Kirchoff method and/or FWH method.

GOLDBERG, U. and OTA, D. (1990).
A k-epsilon near-wall formulation for separated flows.

Gölling, B. (2006).
Experimental investigations of separating boundary-layer flow from circular cylinder at reynolds numbers from 105 up to 107.
pages 455–462.

Jaeger, M. and Dhatt, G. (1992).
An extended k-epsilon finite element model.
International Journal for Numerical Methods in Fluids, 14(11) :1325–1345.

Lourenço, L. M. (1993).
Characteristics of the plane turbulent near wake of a circular cylinder.

Norberg, C. (1994).
An experimental investigation of the flow around a circular cylinder : influence of aspect ratio.
Journal of Fluid Mechanics, 258 :287–316.

Parnaudeau, P., Carlier, J., Heitz, D., and Lamballais, E. (2008).
Experimental and numerical studies of the flow over a circular cylinder at reynolds number 3900.
Physics of Fluids, 20(8) :085101.

Schewe, G. (1983).
On the force fluctuations acting on a circular cylinder in crossflow from subcritical up to transcritical reynolds number.
Journal of Fluid Mechanics, 133 :265 – 285.