Updates of hybrid 3D simulations on cylinder at Re=1M and review of transitional model

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Brief description

- DVMS side : Subgrid model
  - Smagorinsky :

$$\mu_{sgs} = \overline{\rho}(C_S\Delta)^2\sqrt{2S:S}, \quad \Delta = \left(\int_T d\mathbf{x}\right)^{1/3}$$

with S the deformation tensor.

  - Wall Adapting Local Eddy

$$\mu_{sgs} = \overline{\rho}(C_W\Delta)^2 \frac{(S^d:S^d)^{5/2}}{(S:S)^{5/2} + (S^d:S^d)^{5/4}},$$

$$S^d_{ij} = \frac{1}{2}(g^2_{ij} + g^2_{ji}) - \frac{1}{3}\delta_{ij}g^2_{kk},\ g^2_{ij} = \sum_k \frac{\partial \mathbf{u}_i}{\partial \mathbf{x}_k}\frac{\partial \mathbf{u}_k}{\partial \mathbf{x}_j}$$

Brief description : suite of previous presentation

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- RANS side : $k - \varepsilon$ with menter correction

$$\mu_t = \frac{\overline{\rho} k \sqrt{c_\mu f_\mu}}{\max\left(\frac{\varepsilon}{k\sqrt{c_\mu f_\mu}}, \left|\frac{\partial \mathbf{u}}{\partial y}\right| \tanh(\psi^2) \right)},$$

$$\psi = \max\left(2\frac{k^{3/2}}{y\varepsilon}, \frac{500 \mu c_\mu k}{\overline{\rho} \varepsilon y^2 Re} \right)$$

- Model used : RANS/DVMS with :
  - Blending : $\theta = 1 - f_d \times (1 - \overline{\theta})$; $\overline{\theta} = \tanh\left(\left(\frac{\Delta_T}{k^{3/2}}\varepsilon\right)^2\right)$,
  - Subgrid model for VMS : WALE,
  - Closure model for RANS : $\begin{cases} k - \varepsilon & \text{Goldberg} \\ k - \varepsilon & \text{with Menter correction} \end{cases}$
- Simulation set up :
  - mach number : 0.1 (subsonic flow)
  - reference pressure : 101300 $[\mathrm{N/m^2}]$
  - density : 1.225 $[\mathrm{kg/m^3}]$
  - minimal mesh size is such that $y_w^+ = 1$

Name	Mesh size	y_w^+	y_m^+	$\overline{C}_d$	C_l'	$-\overline{C}_{pb}$	L_r	$\overline{\theta}$
Present simulation								
URANS $k-\varepsilon$	4.8M	1	0	0.50	0.24	0.61	0.77	109
DDES $k-\varepsilon$ Goldberg WL	4.8M	20	100	0.20	0.04	0.22	0.87	138
DDES $k-\varepsilon$ Goldberg WL	4.8M	20	25	0.40	0.05	0.56	1.46	113
DDES $k-\varepsilon$ Goldberg ITW	4.8M	1	0	0.50	0.07	0.54	1.22	103
DDES $k-\varepsilon$ menter ITW	4.8M	1	0	0.69	0.21	0.80	1.26	102
DVMS								
cubic Smagorinsky ITW	4.8M	20	0	0.49	0.17	0.42	0.71	92
DDES/ DVMS								
k - ε / cubic WL Smagorinsky	4.8M	20	100	0.20	0.02	0.22	0.82	135
k - ε / cubic WALE WL	4.8M	1	100	0.20	0.02	0.26	0.80	132
k - ε / cubic WALE ITW	4.8M	1	0	0.49	0.06	0.60	1.56	104
k - ε menter/ cubic WALE ITW	4.8M	1	0	0.57	0.11	0.69	1.80	103
RANS / DVMS								
k - ε / cubic Smagorinsky WL	4.8M	20	100	0.24	0.05	0.22	0.62	133
k - ε / cubic Smagorinsky WL	4.8M	1	100	0.25	0.09	0.25	0.64	132
k - ε / cubic WALE WL	4.8M	1	100	0.26	0.11	0.22	0.65	134
k - ε / cubic WALE ITW	4.8M	1	0	0.48	0.11	0.55	1.14	109
k - ε menter/ cubic WALE ITW	4.8M	1	0	0.54	0.15	0.64	1.16	106
Other simulations								
RANS Catalano [1]	2.3M	-	-	0.39	-	0.33		
LES Catalano [1]	2.3M	-	-	0.31	-	0.32		
LES Ono [2]	4.5M	-	-	0.27	0.13		-	
LES Kim [3]	6.8M	-	-	0.27	0.12	0.28	-	108
Expériences								
Shih et *al* [5]				0.24	-	0.33		
Schewe [4]				0.22	-	-		
Szechenyi [6]				0.25	-	0.32		
Gölling [9]							-	130
Zdravkovich [8]				0.2-0.4	0.1-0.15	0.2-0.34		

Table – Bulk coefficient of the flow around a circular cylinder at Reynolds number 1M, $\overline{C}_d$ holds for the mean drag coefficient, C_l' is the root mean square of lift time fluctuation, $\overline{C}_{p_b}$ is the pressure coefficient at cylinder basis, L_r is the mean recirculation lenght, $\overline{\theta}$ is the mean separation angle.

Pressure coefficient

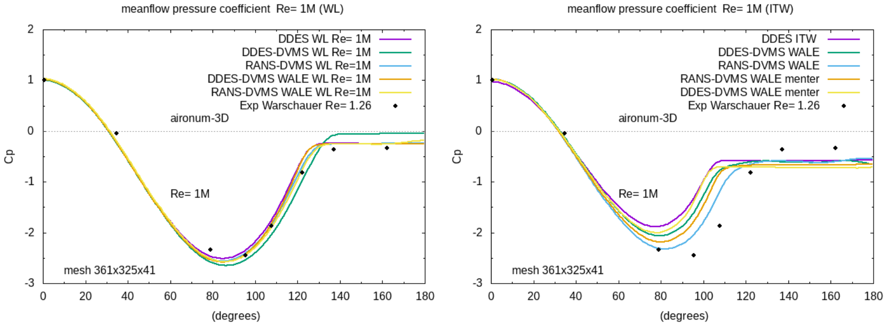


Figure – Distribution of mean pressure as a function of polar angle. Comparaison between experiment. Wall law on the left and integration to the wall on the right.

■ Skin friction coefficient :

$$C_f = \frac{\tau_w}{1/2\rho_\infty U_\infty^2}, \quad \text{with } \tau_w \text{ the wall shear stress} \tag{1}$$

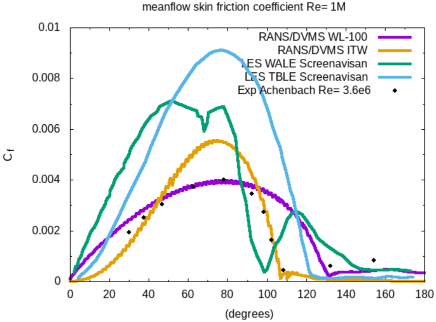


Figure – Distribution of skin friction coefficient as a function of polar angle. Comparaison between experiment.

Review of transition modeling

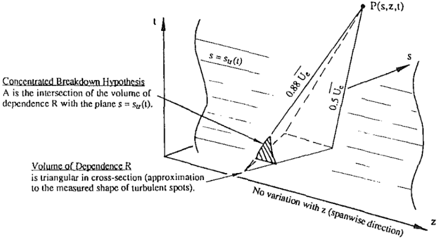
- Algebraic relation [11][13][10][12], main work of Addison, Narasimha :

$$\text{Narasimha1985}: \quad \gamma(\mathbf{x}) = 1 - \exp\left(-0.412\frac{(x-x_t)^2}{\lambda^2}\right) \quad x \geq x_t, \gamma = 0 \text{ otherwise}$$

$$\text{Addison1992}: \quad \gamma(\mathbf{x}, t) = 1 - \exp\left(-\int_{A(\mathbf{x},t)} \frac{\nu N}{\sigma\theta}(x_t, z, t)dzdt\right)$$

x_t : start of transition point ($\gamma(x_t) = \frac{1}{2}$), $\sigma = 0.25$ and N depend of λ_θ : the pressure gradient of boundary layer.

$$\mu_{eff} = \mu + \overline{\gamma}(\mathbf{x})\mu_T, \quad \mu_T \text{ turbulente viscosity} \qquad (2)$$



■ Arround $k - \varepsilon - \gamma$, main works are Kollman 1986 [16], Cho-Chung 1992 [17], Steelant 1996 [19], Suzen 2000 [18], we present work of Akter 2007 [14] :

$$\text{Akter 2007} : \frac{\partial \rho\gamma}{\partial t} + \nabla \cdot \rho \mathbf{u} \gamma = C_{g1}\gamma(1-\gamma)\frac{P_k}{k} + \rho C_{g2}\frac{k^2}{\varepsilon}\nabla\gamma \cdot \nabla\gamma - C_{g3}\rho\gamma(1-\gamma)\frac{\varepsilon}{k}\Gamma + \nabla \cdot [\sigma_\gamma(1-\gamma)(\mu+\mu_t)\nabla\gamma]$$

- The first term represent the kinetic turbulent energy production : $P_k = \mu_t \| \frac{\partial \mathbf{u}}{\partial y} \|^2$
- The increase of γ is modeled by $\rho C_{g2}\frac{k^2}{\varepsilon}\nabla\gamma \cdot \nabla\gamma$
- The third term is an inertial term where :

$$\Gamma = \frac{k^{5/2}}{\varepsilon^2}\frac{1}{\|\mathbf{u}\|}(\mathbf{u}\nabla\mathbf{u}) \cdot \nabla\gamma. \tag{3}$$

- The last term is a diffusion term

We have the following constants $C_{g1} = 0.19$, $C_{g2} = 0.1$, $C_{g3} = 0.01$, $\sigma_\gamma = 1$.

- Akter 2007 [14] viscosity : Thus, the transition concept has been incorporated by modifying expression of eddy viscosity, as following :

$$\mu_t^* = \left[1 + C_{\mu g}\frac{k^3}{\varepsilon^2}\gamma^{-3}(1-\gamma)\|\nabla\gamma\|^2\right]\mu_t, \tag{4}$$

with $C_{\mu g} = 0.1$ and $\mu_t = C_\mu f_\mu \frac{k^2}{\varepsilon}$ and $f_\mu = \left(1 + \frac{3.45}{\sqrt{Re_t}}\right)\left[1 - \exp\left(-\frac{y^+}{70}\right)\right]$

- Initial and boundary condition :

$$\begin{cases} \gamma(\mathbf{x}, 0) & = & 10^{-3}, \\ \gamma_{|\partial\mathcal{C}} & = & 1. \end{cases} \tag{5}$$

- A recent model : $k - R$ of Zhang 2020 [20] :
  - A two equation model equation on k and $R \simeq \frac{k^2}{\varepsilon}$
  - An original production term : $\mathfrak{S} \simeq s - \frac{|\eta_1| - \eta_1}{c_T}; \quad \eta_1 = s - v,$
  - $\gamma = \tanh\left(\max\left(0, \frac{s}{v} - 1\right)\right)$ where $s = \sqrt{2S : S}$ and $v = \sqrt{2\Omega : \Omega}$:

$$\mu_t = \rho c_\mu f_\mu \left[\underbrace{k T_t (1 - \gamma)}_{\mu_t^{(1)}} + \underbrace{R\gamma}_{\mu_t^{(2)}} \right]. \qquad (6)$$

  - The main particularity is $0 < \gamma < 1$ which create a transition between laminar to turbulent flow

- Other models has been developed :
  - $k - \zeta - \gamma$ of Warren 1997 [22] developed for high mach computation
  - $k - k_L - \gamma$ of Lardeau 2004 [23] blend laminar and turbulent kinetic energy
  - $\gamma - \widetilde{Re}_\theta$ of Langtry and Menter [21] two transport equation coupled with $k - \omega$ SST model
  - $k - \widetilde{\omega} - \gamma$ of Lorini 2014 [?] separate natural bypass transition and separation induced mode

Conclusions and things to do

- WALE model with $y_w^+ = 1$ gives better results,
- Bulk coefficients are close to experimental data for WL,
- Drag coefficients are over estimated,
- We are implementing $k - R$, $k - \varepsilon - \gamma$ model to improve the transition from laminar to turbulent.

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Annexe

Definition

(Momentum thickness)
Let ρ_∞, u_∞ respectively free stream density and velocity, we define the momentum thickness $\theta : \Omega \times [0, T] \longrightarrow \mathbb{R}$ such that :

$$\theta(\mathbf{x}, t) = \int_0^{\overline{y}} \frac{\rho(\mathbf{x}, t)\mathbf{u}(\mathbf{x}, t)}{\rho_\infty u_\infty} \left(1 - \frac{\mathbf{u}(\mathbf{x}, t)}{u_\infty}\right) dy$$

where direction y is the free stream direction.

Definition

(Pressure gradient of boundary layer) We call pressure gradient of boundary layer, the following quantity :

$$\lambda_\theta = \frac{\rho\theta^2}{\mu}\frac{\partial u_x}{\partial s}, \quad (7)$$

where the derivative is the acceleration in the streamwise direction, and θ is the momentum thickness.