Computations of a circular cylinder at Reynolds numbers 140K and 1M using hybrid turbulence models

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(Drag crisis : Lehmkuhl et al., 2014)
Hybrid turbulence models favored by INRIA and univ. of Montpellier in the NORMA project

Three hybrid turbulence models belonging to the category of hybrid RANS/LES approaches (i.e. LES does not extend all the way to the wall as for wall-stress-models), seamless or zonal, used/developed by the french partners in the NORMA project:

- DDES
- RANS/DVMS
- DDES/DVMS

In numerical applications, use of these models in their natural mode (RANS in the entire boundary layer, i.e. no wall-modeled LES) in order to avoid log-layer mismatch.
DDES

DDES/$k - \varepsilon$

Based on the low Reynolds $k - \varepsilon$ model proposed by Goldberg, Peroomian and Chakravarthy (1998), which can be written briefly:

$$\frac{\partial W}{\partial t} + \nabla \cdot F_c(W) + \nabla \cdot F_v(W) + \nabla \cdot F_{v}^{RANS}(W) = \Omega(W) \quad \text{(RANS eq.)}$$

The dissipation term $D_{RANS}^{k} = \rho \varepsilon$ in the RHS of the $k - \varepsilon$ equations is replaced by:

$$D_{k}^{DDES} = \rho \frac{k^{3/2}}{l_{DDES}}$$

with $l_{DDES} = \frac{k^{3/2}}{\varepsilon} - f_{d} \max\left(0, \frac{k^{3/2}}{\varepsilon} - C_{DDES} \Delta \right)$, $C_{DDES} = 0.65$, $\Delta$ is a measure of local mesh size, and $f_{d}$ is the shielding function ($f_{d} \simeq 0$ in the BL).

Resulting DDES eq.:  

$$\frac{\partial W}{\partial t} + \nabla \cdot F_c(W) + \nabla \cdot F_v(W) + \nabla \cdot F_{v}^{RANS}(W) = \Omega^{DDES}(W)$$
Dynamic Variational Multiscale (DVMS)

For turbulent wakes, many LES models are well performing. A particular one, the Variational Multiscale (VMS) model, can be built in order to dissipate solely the numerical scales which are the smallest represented by the mesh and not the larger ones. In this approach, the effects of the unresolved structures are only modeled in the equations governing the small resolved scales:

\[
\left( \frac{\partial W_h}{\partial t}, \Psi_i \right) + (\nabla \cdot F_c(W_h), \Psi_i) + (\nabla \cdot F_v(W_h), \Phi_i) = - \left( \tau^{LES}(W'_h), \Phi'_i \right)
\]

\[W'_h = W_h - \overline{W_h} = \text{small resolved scales, where } \overline{W_h} = \text{spatial averaged of } W_h \text{ on agglomerated cells}.
\]
VMS still slightly depends on the uniform SGS coefficient used for this dissipation.
In previous works we identified **DVMS**, a combination of VMS with Germano-type dynamic algorithm adapting in space and time the SGS coefficient ($C_{SGS} \rightarrow C_{SGS}(x, t)$), as more accurate than VMS.

Flow around a circular cylinder at Reynolds number 20,000: viscosity ratio for VMS (left) and for DVMS (right)

**DVMS** introduces less dissipation than classical LES $\Rightarrow$ good candidate for aeroacoustic computation.
Hybrid RANS/DVMS model

- Seamless hybridization of Goldberg $k - \varepsilon$ model and DVMS through a blending function.

- Hybrid RANS/DVMS governing equations:

$$
\left( \frac{\partial W_h}{\partial t}, \Psi_i \right) + (\nabla \cdot F_c(W_h), \Psi_i) + (\nabla \cdot F_v(W_h), \Phi_i) = -\theta \left( \tau^{RANS}(W_h), \Phi_i \right) - (1 - \theta) \left( \tau^{LES}(W_h'), \Phi_i' \right)
$$

- $\theta = 1 - f_d(1 - \bar{\theta}) \in [0, 1]$ is a blending function where
  - $f_d$ is the DDES shielding function
  - $f_d \simeq 0$ in the BL $\Rightarrow$ RANS mode activated ($\theta \simeq 1$)
  - $f_d \simeq 1$ outside the BL $\Rightarrow$ $\theta = \bar{\theta}$ with hybridization parameter $\bar{\theta} \simeq 0$ if the fineness of the grid is sufficient for DVMS $\Rightarrow$ DVMS mode activated in this case ($\theta \simeq 0$).
DDES/DVMS

Zonal combination of DDES and DVMS

- Compute all DDES fluxes on the whole computational domain.
- Define the DVMS region: \( Y^+ \geq 1000 \Rightarrow \text{DVMS activated in the wake.} \)
- In this region, re-evaluate the DDES turbulent viscous fluxes with DVMS.
- DDES/DVMS governing equations:

\[
\left( \frac{\partial W_h}{\partial t}, \Psi_i \right) + (\nabla \cdot F_c(W_h), \Psi_i) + (\nabla \cdot F_v(W_h), \Phi_i) + \\
\theta \left( \nabla \cdot F_{v}^{RANS}(W_h), \Phi_i \right) + (1 - \theta) \left( \nabla \cdot F_{v}^{LES}(W_h'), \Phi_i' \right) = \left( \Omega^{DDES}(W_h), \Phi_i \right)
\]

where \( \theta = 0 \) in the DVMS region, and \( \theta = 1 \) elsewhere, with a smooth fitting between the two regions.
Numerical model

Specificities of the Mixed finite-Element/finite-Volume (MEV) discretization

- Combination of a Finite-Element method (FEM) with a vertex-centered Finite Volume method (FVM)
- Applicable to a general class of tetrahedrizations
- Low order numerical methods: second-order accuracy in space
- A high-derivative model mastering numerical dissipation:
  - a very low numerical dissipation made of sixth-order derivatives and directly controled by a scaling factor $\gamma$
  $\Rightarrow$ further enhance the complementarity between the SGS model and the MUSCL stabilization and further reduce their competition.

Specifities of the time discretization

- Implicit time integration by a second order backward difference scheme
<table>
<thead>
<tr>
<th>NUMERICAL EXPERIMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular cylinder at Reynolds number 140,000 ⇒ subcritical regime.</td>
</tr>
<tr>
<td>Circular cylinder at Reynolds number 1,000,000 ⇒ supercritical regime.</td>
</tr>
</tbody>
</table>
Sub-critical flow (near critical) : Re=140K

<table>
<thead>
<tr>
<th>AIRONUM results Cylinder Re= 140K</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Flow parameters:</strong></td>
</tr>
<tr>
<td>Reynolds = 140K</td>
</tr>
<tr>
<td>Mach = 0.1</td>
</tr>
<tr>
<td>reference density = 1.225 kg/m³</td>
</tr>
<tr>
<td>reference pressure = 101300 N/m²</td>
</tr>
<tr>
<td>Velocity computed from Mach eqn</td>
</tr>
<tr>
<td>mesh 165x165x33 (θ, radial, span)</td>
</tr>
<tr>
<td>Re = 140K mesh is very coarse</td>
</tr>
<tr>
<td>Re = 1M mesh = 256x215x21</td>
</tr>
</tbody>
</table>

**Computational grids:**
0.892M nodes
52M elements
Sub-critical flow (near critical): $Re=140K$

**Flow parameters:**
- Mach = 0.1
- Reynolds = 140K
- Reference density = 1.225 kg/m$^3$
- Reference pressure = 101300 N/m$^2$

Velocity computed from Mach eqn

**Computational grids:**
- 0.892M nodes
- 52M elements
Sub-critical flow (near critical) : Re=140K

AIRONUM results Cylinder Re= 140K

- **Flow parameters:**
  - Reynolds = 140K
  - Mach = 0.1
  - reference density = 1.225 kg/m³
  - reference pressure = 101300 N/m²

  Velocity computed from Mach eqn

- **Computational grids:**
  - 0.892M nodes
  - 52M elements
Hybrid turbulence models

Numerical Model

Numerical applications

Future works for improvement

Sub-critical flow (near critical) : Re=140K

AIRONUM results Cylinder Re= 140K

- **Flow parameters:**
  - Mach = 0.1
  - Reynolds = 140K
  - mesh (θ,radial,z-direction) mesh = 165x165x33
  - $Y_{+}$ surface = 20
  - Δradial at surface = 0.002
  - Time steps = 140000
  - cfl = 40
  - $Δt$ (adimensional) = 0.0014
  - $V_6\ \gamma = 0.3$ (3rd-order space)
  - Reference density = 1.225 $kg/m^3$
  - Reference pressure = 101300 $N/m^2$

Velocity computed from Mach eqn

- **Computational grids:**
  - 0.892M nodes
  - 52M elements

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Hybrid turbulence models

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Sub-critical flow (near critical) : Re=140K

AIRONUM convergence Cylinder Re= 140K

- **Flow parameters:**
  - 58000 = 1.25 sec
  - 144000 = 5.30 sec
Sub-critical flow (near critical) : $Re=140K$

<table>
<thead>
<tr>
<th>Experiments</th>
<th>$\bar{C}_d$</th>
<th>$-\bar{C}_{pb}$</th>
<th>$L_r$</th>
<th>$S_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantwell-Coles (1983)</td>
<td>1.24</td>
<td>1.21</td>
<td>0.5</td>
<td>0.179</td>
</tr>
<tr>
<td>Son-Hanratty (1969), Zdravkovich (1997)</td>
<td></td>
<td></td>
<td></td>
<td>$\approx 0.2$</td>
</tr>
<tr>
<td>Present simulations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No model</td>
<td>0.43</td>
<td>0.40</td>
<td>0.63</td>
<td>0.142</td>
</tr>
<tr>
<td>URANS $k - \varepsilon$</td>
<td>0.77</td>
<td>0.87</td>
<td>1.05</td>
<td>0.218</td>
</tr>
<tr>
<td>DDES $k - \varepsilon$</td>
<td>0.97</td>
<td>1.01</td>
<td>0.96</td>
<td>0.217</td>
</tr>
<tr>
<td>DDES/DVMS</td>
<td>1.04</td>
<td>1.12</td>
<td>0.91</td>
<td>0.214</td>
</tr>
<tr>
<td>DVMS</td>
<td>1.25</td>
<td>1.33</td>
<td>0.88</td>
<td>0.217</td>
</tr>
</tbody>
</table>

**Table 1:** Bulk quantities for $Re = 140,000$ flow around a cylinder.
Super-critical flow: Re=1M (1.0E+06)

Important paper of Tamura, Ohta, and Kuwahra 1990

- **Flow parameters:**
  - Showed that Supercritical flows are basically two-dimensional
  - Can be computed with two-dimensional codes
Super-critical flow: Re=1M

**AIRONUM centerplane mesh**

- **Flow parameters:**
  Can be computed with two-dimensional codes
  Reynolds = 1M(1.0E + 06)
  reference density = 1.225 kg/m$^3$
  reference pressure = 101300 N/m$^2$

- Velocity computed from Mach eqn

- **Computational grids:**
  1.210M nodes
Super-critical flow: Re = 1M (1.0E+06)

AIRONUM Re = 1M 3D vs 2D-per

- **Flow parameters:**
  Mesh refined for the aft-cylinder
  Reynolds = 1M(1.0E + 06)
  Mach = 0.1
  reference density = 1.225 kg/m³
  reference pressure = 101300 N/m²

  Velocity computed from Mach eqn

  2D-per mesh 256x215x3 vertices
  \( \theta, \text{radial, span} \)

- **Computational grids:**
  1.210M nodes
Super-critical flow: \( \text{Re} = 1\text{M} \ (1.0E+06) \)

**AIRONUM Re= 1M results near drag crisis**

- **Flow parameters:**
  - Reynolds\( = 1\text{M}(1.0E + 06) \)
  - Mach\( = 0.1 \)
  - reference density \( = 1.225 \text{ kg/m}^3 \)
  - reference pressure \( = 101300 \text{ N/m}^2 \)

  Velocity computed from Mach eqn

- **Computational grids:**
  - 1.210M nodes
Super-critical flow: Re=1M (1.0E+06)

Wall functions/wall-law layers

- **Flow parameters:**
  - Reynolds = $1M(1.0E + 06)$
  - Mach = 0.1
  - Reference density = 1.225 kg/m$^3$
  - Reference pressure = 101300 N/m$^2$

- **Computational grids:**
  - 1.56M nodes

Velocity computed from Mach eqn
Super-critical flow: $Re=1M \ (1.0E^{+06})$

AIRONUM-the effect of $Y_{match}^+$ on the drag

- **Flow parameters:**
  - Reynolds $= 1M \ (1.0E^{+06})$
  - Mach $= 0.1$
  - Reference density $= 1.225 \ kg/m^3$
  - Reference pressure $= 101300 \ N/m^2$

- Velocity computed from Mach eqn

- **Computational grids:**
  - 1.56M nodes
Super-critical flow: \( Re=1M \ (1.0E+06) \)

**Flow parameters:**
- Mach = 0.1
- Reynolds = \( 1M(1.0E+06) \)
- Reference density = 1.225 kg/m\(^3\)
- Reference pressure = 101300 N/m\(^2\)

Velocity computed from Mach eqn

**Computational grids:**
- 1.210M Nodes
Super-critical flow: \( \text{Re}=1M \ (1.0E+06) \)

- **Flow parameters:**
  - Mach = 0.1
  - Reynolds = \( 1M(1.0E+06) \)
  - DDES fddes PZ designed for wings
    - attached flow
    - laminar, turbulent flow
    - excludes massive separation like cylinders

- **Computational grids:**
  - 1.210M Nodes
Super-critical flow: Re=1M (1.0E+06)

- **Flow parameters:**
  - Mesh refined for the aft-cylinder
  - Reynolds = 1M(1.0E + 06)
  - Mach = 0.1
  - Reference density = 1.225 kg/m³
  - Reference pressure = 101300 N/m²
  - Velocity computed from Mach eqn

\[ \delta = 20 \times Y_{match}^+ / Re \times D \]

- **Computational grids:**
  - 1.210M nodes
Super-critical flow: Re=1M (1.0E+06)

**AIRONUM Re= 1M geometric protection-zone/shield**

- **Flow parameters:**
  - Mesh refined for the aft-cylinder
  - Reynolds = 1M(1.0E + 06)
  - Mach = 0.1
  - Reference density = 1.225 kg/m³
  - Reference pressure = 101300 N/m²

- **Velocity computed from Mach eqn**

- **Computational grids:**
  - 1.210M nodes

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Numerical applications
Future works for improvement
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Supercritical flow: \( \text{Re} = 1\text{M} \)

**Test case definition**

- **Flow parameters:**
  - Mach = 0.1
  - Reynolds = 1M

- **Computational grids:**
  - 2.85M nodes
  - 16.6M elements
### Supercritical flow: Re=1M

<table>
<thead>
<tr>
<th></th>
<th>$\bar{C}_d$</th>
<th>$C'$</th>
<th>$-\bar{C}_{p_b}$</th>
<th>$S_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Experiments</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Szechenyi (1975)</td>
<td>0.25</td>
<td>0.32</td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td>Goelling (2006)</td>
<td></td>
<td></td>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td>Zdravkovich (1997)</td>
<td>0.2-0.4</td>
<td>.1-.15</td>
<td>.2-.34</td>
<td>0.50</td>
</tr>
<tr>
<td><strong>Present simulations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>URANS $k - \varepsilon$</td>
<td>0.24</td>
<td>0.07</td>
<td>0.26</td>
<td>0.45</td>
</tr>
<tr>
<td>DDES $k - \varepsilon$</td>
<td>0.24</td>
<td>0.04</td>
<td>0.34</td>
<td>0.26</td>
</tr>
<tr>
<td>DDES/DVMS</td>
<td>0.23</td>
<td>0.04</td>
<td>0.30</td>
<td>0.33</td>
</tr>
<tr>
<td><strong>Other simulation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LES of Kim and Mohan (2005)</td>
<td>0.27</td>
<td>0.12</td>
<td>0.28</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2**: Bulk coefficients of the flow around a circular cylinder at Reynolds number $10^6$. 
Some tracks to explore:

- A transition prediction model (RANS component) in order to more accurately compute transitional boundary layers (supercritical regime).
- SST $k − \omega$ model combined with DDES, RANS/DVMS and DDES/DVMS.
- $k − R$ model (Zhang-Rahman-Chen, 2019) combined with DDES, RANS/DVMS and DDES/DVMS.
- Further improve the blending function in the RANS/DVMS approach.
- A seamless DDES/DVMS strategy based on a blending function allowing for an automatic switch from DDES to DVMS and vice versa.
- A DDES variant (limitation of the production term, Reddy-Ryon-Durbin, 2014) which avoids the log-layer mismatch issue.