Progress in anisotropic moving-mesh adaptation in 2D and the status in 3D.

Speaker: Valeriia Tsvetkova

Keldysh Institute of Applied Mathematics of RAS

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Main features of the technique

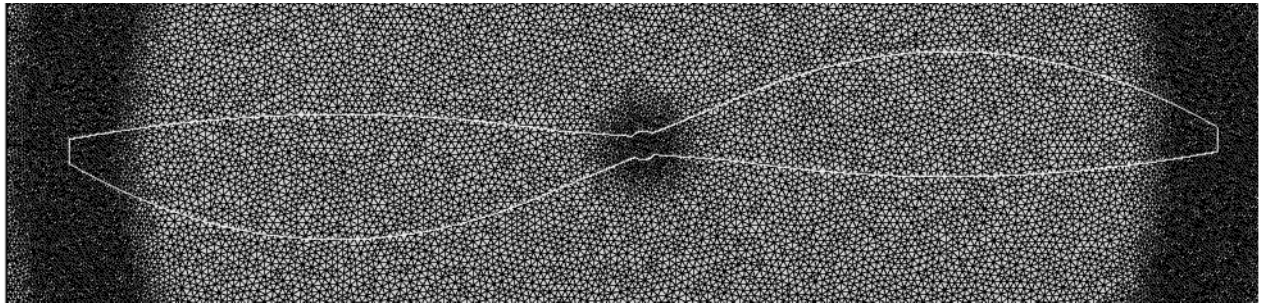
- Simply connected domain

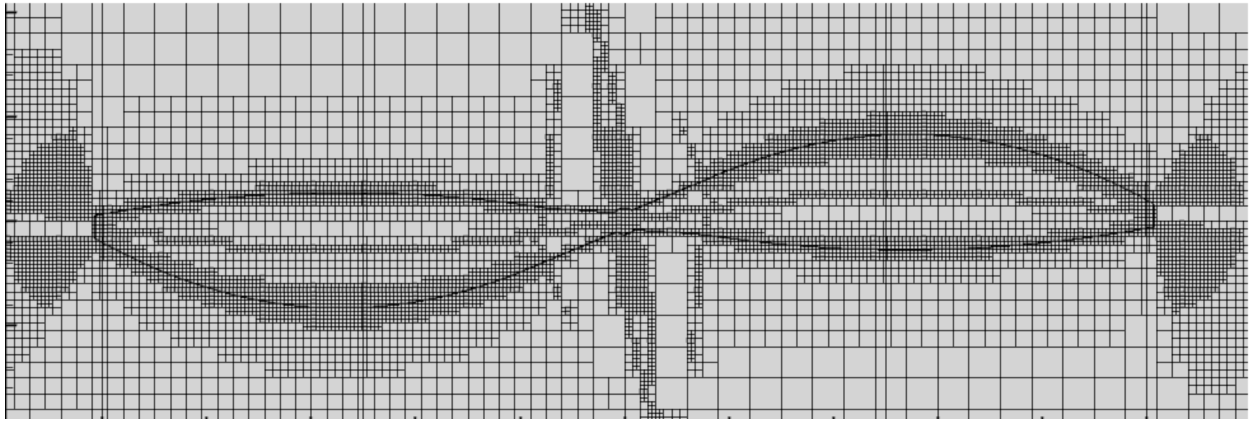
- Geometry is defined by interpolation grid (level-set tree)

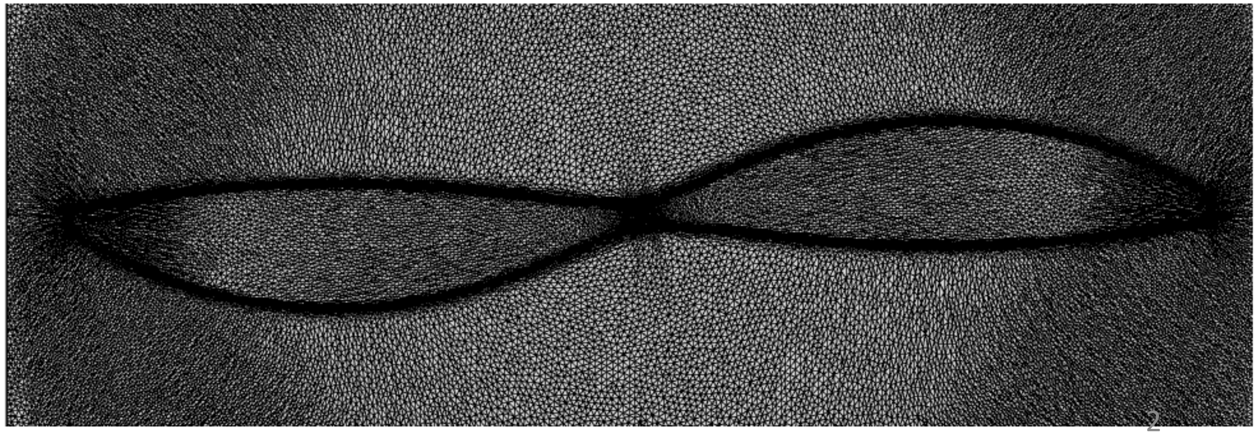
- Immersed boundary method (IBM) – Brinkman penalization

- The shape of the body is approximated using adaptation of r-type (nodes are redistributed while topology remains the same)

- Adaptation produces anisotropic cells







Features of the adaptation technique

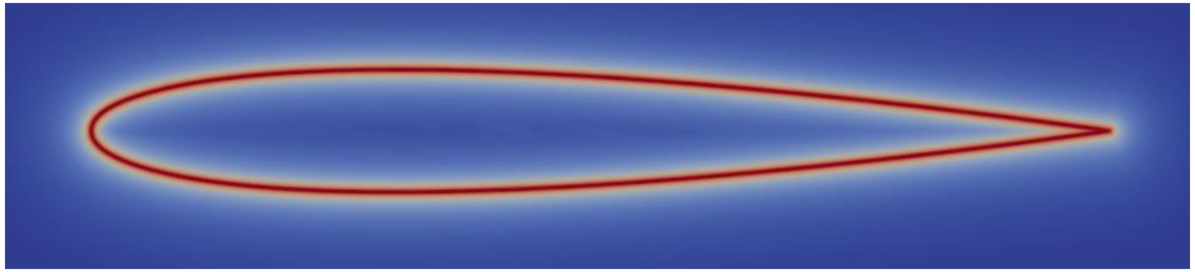
- Level-set function u(x,t) defines the solid body location and is close to signed distance function near the boundary
- Metric tensor G(x,t) is built upon u(x,t) as

$$G(x,t) = \sigma_1^2 I + (\sigma_2^2 - \sigma_1^2)\nabla_x u \nabla_x u^T \frac{1}{|\nabla_x u|^2} \quad \xrightarrow{\sigma_2 = \sigma_1} \quad G(x,t) = \sigma_1^2 I$$

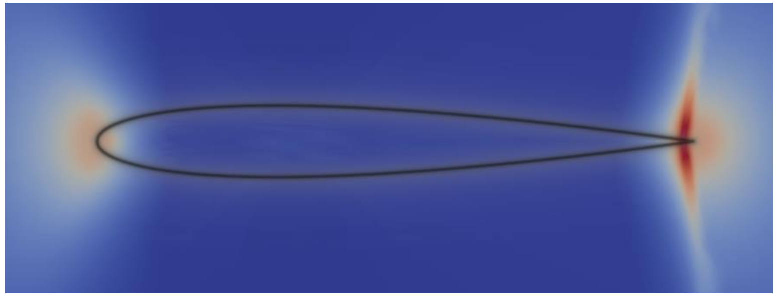
- On highly curved fragments of the boundary or near sharp vertices $\sigma_2 = \sigma_1$, otherwise $\sigma_2 = \sigma_1 / K$. K is user-defined anisotropic ratio.

$\sigma_1 = \sigma_{\text{normal}}(x,t)$ - mesh stretching in the normal direction

$\sigma_2 = \sigma_{\text{tangential}}(x,t)$ ($\sigma_{2,3}$ in 3D) - spatial distribution of the anisotropy

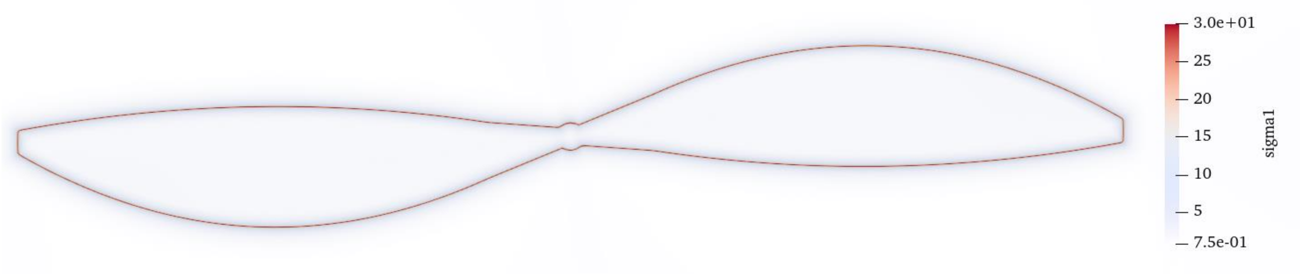


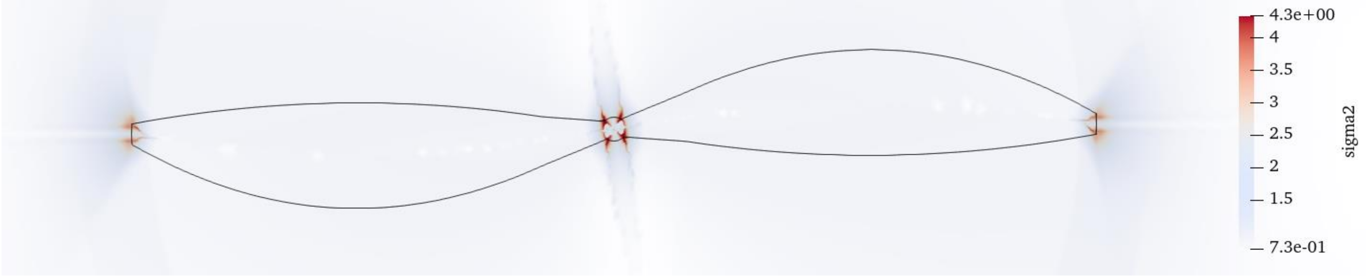
σ_1 distribution

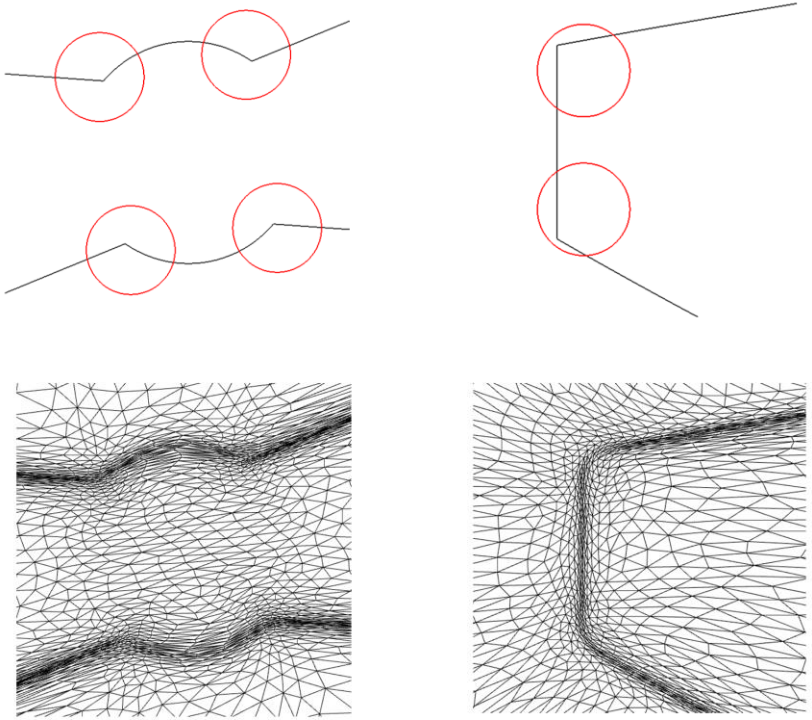


σ_2 distribution

Manual anisotropy control



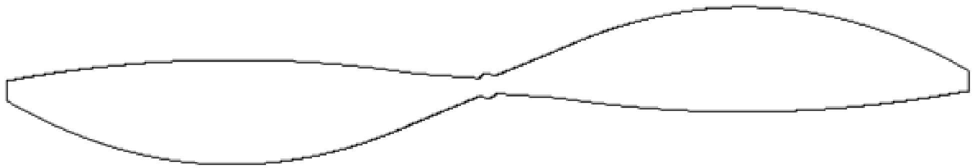




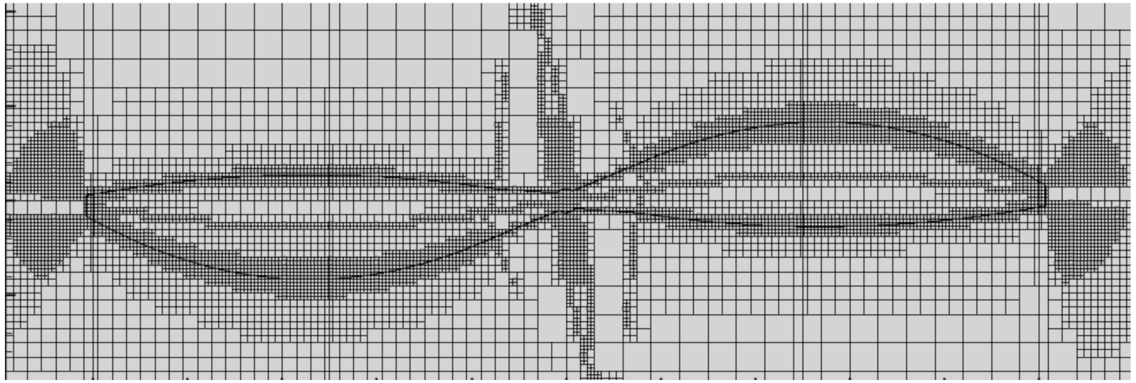
Automatic anisotropy control

Main idea: define curvature on interpolation grid during mesh preparation

Approximated curvature is defined in contour vertices

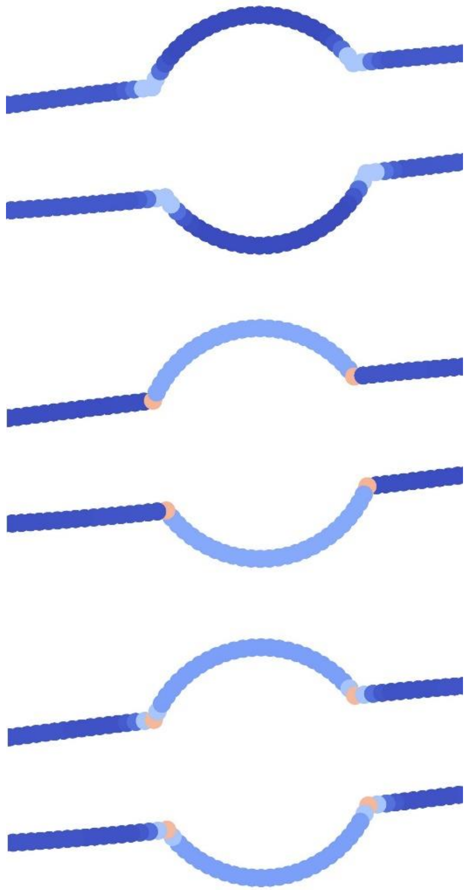


Curvature extrapolated to the entire domain



Adaptation will use the curvature as a predefined value

Calculation of the tangential compression parameter



For each vertex of the contour the approximate curvature and the distance the medial axis are calculated.

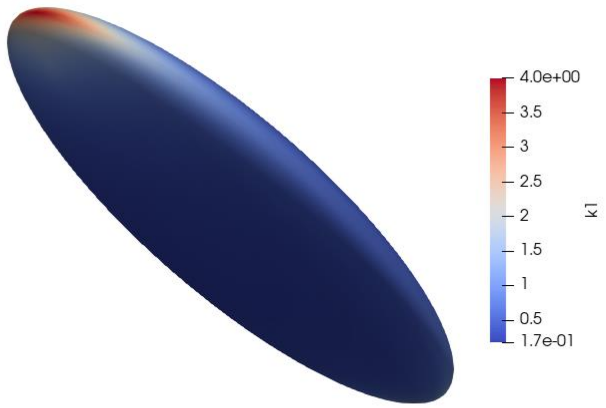
σ_2 is defined by these two parameters.

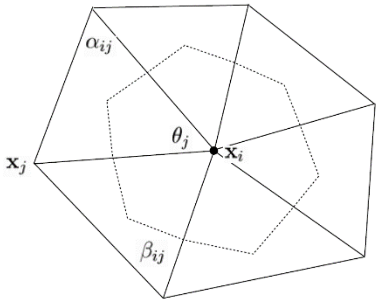
If distance to the medial axis is less than the size of the boundary layer in the logical space, σ_2 is increased to avoid “tearing” the meshed.

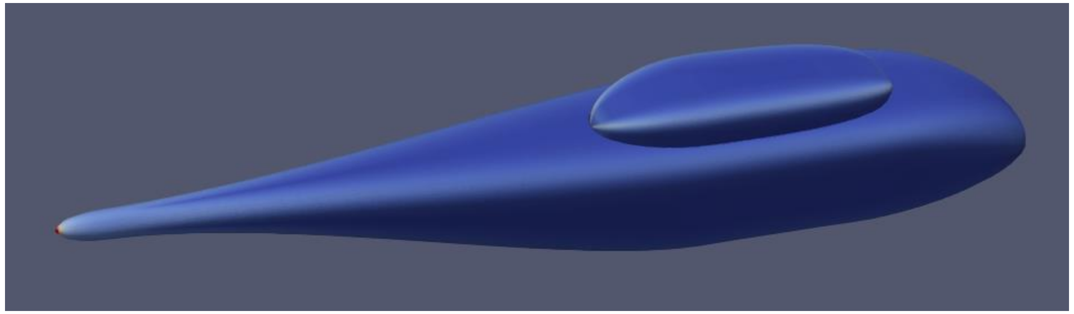
Picture shows the distribution of eigen value σ_2 along the contour of the body defined by 1) medial axis 2) curvature 3) their combination

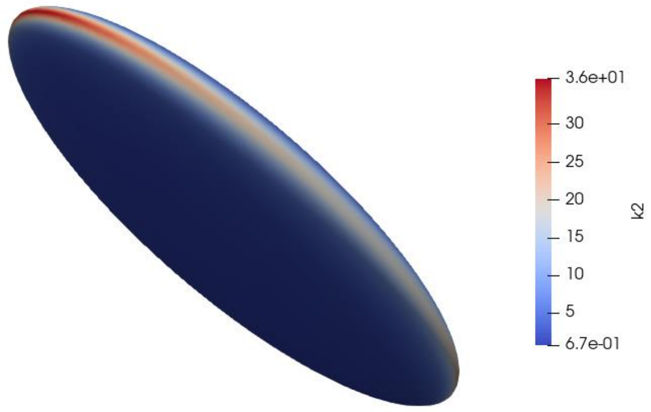
Curvature calculation

- Surface triangulation is used
- For every vertex of the triangulation we build the approximation of the touching paraboloid
- Quadratic fitting is found using vertices of first and second order of neighborhood and the least squared method *
- The tangential coordinate system in every vertex of the surface triangulation is defined iteratively.
- For the quadratic fitting principal curvature and principal directions are calculated



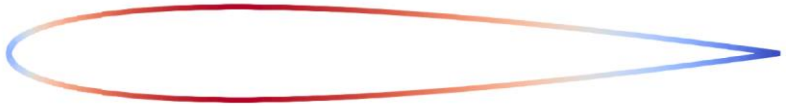


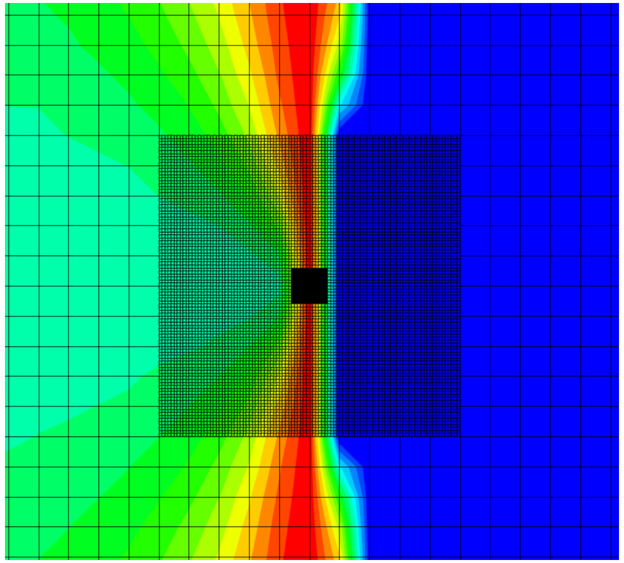


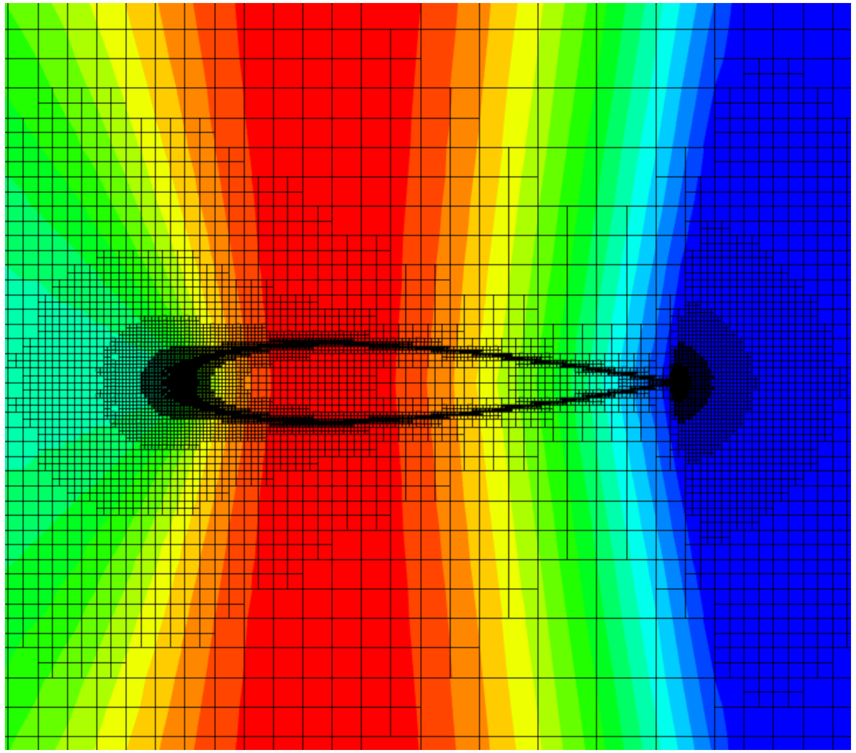


* Garimella, Rao. (2003). Curvature Estimation for Unstructured Triangulations of Surfaces.

Extrapolation to the grid

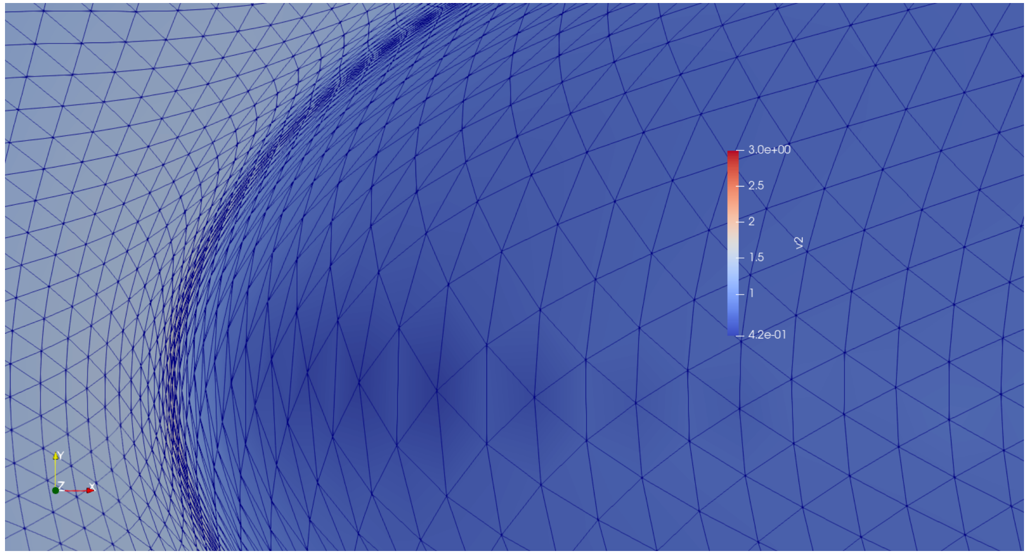


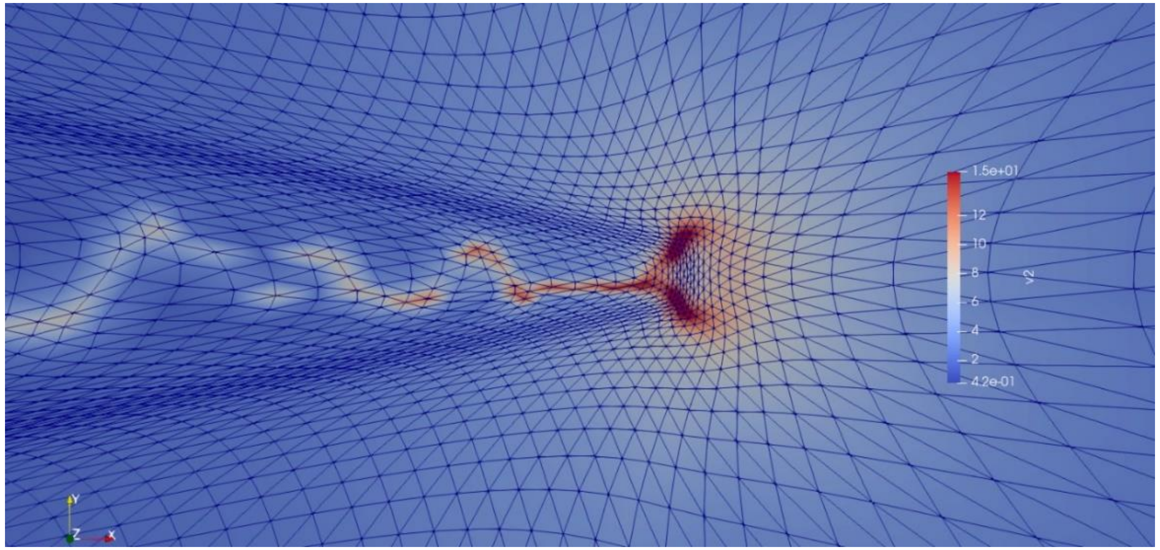




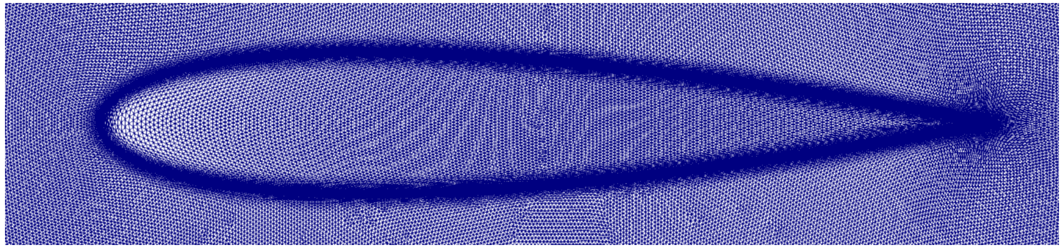
Every vertex of the octree carries the radius of curvature taken from the nearest point of the body contour.

Control parameters accounting curvature



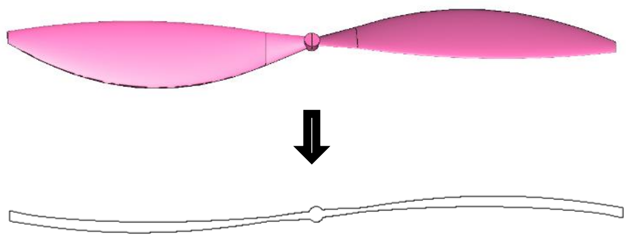


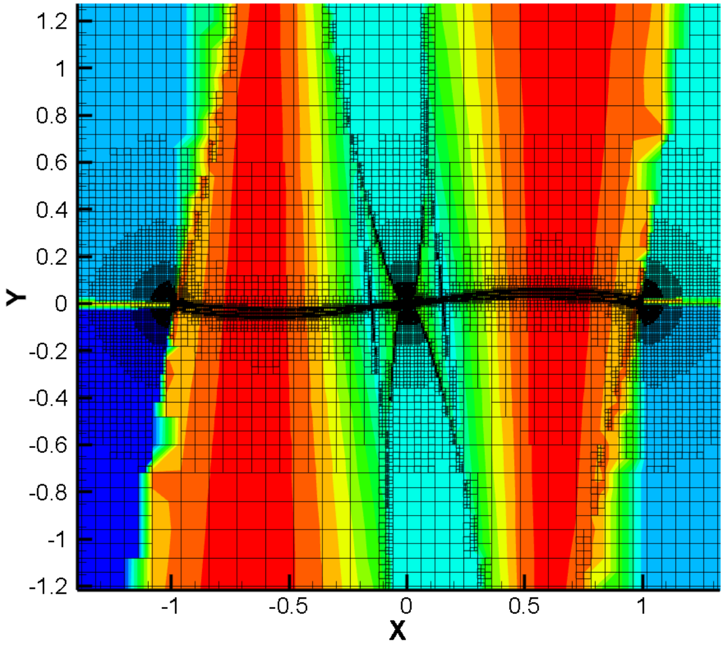
σ_2 destribution

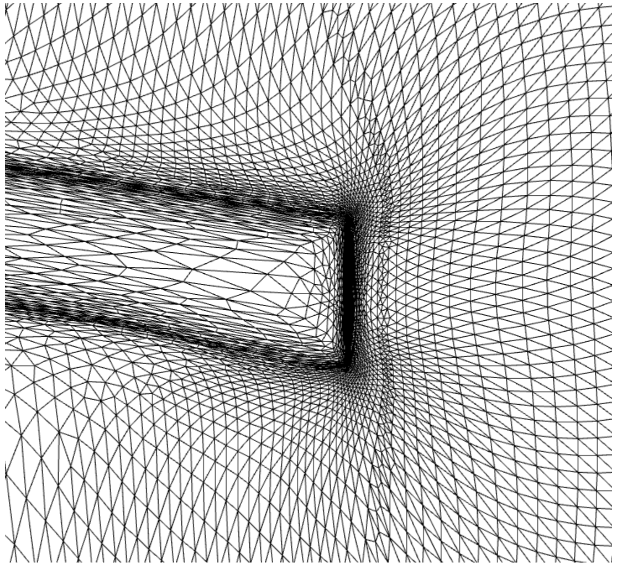


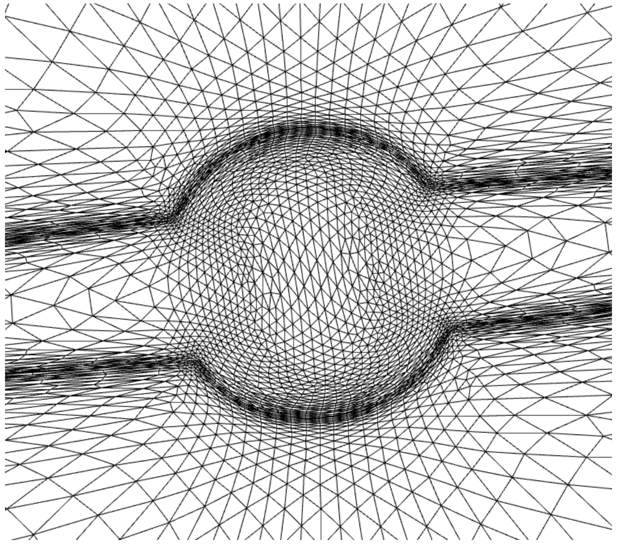
The compression degree in the tangential direction is locally increased if during adaptation “holes” in the mesh appear

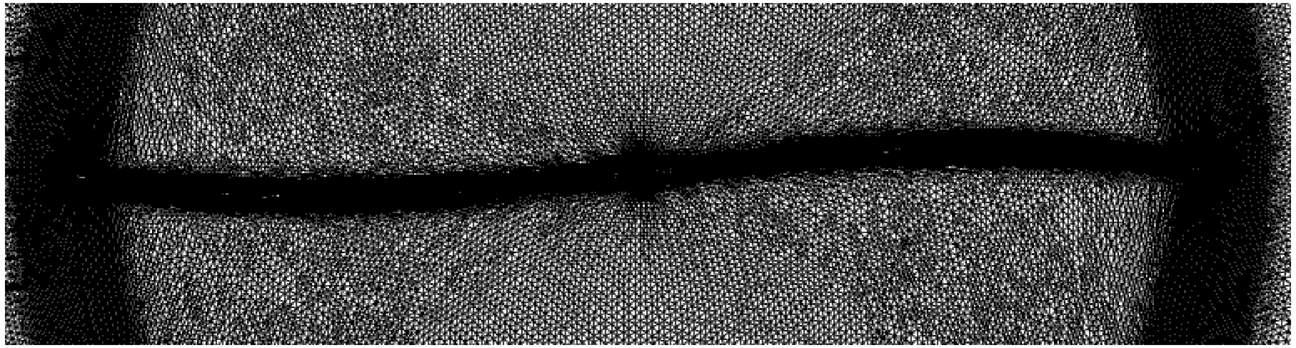
Examples of using of the described method for 2D geometry



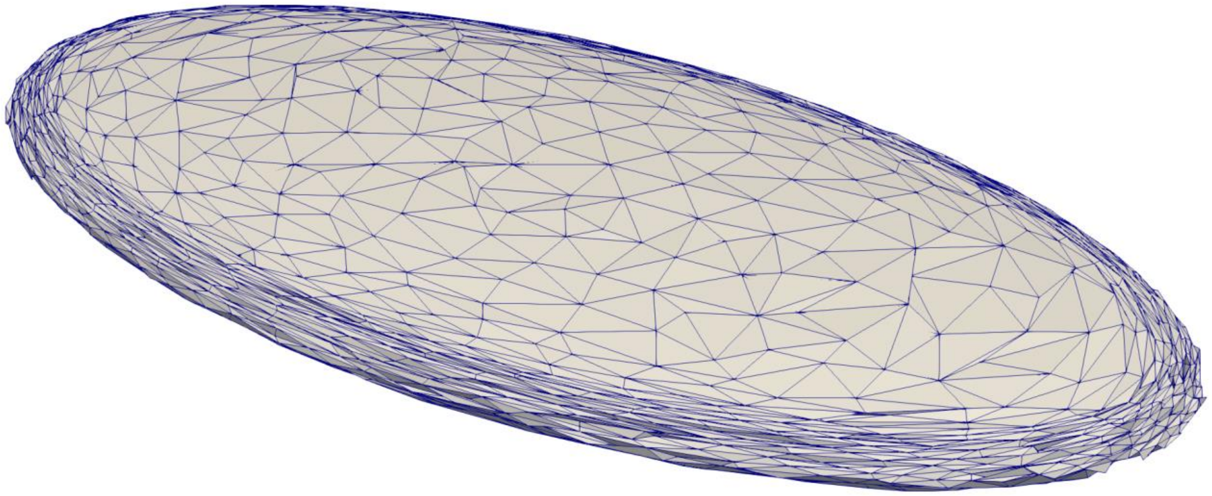


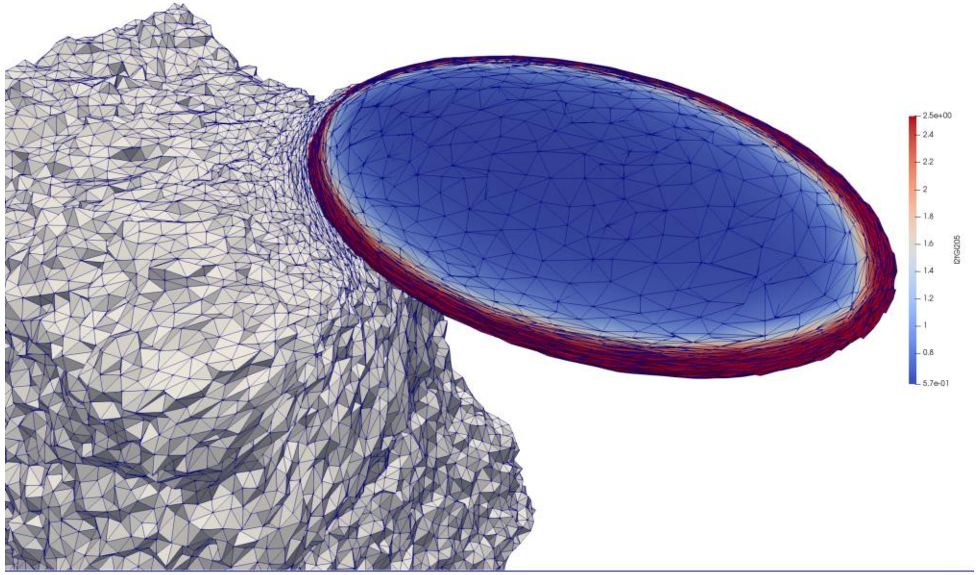


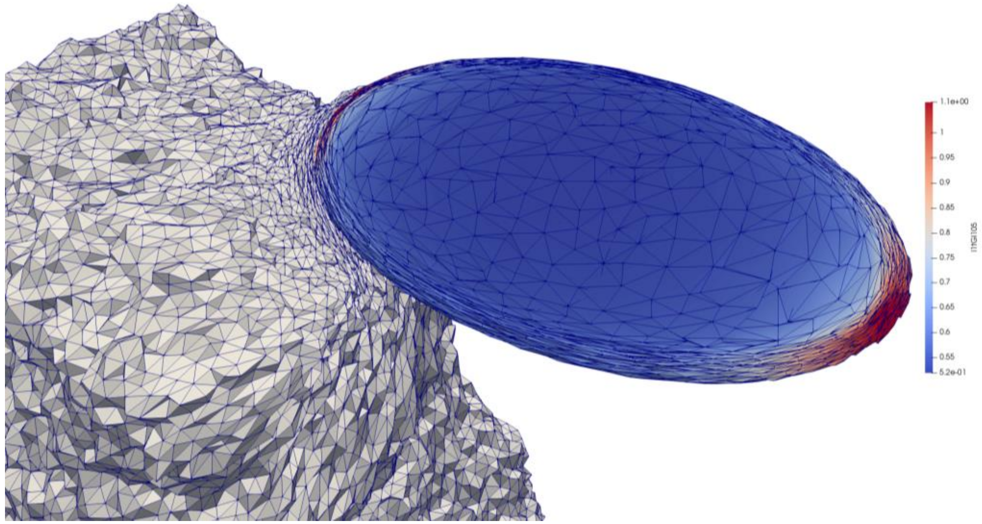




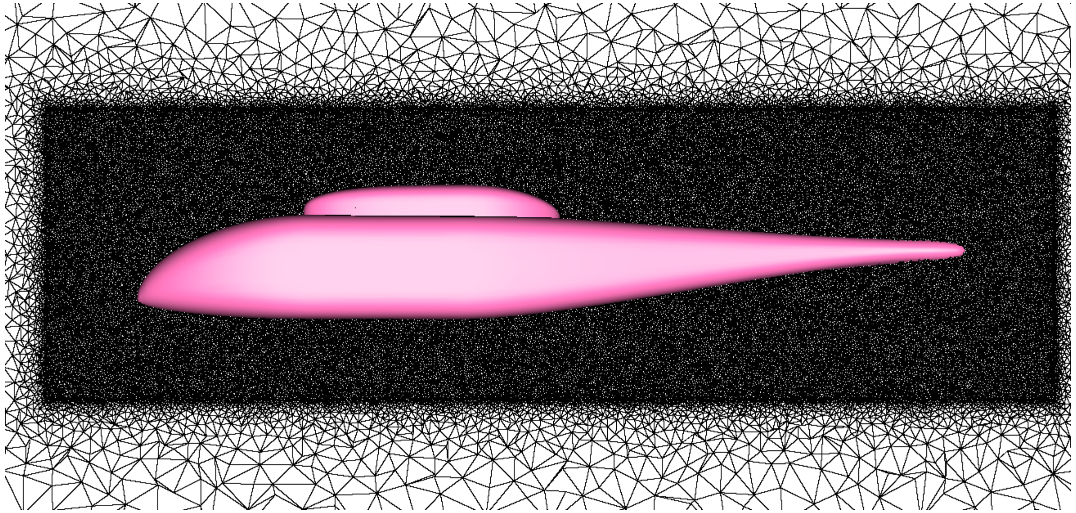
Examples of using the described method for 3D geometry

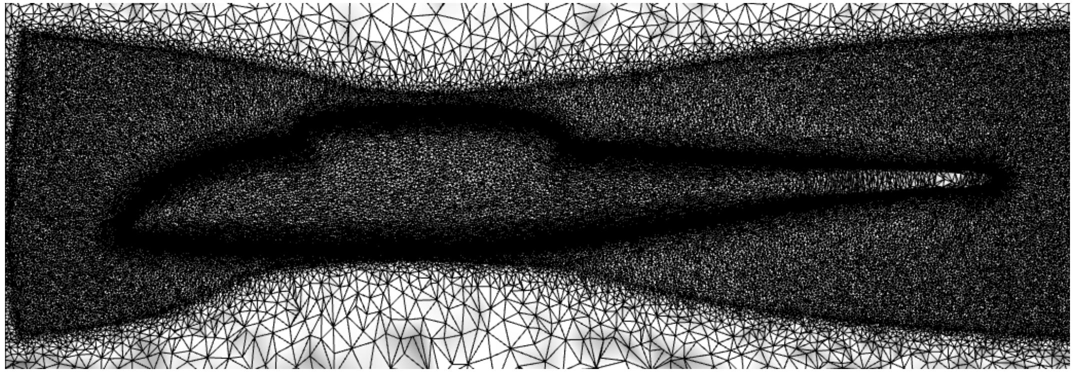


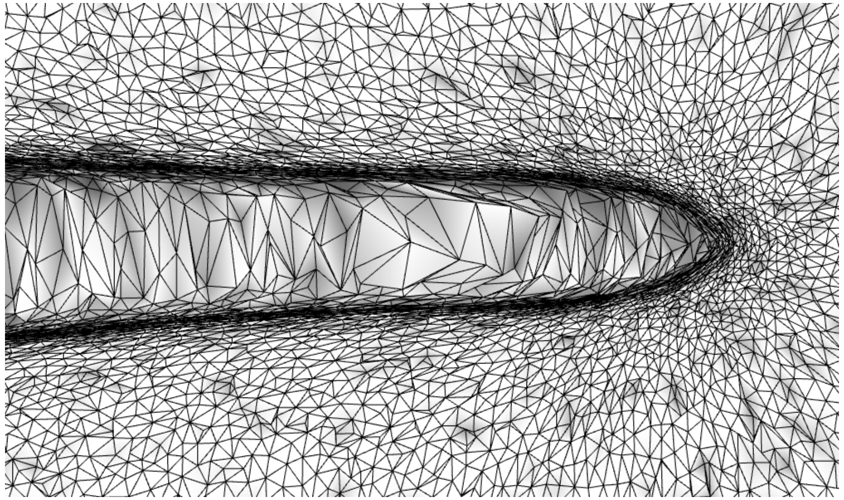


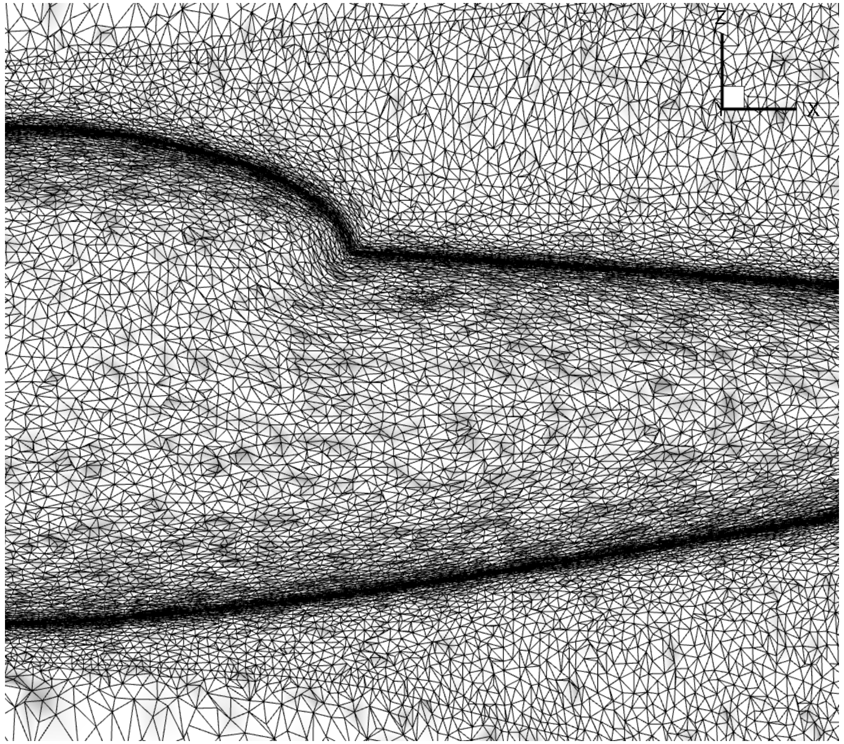


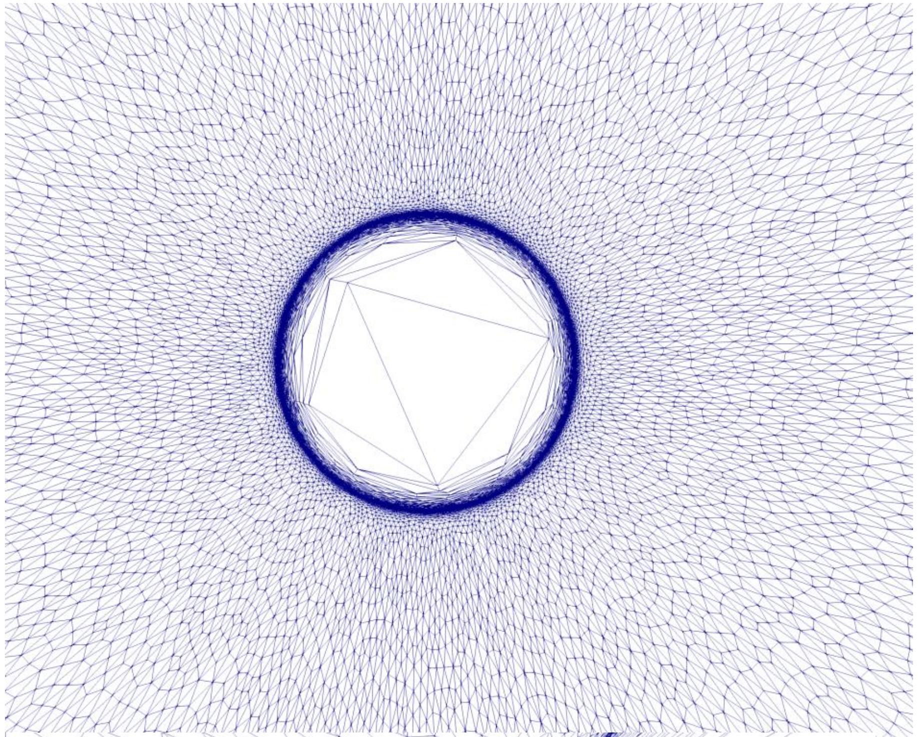
Adaptation for 3D

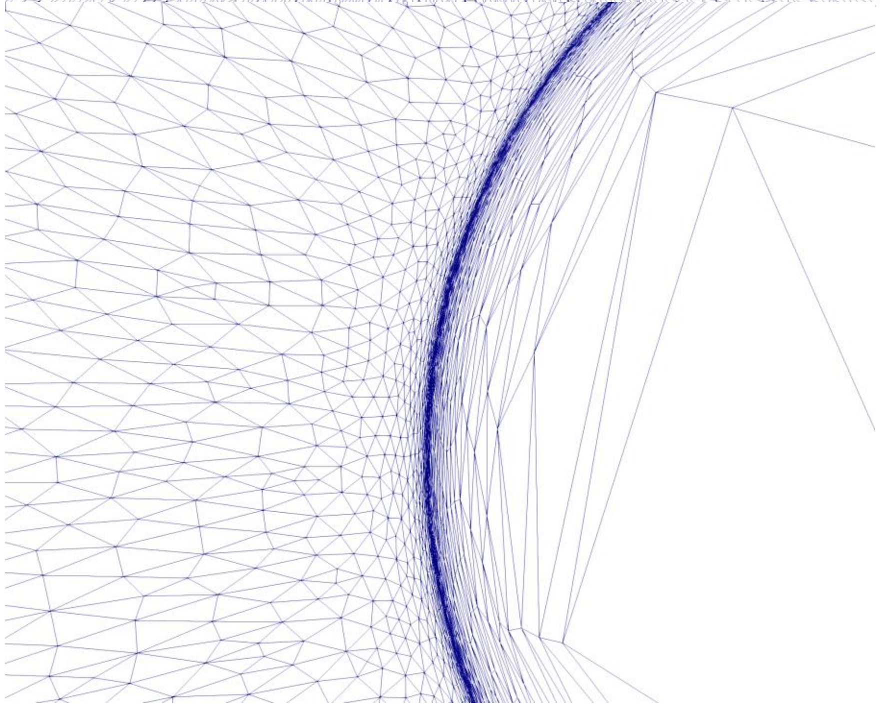


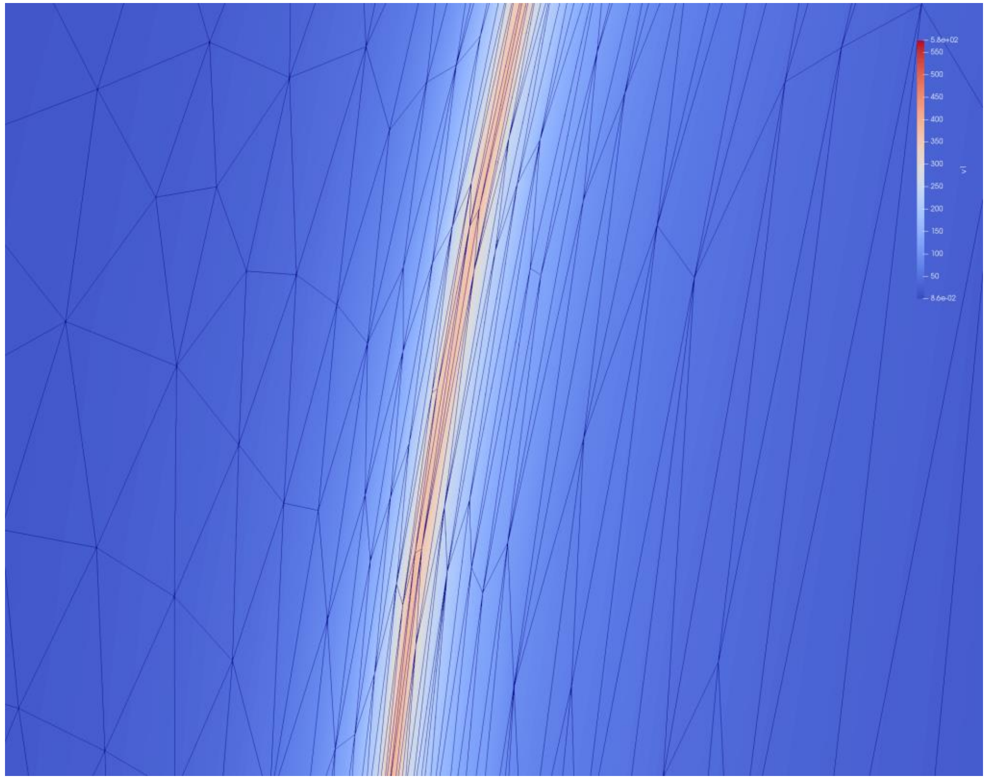






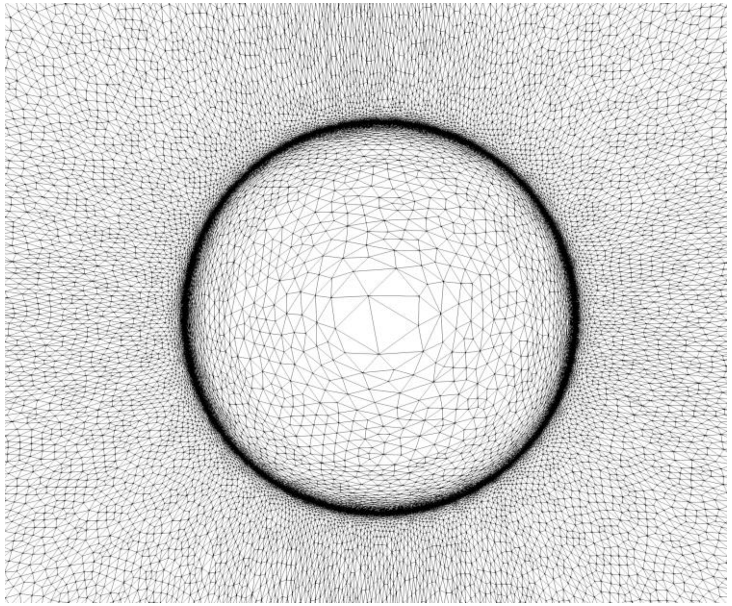


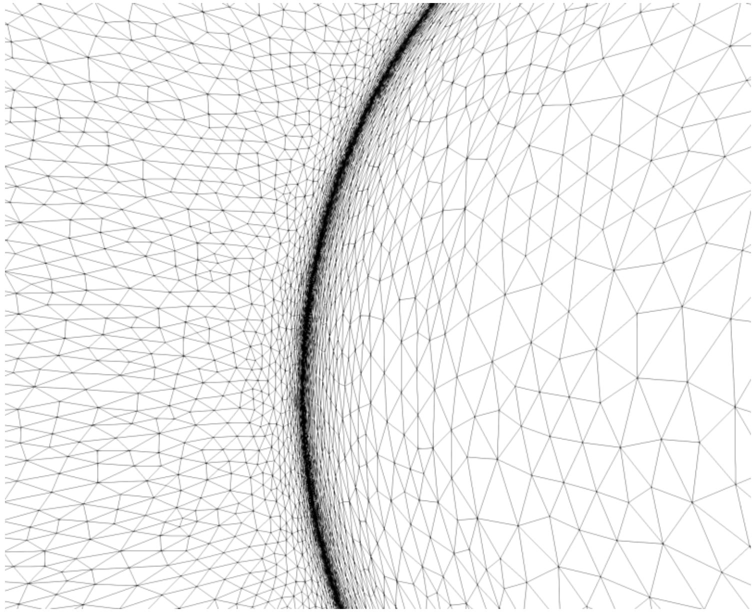


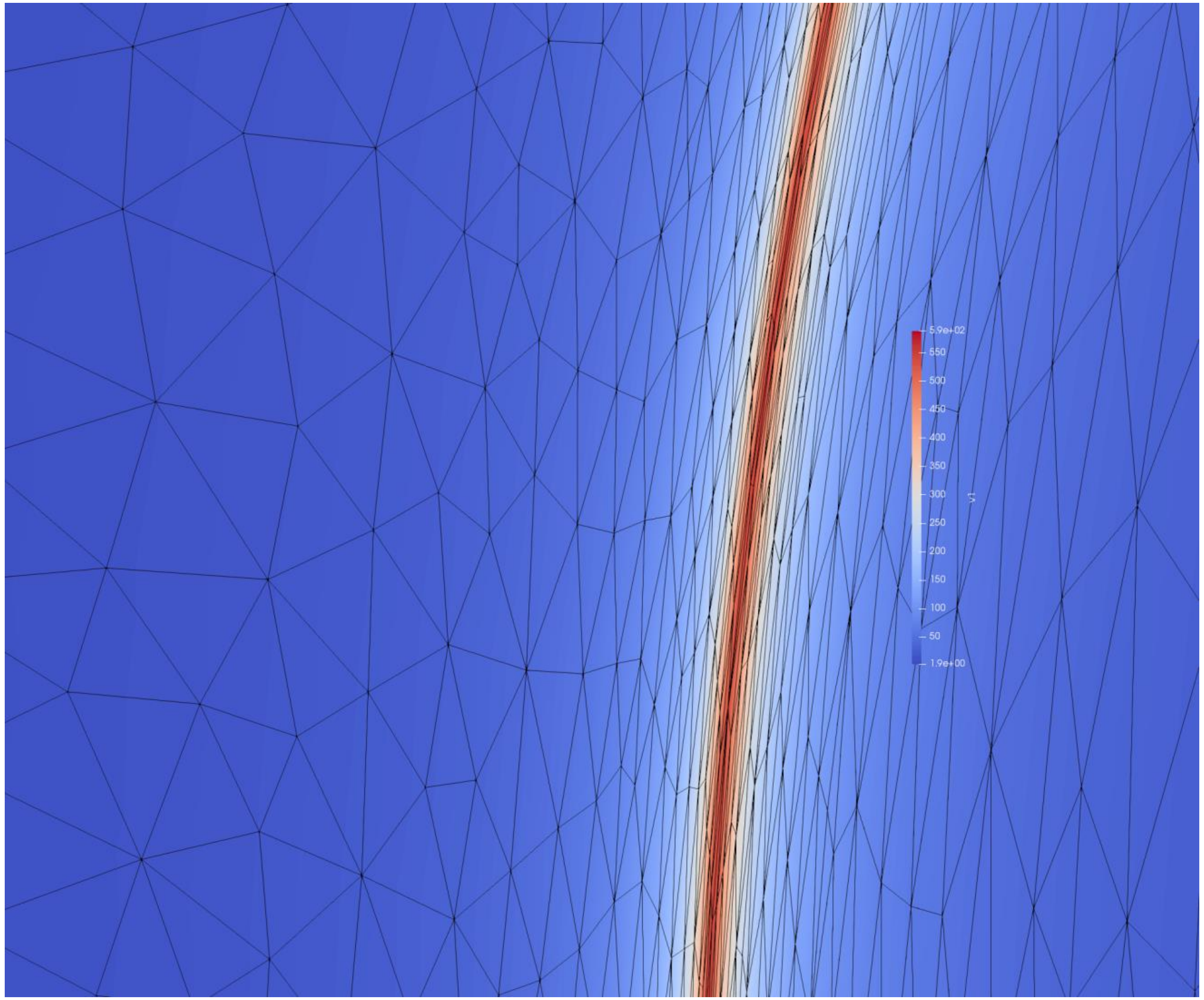


Testing high compression degree on analytical geometry

Accounting distance to medial axis







Problems to solve

- Test adaptation control on complicated 3D objects
- Build high quality adapted mesh for helicopter fuselage and the rotor of a drone
- Test movement of the adapted mesh in 3D
- Simulate flow near fuselage using IBC + adpatation