Space-Time Mesh Adaptation : Reminder and new results

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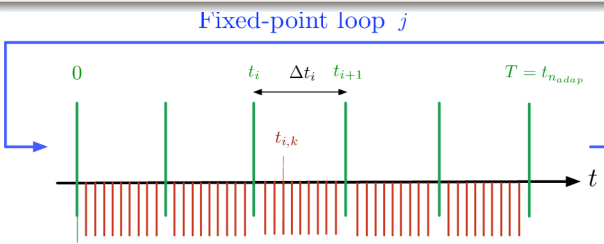
Motivations

Our motivation is the mesh adaptive simulation of unsteady flows, with a particular final target, LES and hybrid RANS/LES flows. We restrict to time advancing methods.

The Transient Fixed Point algorithm(*) was proposed for specifying automatically a *succession of* n_{adap} *meshes* over a decomposition in sub-intervals (in green) used for the transient process (timesteps in red).



(*)F. Alauzet, P.J. Frey, P.-L. George, and B. Mohammadi. 3D transient fixed point mesh adaptation for time-dependent problems : Application to CFD simulations. J. Comp. Phys.,222 :592-623, 2007.

Motivations(2)

In the case of explicit time-advancing, the heuristics (with theoretical proofs for simple models) was to consider maximal Courant number =1 as a good principle for time adaptation (*)(**).

For our target set of flows, only *implicit time advancing* is affordable. Small timesteps are CPU costly. Large timesteps may loose the accuracy of the spatial resolution.

Our purpose is to define a space-time global error and optimize it simultaneously in terms of spatial meshes and timestep length.

(*) F. Alauzet, A. Loseille, G. Olivier, Time-accurate multi-scale anisotropic mesh adaptation for unsteady flows in CFD, Journal of Computational Physics 373 (2018) 28-63.

(**) A. Belme, A. Dervieux, F. Alauzet, Time accurate anisotropic goal-oriented mesh adaptation for unsteady flows, Journal of Computational Physics 231 (2012) 6323-6348.

Space-time metric

In space-time domain $\Omega \times [0, T[$, the Transient Fixed Point mesh-adaptation method relies on the notion of *space-time continuous mesh or space-time metric*, $\mathcal{M}_{st}$,

$$\mathcal{M}_{st} = ((t_i)_{i=0,n_{adap}}, \tau, \mathcal{M}) \qquad (t_0 = 0)$$

defined by :

(i) The *splitting* $(t_i)_i$ of $[0, T]$ into n_{adap} subintervals :

$$[0, T] = \bigcup_{i=1}^{n_{adap}} [t_{i-1}, t_i[.$$

(ii) A *continuous timestep length* $\tau : t \in]0, T[\mapsto \tau(t)$.

(iii)A time-dependant *spatial metric* $\mathcal{M}(t) = \mathcal{M}_i$ for $t \in [t_{i-1}, t_i[$, where $\mathcal{M}_i$ is defined as the field $(\mathcal{M}_i(\mathbf{x}), \mathbf{x} \in \Omega)$ with $\mathcal{M}_i(\mathbf{x})$ a positive definite symmetric 3×3 matrix.

Complexity of a space-time metric

The *complexity* $\mathcal{C}_{st}(\mathcal{M}_{st})$, or computational effort, of a space-time metric

$$\mathcal{M}_{st} = ((t_i)_{i=0,n_{adap}}, \tau, \mathcal{M})$$

is the sum of complexities $\mathcal{C}_i$ on each time sub-interval $[t_{i-1}, t_i[$, each $\mathcal{C}_i$ being evaluated as the product of the *spatial complexity*,

$$\mathcal{C}_{space}(\mathcal{M}_i) = \int_\Omega \sqrt{det(\mathcal{M}_i(x))}\mathrm{d}x$$

which is the continuous analog of the number of vertices of spatial discretization, by the *time complexity*, namely the number of timesteps, therefore :

$$\mathcal{C}_{st}(\mathcal{M}_{st}) = \sum_{i=1}^{i=n_{adap}} \mathcal{C}_{space}(\mathcal{M}_i) \int_{t_{i-1}}^{t_i} \tau(t)^{-1}\mathrm{d}t.$$

Error model

Given a metric $\mathcal{M}_{st} = ((t_i)_{i=0,n_{adap}}, \tau, \mathcal{M})$ and a unit mesh $((t_i)_i, \tau, \mathcal{H})$ of it. We can compute on $((t_i)_i, \tau, \mathcal{H})$ a discrete solution $W(\mathbf{x}, t)$ of the Navier-Stokes equations in $\Omega \times [0, T[$, with an *approximation error* $\mathcal{E}$ which we consider as a function of $\mathcal{M}_{st}$.

The *error model* can be based on a goal-oriented analysis, with functional and adjoint. Instead, for simplicity, we consider a L^p feature-based analysis with a *sensor* M (typically the Mach number).

$$\mathcal{E}(\mathcal{M}_{st}) = \mathcal{E}_{time}(\mathcal{M}_{st}) + \mathcal{E}_{space}(\mathcal{M}_{st})$$

$$\mathcal{E}_{time}(\mathcal{M}_{st}) = \int_0^T \int_{-\infty}^{+\infty} [\tau^2 | \frac{\partial^3 M}{\partial t^3} ||]^p \mathrm{d}t\mathrm{d}x$$

$$\mathcal{E}_{space}(\mathcal{M}_{st}) = \sum_{i=1}^{n_{adap}} \int_{t_{i-1}}^{t_i} \mathcal{E}^i(t)\mathrm{d}t \quad \text{with}$$

$$\mathcal{E}^i(t) = \int_\Omega \left[\text{trace}\left((\mathcal{M}^i)^{-\frac{1}{2}}(\mathbf{x})\, \mathbf{H}_M(\mathbf{x}, t)\, (\mathcal{M}^i)^{-\frac{1}{2}}(\mathbf{x}) \right) \right]^p \mathrm{d}x$$

and $\mathbf{H}_M = |Hessian(M)|$.

Adaptation optimality

We solve the following optimal problem :

$$\begin{cases} \min\limits_{\mathcal{M}_{st}} \mathcal{E}(\mathcal{M}_{st}), \\ \mathcal{C}_{st}(\mathcal{M}_{st}) = N_{st}. \end{cases}$$

We prescribe :

(a) the time subintervals : n_{adap} and $(t_i)_{i=0,n_{adap}}$ and

(b) an integer N_{st} (prescribed space-time complexity).

There exists

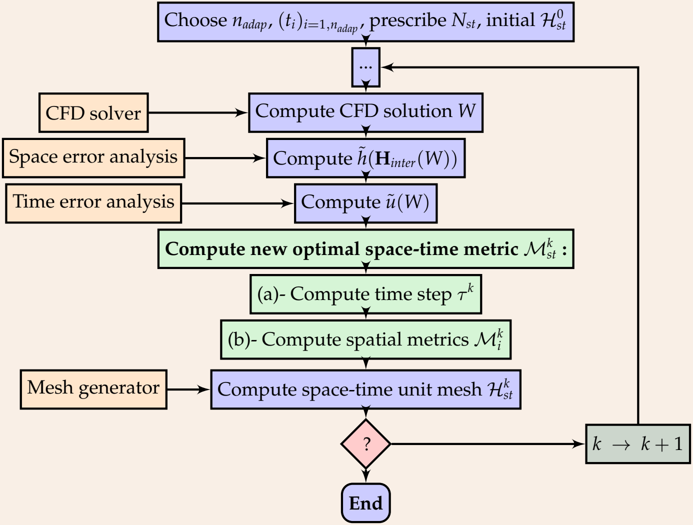
$$\mathcal{M}_{st}^{opt} = \Big((t_i)_{i=0,n_{adap}}, \tau^{opt}, (\mathcal{M}_i^{opt})_i \Big),$$

where τ^{opt} and $\mathcal{M}_i^{opt}$ can be expressed in terms of the error data :

$$|\frac{\partial^3 M}{\partial t^3}| \quad \textit{and} \quad \mathbf{H}_M,$$

which minimizes the error $\mathcal{E}(\mathcal{M}_{st})$ under the constraint $\mathcal{C}_{st}(\mathcal{M}_{st}) = N_{st}$.

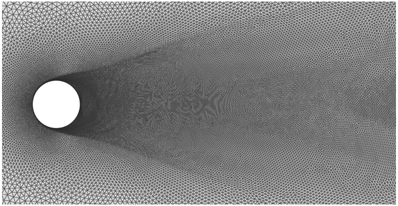
Space-time Transient Fixed-Point Algorithm



Flow past a cylinder at Reynolds 3900

2D and 3D computation of a flow around a cylinder at Reynolds number 3900, Mach number 0.3, with Spalart-Allmaras turbulence model.

Mesh adaptation options are :
- only one adapted spatial mesh, i.e. $n_{adap} = 1$
- Space-Time complexity N_{st} is prescribed to 5M, 10M, 20M, 40M in 2D and 750M in 3D.



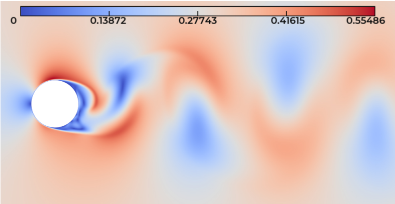


Figure – Flow past a circular cylinder at Re = 3900 : adapted mesh (left) and Mach number field (right) in cross-section.

Flow past a cylinder at Reynolds 3900 : Results (2D)

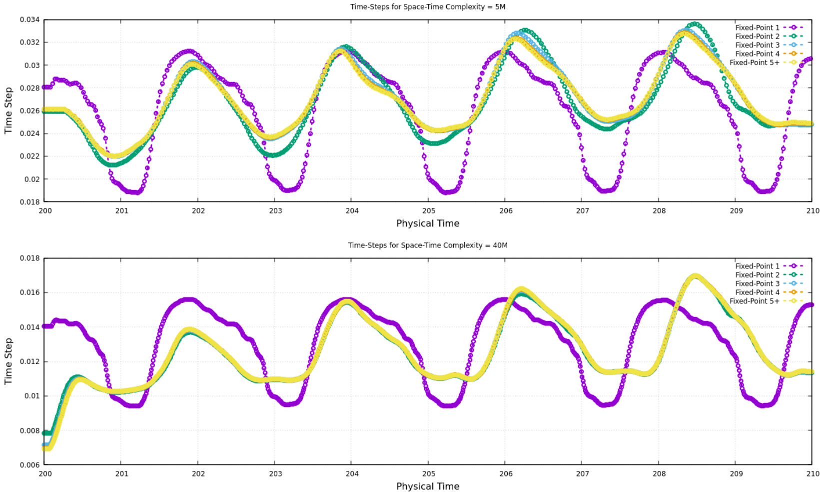


Figure – 2D cylinder flow at Reynolds number 3900. Time Steps as functions of time iteration, for complexities 5M, 40M. On each figure the successive iterations of the fixed point are depicted.

Flow past a cylinder at Reynolds 3900 : Results (2D)

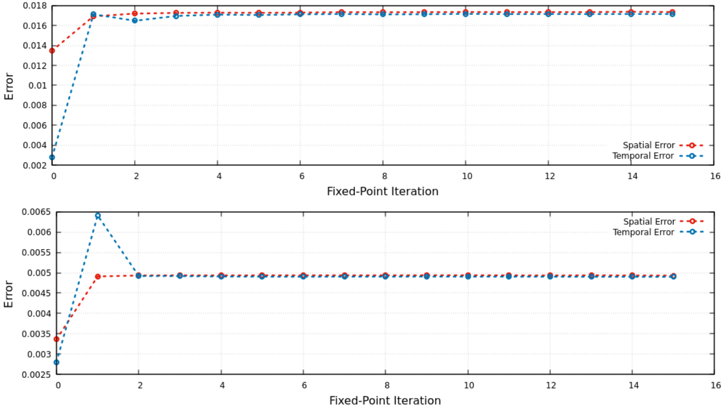


Figure – 2D cylinder flow at Reynolds number 3900. Error functionals for complexities 5M, 40M. Complete convergence between time error and space error is expected only in the continuous case. The corresponding error levels at convergence of fixed point are respectively 0.017, 0.005.

Flow past a cylinder at Reynolds 3900 : Results (3D)

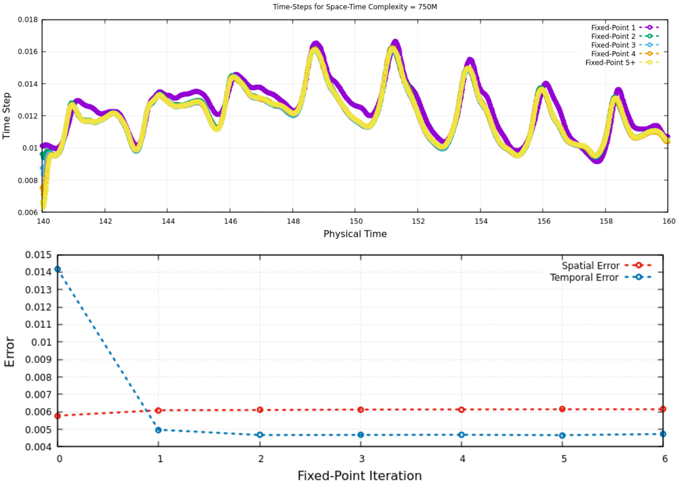


Figure – 3D cylinder flow at Reynolds number 3900. Time Steps (top) and error functionals (down) for complexity 750M.

Flow past a cylinder at Reynolds 1M

2D and 3D computation of a flow around a cylinder at Reynolds number 1M, Mach number 0.3, with Spalart-Allmaras turbulence model.

Mesh adaptation options are :
- only one adapted spatial mesh,
- Space-Time complexity is prescribed to 12.5M, 25M, 50M, 100M in 2D and 725M in 3D.

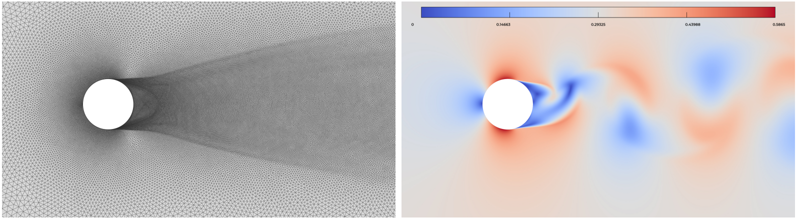


Figure – Flow past a circular cylinder at Re = 1M : adapted mesh (left) and the Mach number (right) in cross-section.

Flow past a cylinder at Reynolds 1M : Results (2D)

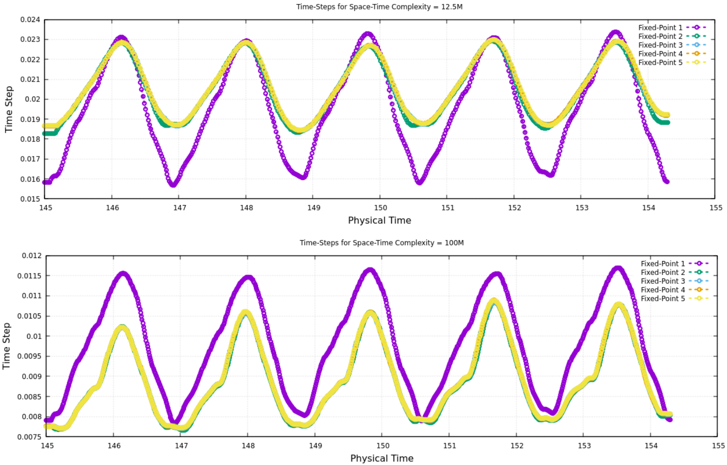


Figure – 2D cylinder flow at Reynolds number 1M. Time Steps as functions of time iteration, for complexities 12.5M and 100M. On each figure the successive iterations of the fixed point are depicted.

Flow past a cylinder at Reynolds 1M : Results (2D)

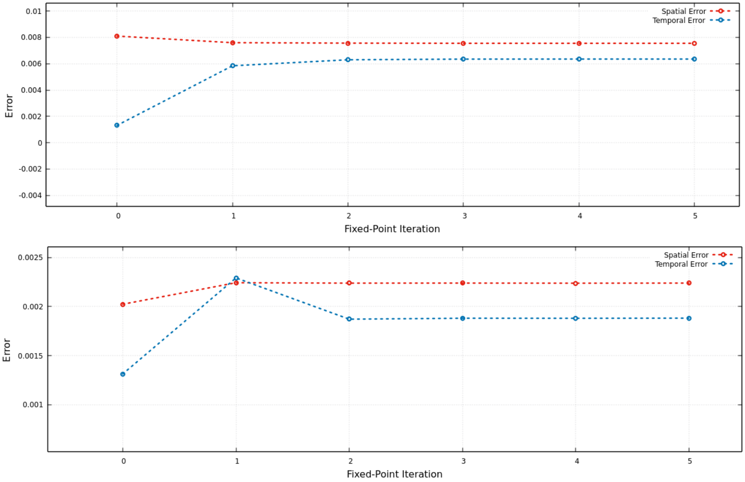


Figure – 2D cylinder flow at Reynolds number 1M. Error functionals for complexities 12.5M and 100M. Complete convergence between time error and space error is expected only in the continuous case. The corresponding error levels at convergence of fixed point are respectively 0.014, 0.004.

Flow past a cylinder at Reynolds 1M : Results (3D)

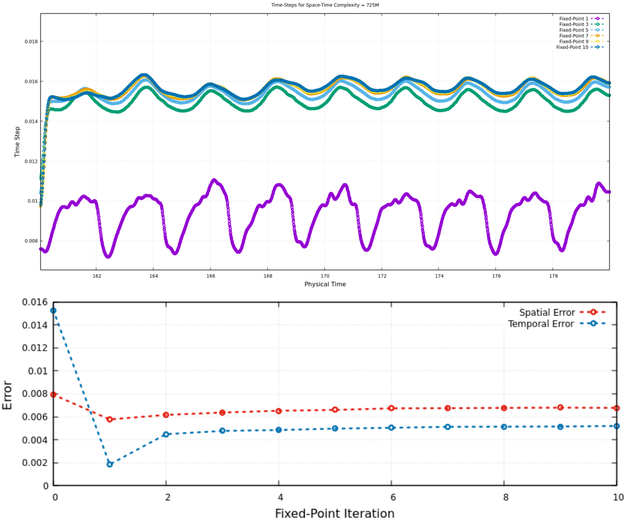


Figure – 3D cylinder flow at Reynolds number 1M. Time Steps (top) and error functionals (down) for complexity 725M.

Flow past a cylinder : Results (3D)

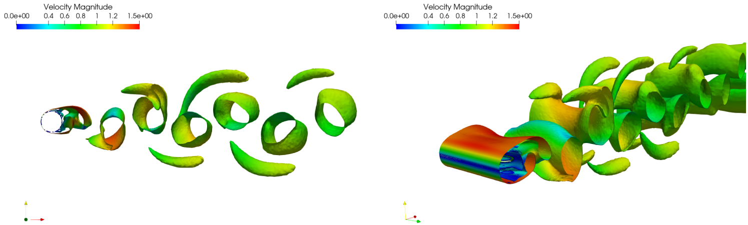


Figure – 3D flow past a cylinder at Reynolds number 3900 : views of the Q-criterion.

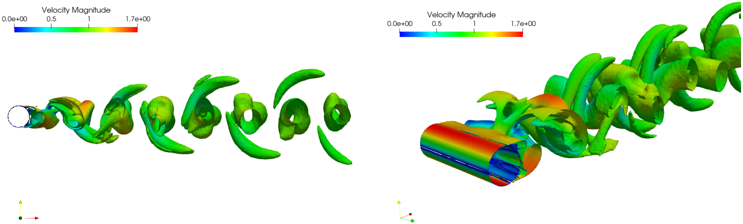


Figure – 3D flow past a cylinder at Reynolds number 1M : views of the Q-criterion.