NORMA

CENO HO Scheme
Norma, February 9, 2021

M. Gschwend, A. Dervieux

Contents

1. Bibliography about High Order

Up to now 80 references are gathered.

Essentially five identifiable subsets:

- Edge-Based-Reconstruction with line-reconstruction,

- Central-ENO: edge based flux, molecule-based reconstruction,

- NASA-Flux-Correction, third-order, quadratic nodal reconstruction (Katz,...),

- DG, Hybrid-DG.

- Various spectral non-DG : Spectral Differences, Spectral Volume, Flux-Reconstruction, Correction-Procedure-with-Reconstruction,...

pdf files are in Google Drive.

2. About anisotropic hp adaptation (prospective)

(a) In contrast to usual h-p approaches, we rely on a CENO scheme which involves its own p-choice on each cell.

(b) As in a previous work in 2D(*), the anisotropic adaptation is obtained thanks to a cell-by-cell conversion to second-order metric (Hessian-type).

(*)Carabias, A., Belme, A., Loseille, A. and Dervieux, A., Anisotropic goal-oriented error analysis for a third-order accurate CENO Euler discretization, International Journal for Numerical Methods in Fluids, Volume 86, Issue 6, 28 February 2018, Pages 392-413.

hp adaptation (2): (a)

(a) CENO with p-choice

For every cell :

Step 1: A *potential* reconstruction is performed up to third or fourth derivatives (details given in Matthieu Gschwend's talk).

Step 2: Depending of Taylor series truncated to p, the local error is a tensor of order p times the local mesh size, with $p = 1, 2, ..4$. For choosing the degree $p_{effective}$ of reconstruction we shall use a criterion inspired by the work of Dolejsi(*). $p_{effective}$ corresponds to smallest error term in the Taylor series. The *effective* reconstruction used for advancing in time relies on this term.

(*)V. Dolejsi. Anisotropic hp-adaptive method based on interpolation error estimates in the Lq-norm. Applied Numerical Mathematics, 82:80-114, 2014.

hp adaptation (3): (b)

(b) Anisotropic adaptation

Step 1: For every cell: we start from the error tensor $\mathbb{T} = \mathbb{T}_{p_{adapt}}$, with $p_{adapt} = max(2, p_{effective})$.

Step 2: Passing to second-order metric: let us define:

$$\tilde{H}_i \; = \; Argmin \sum_{j=1}^{N(i)} \left(\, \tilde{H}_i(\vec{ij}^{\,2}) \; - \; \left(\mathbb{T}(\vec{ij}^{\,p})\right)^{2/p} \, \right)^2$$

Then, relying on:

$$\sup_{\delta \mathbf{x}} \mathbb{T}\left((|\delta \mathbf{x}|)^p\right) \approx \left(trace(\mathcal{M}^{-\frac{1}{2}} |\tilde{H}_i| \mathcal{M}^{-\frac{1}{2}}) \right)^{\frac{p}{2}}$$

we replace for evaluating the optimal metric the $\mathbb{T}$-based error by the $\tilde{H}$-based one. The cell error is expressed in terms of the metric $\mathcal{M}$.

Step 3: By minimisation of the sum of cell-errors, we get the anisotropic optimal metric $\mathcal{M}_{opt}$.

State of progress

The text of the bibliography is in progress (2 months?). Please contribute to the list.

The adaptation project will start after insertion of CENO in the CFD code: september.