Computations of a circular cylinder at Reynolds numbers 140K and 1M using hybrid turbulence models

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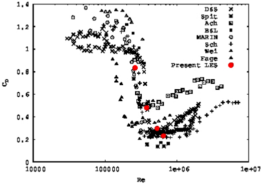


(Drag crisis : Lehmkuhl et al., 2014)

Hybrid turbulence models favored by INRIA and univ. of Montpellier in the NORMA project

Three hybrid turbulence models belonging to the category of hybrid RANS/LES approaches (i.e. LES does not extend all the way to the wall as for wall-stress-models), seamless or zonal, used/developed by the french partners in the NORMA project :

- DDES
- RANS/DVMS
- DDES/DVMS

In numerical applications, use of these models in their natural mode (RANS in the entire boundary layer, i.e. no wall-modeled LES) in order to avoid log-layer mismatch.

DDES

DDES/$k-\varepsilon$

Based on the low Reynolds $k-\varepsilon$ model proposed by **Goldberg, Peroomian and Chakravarthy (1998)**, which can be written briefly :

$$\frac{\partial W}{\partial t} + \nabla \cdot F_c(W) + \nabla \cdot F_v(W) + \nabla \cdot F_v^{RANS}(W) = \Omega(W) \quad \text{(RANS eq.)}$$

The dissipation term $D_k^{RANS} = \rho\varepsilon$ in the RHS of the $k-\varepsilon$ equations is replaced by:

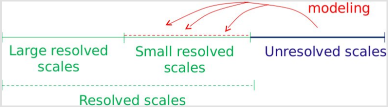
$$D_k^{DDES} = \rho \frac{k^{3/2}}{l_{DDES}}$$

with $l_{DDES} = \frac{k^{3/2}}{\varepsilon} - f_d max\left(0, \frac{k^{3/2}}{\varepsilon} - C_{DDES}\Delta\right)$, $C_{DDES} = 0.65$, Δ is a measure of local mesh size, and f_d is the shielding function ($f_d \simeq 0$ in the BL).

Resulting DDES eq.: $\frac{\partial W}{\partial t} + \nabla\cdot F_c(W) + \nabla\cdot F_v(W) + \nabla\cdot F_v^{RANS}(W) = \Omega^{DDES}(W)$

Dynamic Variational Multiscale (DVMS)

For turbulent wakes, many LES models are well performing.
A particular one, the Variational Multiscale (VMS) model, can be built in order to dissipate solely the numerical scales which are the smallest represented by the mesh and not the larger ones.
In this approach, the effects of the unresolved structures are only modeled in the equations governing the small resolved scales :



VMS governing equations :

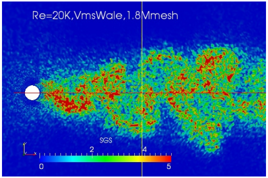
$$\left(\frac{\partial W_h}{\partial t}, \Psi_i\right) + (\nabla \cdot F_c(W_h), \Psi_i) + (\nabla \cdot F_v(W_h), \Phi_i) = -\left(\tau^{LES}(W_h'), \Phi_i'\right)$$

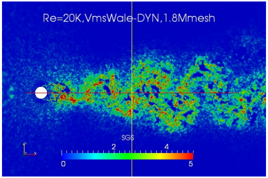
$W_h' = W_h - \overline{W_h}$ = small resolved scales, where $\overline{W_h}$=spatial averaged of W_h on agglomerated cells : 

.

Dynamic Variational Multiscale (DVMS)

VMS still slightly depends on the uniform SGS coefficient used for this dissipation.
In previous works we identified **DVMS**, a combination of VMS with Germano-type dynamic algorithm adapting in space and time the SGS coefficient ($C_{SGS} \rightarrow C_{SGS}(x,t)$), as more accurate than VMS.





Flow around a circular cylinder at Reynolds number 20,000: viscosity ratio for VMS (left) and for DVMS (right)

DVMS introduces less dissipation than classical LES $\Rightarrow$ good candidate for aeroacoustic computation.

Hybrid RANS/DVMS model

- Seamless hybridization of Goldberg $k - \varepsilon$ model and DVMS through a blending function.
- Hybrid RANS/DVMS governing equations :

$$\left(\frac{\partial W_h}{\partial t}, \Psi_i\right) + (\nabla \cdot F_c(W_h), \Psi_i) + (\nabla \cdot F_v(W_h), \Phi_i) =$$
$$-\theta \left(\tau^{RANS}(W_h), \Phi_i\right) - (1-\theta)\left(\tau^{LES}(W_h'), \Phi_i'\right)$$

- $\theta = 1 - f_d(1 - \bar{\theta}) \in [0, 1]$ **is a blending function** where
  - f_d is the DDES shielding function
  - $f_d \simeq 0$ in the BL $\Rightarrow$ RANS mode activated ($\theta \simeq 1$)
  - $f_d \simeq 1$ outside the BL $\Rightarrow$ $\theta = \bar{\theta}$ with hybridization parameter $\bar{\theta} \simeq 0$ if the fineness of the grid is sufficient for DVMS $\Rightarrow$ DVMS mode activated in this case ($\theta \simeq 0$).

DDES/DVMS

Zonal combination of DDES and DVMS

- Compute all DDES fluxes on the whole computational domain.
- Define the DVMS region : $Y^+ \geq 1000 \Rightarrow$ DVMS activated in the wake.
- In this region, re-evaluate the DDES turbulent viscous fluxes with DVMS.
- DDES/DVMS governing equations :

$$\left(\frac{\partial W_h}{\partial t}, \Psi_i\right) + (\nabla \cdot F_c(W_h), \Psi_i) + (\nabla \cdot F_v(W_h), \Phi_i) +$$
$$\theta \left(\nabla \cdot F_v^{RANS}(W_h), \Phi_i\right) + (1-\theta)\left(\nabla \cdot F_v^{LES}(W_h'), \Phi_i'\right) = \left(\Omega^{DDES}(W_h), \Phi_i\right)$$

where $\theta = 0$ in the DVMS region, and $\theta = 1$ elsewhere, with a smooth fitting between the two regions.

Numerical model

Specificities of the **M**ixed finite-**E**lement/finite-**V**olume (MEV) discretization

- Combination of a **Finite-Element** method (FEM) with a **vertex-centered Finite Volume** method (FVM)
- Applicable to a general class of tetrahedrizations
- **Low order numerical methods**: second-order accuracy in space
- A **high-derivative model** mastering **numerical dissipation**:
  - a very low numerical dissipation made of sixth-order derivatives and directly controled by a scaling factor γ
    $\Rightarrow$ further enhance the complementarity between the SGS model and the MUSCL stabilization and further reduce their competition.

Specifities of the time discretization

- Implicit time integration by a second order backward difference scheme

NUMERICAL EXPERIMENTS

- Circular cylinder at Reynolds number 140,000 $\Rightarrow$ subcritical regime.

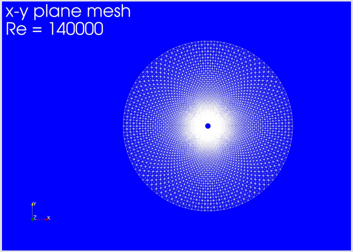
- Circular cylinder at Reynolds number 1,000,000 $\Rightarrow$ supercritical regime.

Sub-critical flow (near critical) : Re=140K

AIRONUM results Cylinder Re= 140K

- **Flow parameters:**
  Reynolds $= 140K$
  Mach $= 0.1$
  reference density $= 1.225\ kg/m^3$
  reference pressure $= 101300\ N/m^2$

  Velocity computed from Mach eqn
  mesh 165x165x33 (θ, radial, span)
  Re = 140K mesh is very coarse
  Re = 1M mesh = 256x215x21



- **Computational grids:**
  0.892M nodes
  52M elements

Sub-critical flow (near critical) : Re=140K

AIRONUM results Cylinder Re= 140K

- **Flow parameters:**

  Mach $= 0.1$

  Reynolds $= 140K$

  reference density $= 1.225\ kg/m^3$

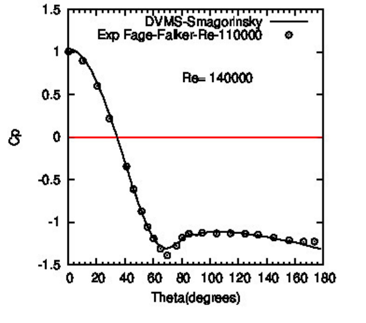
  reference pressure $= 101300\ N/m^2$

  Velocity computed from Mach eqn

- **Computational grids:**

  0.892M nodes

  52M elements



Sub-critical flow (near critical) : Re=140K

AIRONUM results Cylinder Re= 140K

- **Flow parameters:**

  Reynolds $= 140K$
  Mach $= 0.1$
  reference density $= 1.225\ kg/m^3$
  reference pressure $= 101300\ N/m^2$

  Velocity computed from Mach eqn

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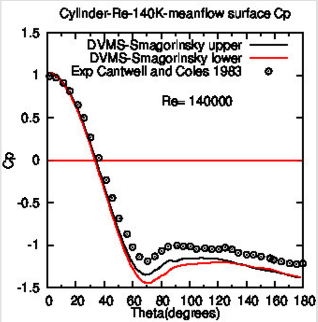


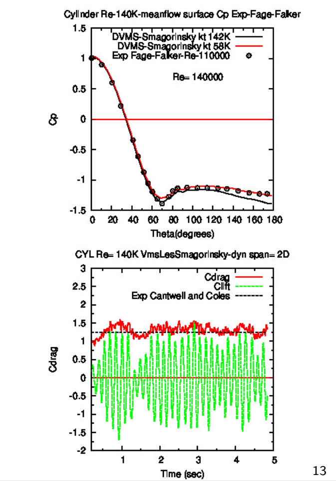
Sub-critical flow (near critical) : Re=140K

AIRONUM results Cylinder Re= 140K

- **Flow parameters:**

  Mach $= 0.1$
  Reynolds $= 140K$
  mesh (θ,radial,z-direction)
  mesh $=$ 165x165x33
  $Y^+_{surface} = 20$
  Δradial at surface $= 0.002$
  time steps $= 140000$
  cfl $= 40$
  Δt(adimensional) $= 0.0014$
  V6 $\gamma = 0.3$ (3rd-order space)
  reference density $= 1.225\ kg/m^3$
  reference pressure $= 101300\ N/m^2$

  Velocity computed from Mach eqn

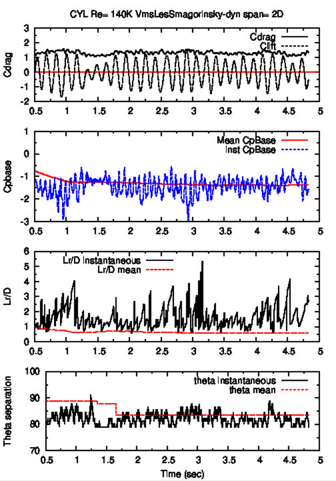


- **Computational grids:**

Sub-critical flow (near critical) : Re=140K

AIRONUM convergence Cylinder Re= 140K

- **Flow parameters:**
  58000 = 1.25 sec
  144000 = 5.30 sec



Sub-critical flow (near critical) : Re=140K

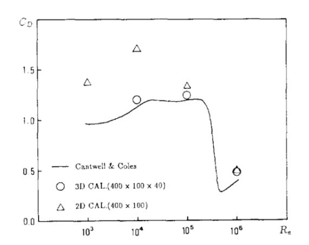
	$\overline{C}_d$	$-\overline{C}_{p_b}$	L_r	S_t
Experiments				
Cantwell-Coles (1983)	1.24	1.21	0.5	0.179
Son-Hanratty (1969), Zdravkovich (1997)				$\simeq 0.2$
Present simulations				
No model	0.43	0.40	0.63	0.142
URANS $k-\varepsilon$	0.77	0.87	1.05	0.218
DDES $k-\varepsilon$	0.97	1.01	0.96	0.217
DDES/DVMS	1.04	1.12	0.91	0.214
DVMS	1.25	1.33	0.88	0.217

Table 1: Bulk quantities for $Re = 140,000$ flow around a cylinder.

Super-critical flow: Re=1M (1.0E+06)

Important paper of Tamura, Ohta, and Kuwahra 1990

- **Flow parameters:**
  Showed that Supercritical flows are basically two-dimensional
  Can be computed with two-dimensional codes

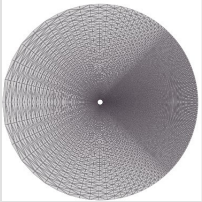


Super-critical flow: Re=1M

AIRONUM centerplane mesh

- **Flow parameters:**
  Can be computed with two-dimensional codes
  Reynolds $= 1M(1.0E + 06)$
  reference density $= 1.225\ kg/m^3$
  reference pressure $= 101300\ N/m^2$

  Velocity computed from Mach eqn



- **Computational grids:**
  1.210M nodes

Super-critical flow: Re=1M (1.0E+06)

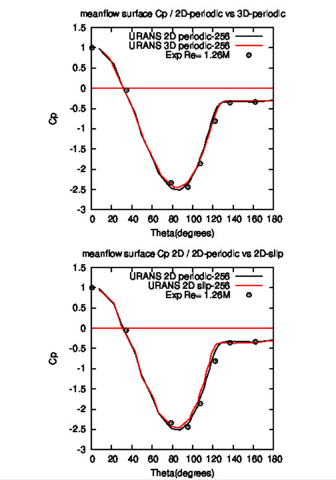
AIRONUM Re= 1M 3D vs 2D-per

- **Flow parameters:**
  Mesh refined for the aft-cylinder
  Reynolds $= 1M(1.0E + 06)$
  Mach $= 0.1$
  reference density $= 1.225\ kg/m^3$
  reference pressure $= 101300\ N/m^2$

  Velocity computed from Mach eqn

  2D-per mesh 256x215x3 vertices
  θ, radial, span

- **Computational grids:**
  1.210M nodes



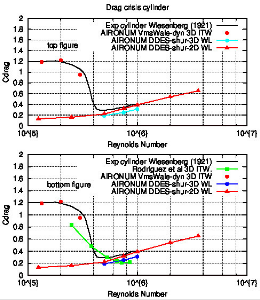
Super-critical flow: Re=1M (1.0E+06)

AIRONUM Re= 1M results near drag crisis

- **Flow parameters:**
  Reynolds $= 1M(1.0E + 06)$
  Mach $= 0.1$
  reference density $= 1.225\ kg/m^3$
  reference pressure $= 101300\ N/m^2$

  Velocity computed from Mach eqn

- **Computational grids:**
  1.210M nodes

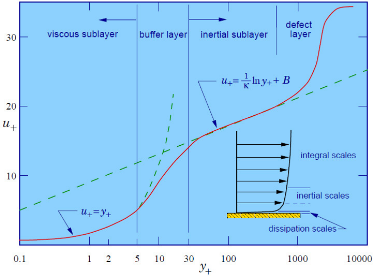


Super-critical flow: Re=1M (1.0E+06)

Wall functions/wall-law layers

- **Flow parameters:**
  Reynolds $= 1M(1.0E+06)$
  Mach $= 0.1$
  reference density $= 1.225\ kg/m^3$
  reference pressure $= 101300\ N/m^2$

  Velocity computed from Mach eqn
- **Computational grids:**
  1.56M nodes



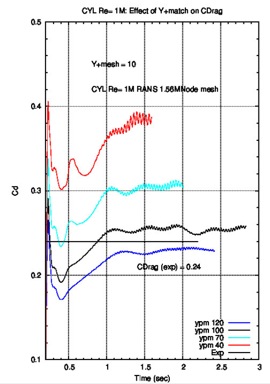
Super-critical flow: Re=1M ($1.0E^{+06}$)

AIRONUM-the effect of Y^+_{match} on the drag

- **Flow parameters:**
  Reynolds $= 1M(1.0E^{+06})$
  Mach $= 0.1$
  reference density $= 1.225\ kg/m^3$
  reference pressure $= 101300\ N/m^2$

  Velocity computed from Mach eqn

- **Computational grids:**
  1.56M nodes

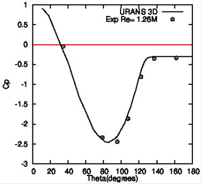


Super-critical flow: Re=1M (1.0E+06)

AIRONUM Re= 1M URANS surface pressure

- **Flow parameters:**
  Mach $= 0.1$
  Reynolds $= 1M(1.0E+06)$
  reference density $= 1.225\ kg/m^3$
  reference pressure $= 101300\ N/m^2$

  Velocity computed from Mach eqn



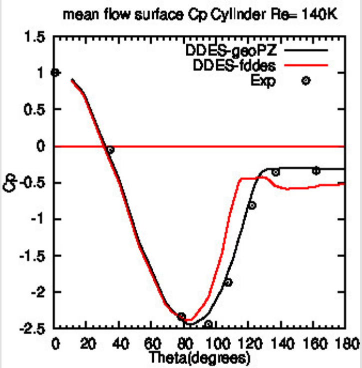
- **Computational grids:**
  1.210MNodes

Super-critical flow: Re=1M (1.0E+06)

AIRONUM Re= 1M DDES effect of protection shield on surface pressure

- **Flow parameters:**
  Mach $= 0.1$
  Reynolds $= 1M(1.0E + 06)$
  DDES fddes PZ designed for wings
  - attached flow
  - laminar, turbulent flow
  - excludes massive separation like cylinders

- **Computational grids:**
  1.210MNodes



Super-critical flow: Re=1M (1.0E+06)

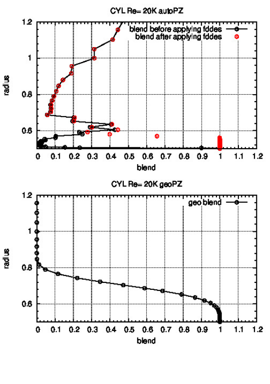
AIRONUM Re= 1M Effect of protection-zone/shield

- **Flow parameters:**
  Mesh refined for the aft-cylinder
  Reynolds $= 1M(1.0E+06)$
  Mach $= 0.1$
  reference density $= 1.225\ kg/m^3$
  reference pressure $= 101300\ N/m^2$

  Velocity computed from Mach eqn

  $\delta = 20 \times Y^+_{match}/Re \times D$

- **Computational grids:**
  1.210M nodes

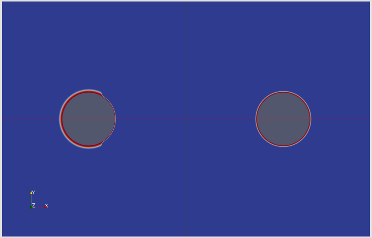


Super-critical flow: Re=1M (1.0E+06)

AIRONUM Re= 1M geometric protection-zone/shield

- **Flow parameters:**
  Mesh refined for the aft-cylinder
  Reynolds $= 1M(1.0E+06)$
  Mach $= 0.1$
  reference density $= 1.225\ kg/m^3$
  reference pressure $= 101300\ N/m^2$

  Velocity computed from Mach eqn

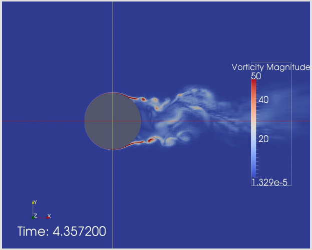


- **Computational grids:**
  1.210M nodes

Supercritical flow : Re=1M

Test case definition

- **Flow parameters:**
  Mach = 0.1
  Reynolds = 1M
- **Computational grids:**
  2.85M nodes
  16.6M elements



Supercritical flow : Re=1M

	$\overline{C_d}$	C_l'	$-\overline{C}_{p_b}$	S_t
Experiments				
Szechenyi (1975)	0.25		0.32	0.35
Goelling (2006)				0.35
Zdravkovich (1997)	0.2-0.4	.1-.15	.2-.34	0.50
Present simulations				
URANS $k-\varepsilon$	0.24	0.07	0.26	0.45
DDES $k-\varepsilon$	0.24	0.04	0.34	0.26
DDES/DVMS	0.23	0.04	0.30	0.33
Other simulation				
LES of Kim and Mohan (2005)	0.27	0.12	0.28	

Table 2: Bulk coefficients of the flow around a circular cylinder at Reynolds number 10^6.

Hybrid turbulence models : future works for improvement

- Some tracks to explore :
  - A transition prediction model (RANS component) in order to more accuratly compute transitional boundary layers (supercritical regime).
  - SST $k-\omega$ model combined with DDES, RANS/DVMS and DDES/DVMS.
  - $k-R$ model (Zhang-Rahman-Chen, 2019) combined with DDES, RANS/DVMS and DDES/DVMS.
  - Further improve the blending function in the RANS/DVMS approach.
  - A seamless DDES/DVMS strategy based on a blending function allowing for an automatic switch from DDES to DVMS and vice versa.
  - A DDES variant (limitation of the production term, Reddy-Ryon-Durbin, 2014) which avoids the log-layer mismatch issue.