Simple IBM implementation test

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■ Problem Navier-Stokes incompressible équation with convective heat :
Find $(T, \mathbf{u}, p, k, \epsilon)$ solution of :

$$
\left\{
\begin{array}{rcl}
\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T - \nabla \cdot (\kappa_T \nabla T) & = & 0 \\
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \nabla \cdot (\mu_T \nabla \mathbf{u}) + \nabla p + \epsilon_0 (T - T_0) \vec{e}_2 & = & 0 \\
\frac{\partial k}{\partial t} + (\mathbf{u} \cdot \nabla) k + \epsilon - \nabla \cdot (\mu_T \nabla k) & = & \frac{\mu_T}{2} \tau^2 \\
\frac{\partial \epsilon}{\partial t} + (\mathbf{u} \cdot \nabla) \epsilon + c_2 \frac{\epsilon^2}{k} - \frac{c_\epsilon}{c_\mu} \nabla \cdot (\mu_T \nabla \epsilon) & = & \frac{c_1}{2} k \frac{\mu_T}{2} \tau^2
\end{array}
\right.
$$

with $\nabla \cdot \mathbf{u} = 0$, $\mu_T = c_\mu \frac{k^2}{\epsilon}$, $\kappa_T = \kappa \mu_T$.

■ Problem Navier-Stokes incompressible équation with convective heat and *Brinkman Penalisation* :

Find $(T, \mathbf{u}, p, k, \epsilon)$ solution of :

$$\left\{ \begin{array}{rcl} \frac{\partial T}{\partial t} + (\mathbf{u}\cdot\nabla)T - \nabla\cdot(\kappa_T \nabla T) & = & -\frac{\chi}{\eta_T}(T - T_{\partial C}) \\ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla\cdot(\mu_T \nabla \mathbf{u}) + \nabla p + \epsilon_0 (T - T_0)\vec{e}_2 & = & -\frac{\chi}{\eta}\mathbf{u} \\ \frac{\partial k}{\partial t} + (\mathbf{u}\cdot\nabla)k + \epsilon - \nabla\cdot(\mu_T \nabla k) & = & \frac{\mu_T}{2}\tau^2 \\ \frac{\partial \epsilon}{\partial t} + (\mathbf{u}\cdot\nabla)\epsilon + c_2\frac{\epsilon^2}{k} - \frac{c_\epsilon}{c_\mu}\nabla\cdot(\mu_T \nabla \epsilon) & = & \frac{c_1}{2}k\frac{\mu_T}{2}\tau^2 \end{array} \right.$$

with $\nabla\cdot\mathbf{u} = 0$, $\mu_T = c_\mu \frac{k^2}{\epsilon}$, $\kappa_T = \kappa\mu_T$. and $\chi = \left\{ \begin{array}{ll} 1 & \textit{if } \mathbf{x} \in C, \\ 0 & \textit{otherwise.} \end{array} \right.$

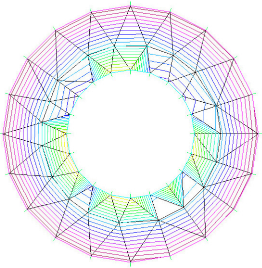
References : I.V.Abalakin,A.P.Duben, N.S.Zhdanova, T.K.Kozubskaya, Simulating an unsteady turbulent flow arround a cylinder by the immersed boundary method, *Mathematical Models ans Computer Simulation*, 2019, vol 11, No 1, pp 74-85.

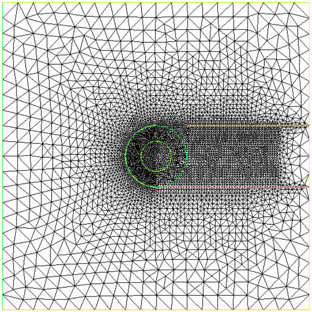
Time discretization : we use the caracteristics method and at every time step :
Find $(T^{n+1}, \mathbf{u}^{n+1}, p^{n+1}, k^{n+1}, \epsilon^{n+1})$ solution of :

$$\left\{\begin{array}{rcl}
\frac{1}{dt}(T^{n+1} - T^n \circ X^n) - \nabla \cdot (\kappa_T^n \nabla T^{n+1}) & = & -\frac{\chi}{\eta_T}(T^n - T_{\partial C} \\
\frac{1}{dt}(\mathbf{u}^{n+1} - \mathbf{u}^n \circ X^n) - \nabla \cdot (\mu_T^n \nabla \mathbf{u}^{n+1}) + \nabla p^{n+1} + \epsilon_0(T^{n+1} - T_0)\vec{e}_2 & = & -\frac{\chi}{\eta}\mathbf{u}^n \\
\frac{1}{dt}(k^{n+1} - k^n \circ X^n) + k^{n+1}\frac{\epsilon^n}{k^n} - \nabla \cdot (\mu_T^n \nabla k^{n+1}) & = & \frac{\mu_T^n}{2}(\tau^n)^2 \\
\frac{1}{dt}(\epsilon^{n+1} - \epsilon^n \circ X^n) + c_2 \epsilon^{n+1}\frac{\epsilon^n}{k^n} - \frac{c_\epsilon}{c_\mu}\nabla \cdot (\mu_T^n \nabla \epsilon^{n+1}) & = & \frac{c_1}{2} k^n \frac{\mu_T^n}{2}(\tau^n)^2
\end{array}\right.$$

with $\nabla \cdot \mathbf{u}^n = 0$, $\mu_T^{n+1} = c_\mu \frac{(k^{n+1})^2}{\epsilon^{n+1}}$, $\kappa_T^{n+1} = \kappa \mu_T^{n+1}$. and $\chi = \left\{\begin{array}{ll} 1 & \textit{if } \mathbf{x} \in C, \\ 0 & \textit{otherwise}. \end{array}\right.$

■ We consider C_h^+ a mesh arround the cylinder :





We use a $\mathbb{P}^1$ interpolation of a wall law :

$$(WL) \qquad u = \phi(y)$$

Then

$$\mathrm{I}_h^1 \circ \phi = \phi_h.$$

Reference :O.Hu,N.Zhao,J.M.Liu, A ghost cell method for turbulent compressible viscous flows on adaptative Cartesian grids, 2013, *Procedia Engeneering* 67 :241-249.

■ Implementation of boundary condition arround Cylinder : Ghost-cell method :

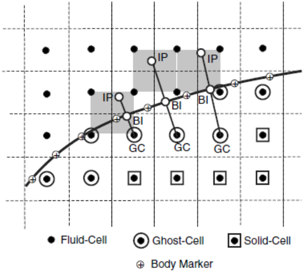


Figure – Ghost cell representation thanks to images points.

Update : $\Phi \in \{T, \mathbf{u}, p, k, \epsilon\}$
At every time step we do :

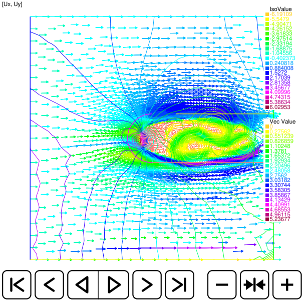
$$\Phi^n_{GC} = 2\Phi_{|\partial C} - \Phi^n_{IP},$$

where $\Phi^n_{IP} = \Phi_h(x_\delta, y_\delta)$, especialy for $\mathbf{u}$:

$$\mathbf{u}^n_{GC} = 2\phi_h(\delta)$$

Reference :R.Ghias, R.Mittal, H.Dong, A sharp interface immersed boundary method for compressible viscous flows, *Jounal of Computational Physics*, 2225 :528-553, 2003.

Results



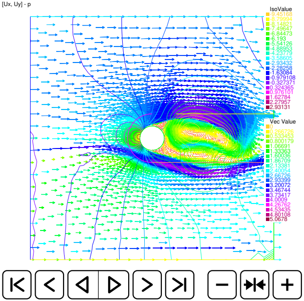


Figure – Flow field velocity with heat convection.

Results

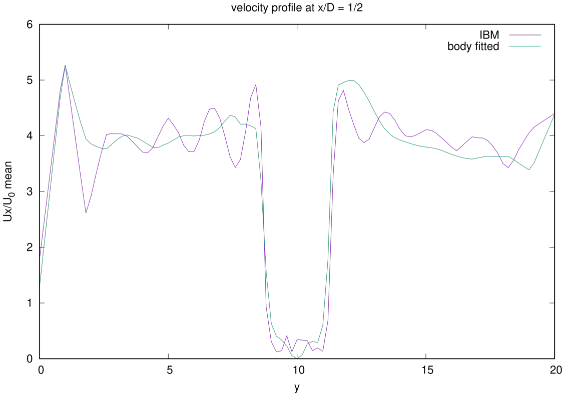


Figure – Velocity profile behind cylinder.

To do :

- Add ghost cell method with Brinkmann penalization.
- Modify the code to take into account a compressible flow.
- Implement a new 2 equations turbulence model.