NORMA

Advancements of the Sophia team
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Motivation for CENO HO

(a) To improve the accuracy of existing software :
tetrahedra, vertex, MUSCL FVM.

(b) To keep its adequation to mesh adaptation: small number of degrees of freedom (d.o.f.) for a given mesh, narrow shock capturing).

⇒ Improve cell polynomial reconstruction, “CENO”.

About complexity

High Order Finite-volume based approximations:

we have to evaluate for a given mesh:
- the number of degrees of freedom,
- the reconstruction/interpolation effort, and
- the number of finite volume fluxes to be evaluated.

By comparison with second-order schemes, e.g. MUSCL, vertex CENO needs :
- reconstructions, and, of most high cost,
- high-order integrated fluxes at finite volume interfaces.

Comparison with a discontinuous Galerkin of degree three (for fourth order):
A DG3 *element* contains 20 d.o.f. and needs 4 high-order integrated fluxes.
The DG3 element can be split in 27 DG1 or P_1 elements.
Counting 6 elements for a vertex, this shows that a vertex CENO of degree 3 will have 27/6= 4.44 d.o.f. on the same DG3 mesh,
but with 14*27/6=62 fluxes to evaluate.
Further, this last figure, 62, is still increased due to the special geometry of vertex-based cell interfaces.

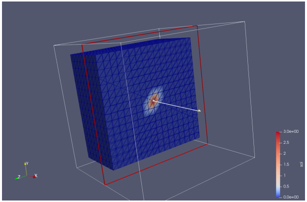
Then it appears that the Achille neel of vertex CENO is the computational cost of the fluxes.

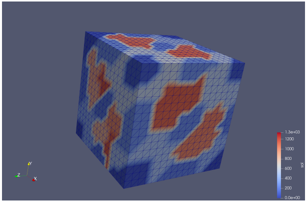
The smaller number of d.o.f. can be advantageous for CENO in case of implicit time advancing.

The treatment of discontinuity can also be easier than for DG.

And for the user, a given mesh of n vertices will not lead to much more than n d.o.f.

Polynomial reconstruction





Reconstruction macromolecules are needed around each of the n cells.

In the *central option*, n macromolecules of k cells are built and n reconstruction are computed.

In the *partition option*, n/k macromolecules are built and n/k reconstruction are computed, at the risk of a degraded accuracy.

Global CENO scheme

For a new mesh:

- build macromolecules
- invert reconstruction matrices

For a new flow, on each cell:

- compute the polynomial with the inverted matrix

Flux assembly, for each interface

- high order (upwind) flux integration

Fourth-order integration

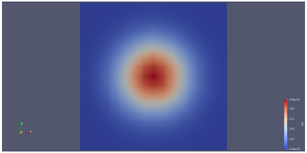
Point location	Weight
Triangles	
$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$	$-\frac{9}{16}$
$(\frac{1}{5},\frac{1}{5},\frac{3}{5})$	$\frac{25}{48}$
$(\frac{1}{5},\frac{3}{5},\frac{1}{5})$	$\frac{25}{48}$
$(\frac{3}{5},\frac{1}{5},\frac{1}{5})$	$\frac{25}{48}$
Tetrahedra	
$(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4})$	$-\frac{4}{5}$
$(\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{2})$	$\frac{9}{20}$
$(\frac{1}{6},\frac{1}{6},\frac{1}{2},\frac{1}{6})$	$\frac{9}{20}$
$(\frac{1}{6},\frac{1}{2},\frac{1}{6},\frac{1}{6})$	$\frac{9}{20}$
$(\frac{1}{2},\frac{1}{6},\frac{1}{6},\frac{1}{6})$	$\frac{9}{20}$

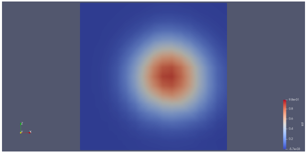
Time advancing

- explicit RK4,

- linearized implicit BDF1 with spatially first-order Jacobian.

Test case : advection of a Gaussian





Explicit RK4 time advancing, CFL=0.5

uniform Cartesian 3D mesh.

Test case : advection of a Gaussian O3

Vertices N_e	Elements N_v	Error(C)	Order(C)	Error(P)	Order(P)
729	3072	0.00954	-	0.013869	-
4913	24576	0.00237	2.0091	0.00356255	1.96088
35937	196608	0.00039	2.603341	0.00079064	2.17180
274625	1572864	5.54e-5	2.81551	0.00013761	2.52239
2146689	12582912	6.9331e-06	2.96158	2.00248e-05	2.78079

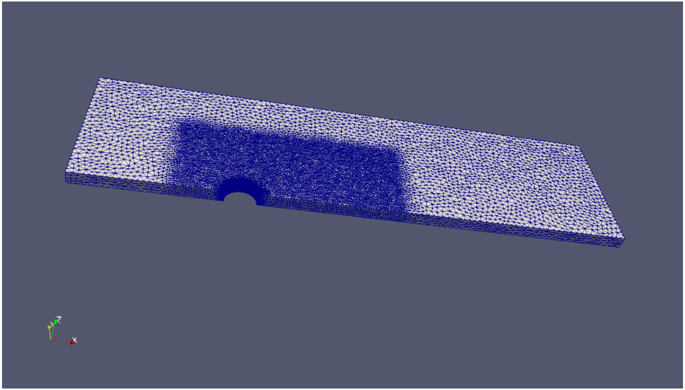
Table: **Propagation of a Gaussian distribution.** Error(C) and convergence order(C) correspond to the error on solution with macromolecules built with the centered algorithm. Error(P) and convergence order(P) correspond to the error on solution with macromolecules built with the partition algorithm.

Test case : advection of a Gaussian O4

Vertices N_e	Elements N_v	Error(C)	Order(C)
729	3072	0.00832135	-
4913	24576	0.00107831	2.94805
35937	196608	8.20648e-05	3.71586
274625	1572864	5.03349e-06	4.02713

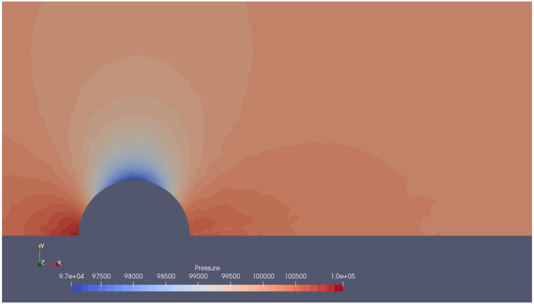
Table: **Propagation of a Gaussian distribution using the fourth-order cubic reconstruction, and a Donor Cell upwind solver $\gamma = 0.5$.** Error(C) and convergence order(C) correspond to the error on solution with macromolecules built with the centered algorithm.

Euler flow past a cylinder



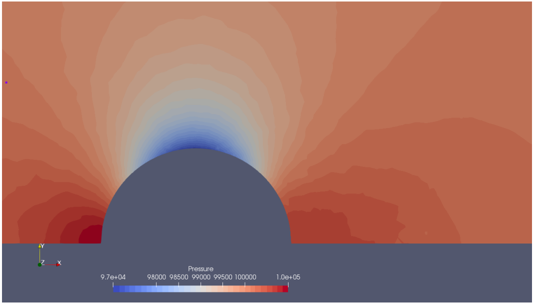
Mach number : 0.3
Implicit time advancing CFL=2
Bounding Box : X: (-5 , 12.5) ; Y : (0, 5) and Z: (0., 0.4)
Vertices: 52665, Tetrahedra : 260 383, Triangles : 36494

Euler flow past a cylinder



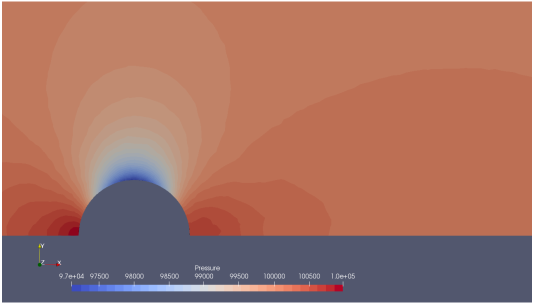
MUSCL

Euler flow past a cylinder



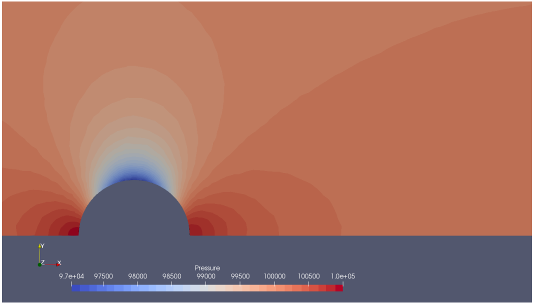
CENO O3 with Partition

Euler flow past a cylinder



CENO 3 central

Euler flow past a cylinder



CENO 4

Conclusions

- CENO4 is implemented in NiceFlow.
- The study restarts in october.
- Efforts for reducing complexity.
- Efforts for increasing flop rate.
- Computation of rotating machines.

Thank you for your attention!