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Space-time adaptation algorithms for time-advancing flows

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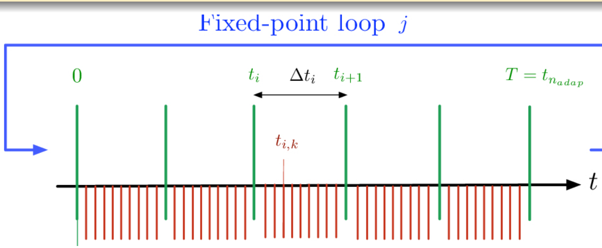
Motivations

Our motivation is the mesh adaptive simulation of unsteady flows, with a particular final target, LES and hybrid RANS/LES flows. We restrict to time advancing methods.

The Transient Fixed Point algorithm(*) was proposed for specifying automatically a *succession of* n_{adap} *meshes* over a decomposition in sub-intervals (in green) used for the transient process (timesteps in red).



(*)F. Alauzet, P.J. Frey, P.-L. George, and B. Mohammadi. 3D transient fixed point mesh adaptation for time-dependent problems: Application to CFD simulations. J. Comp. Phys.,222:592-623, 2007.

Motivations(2)

In the case of explicit time-advancing, the heuristics (with theoretical proofs for simple models) was to consider maximal Courant number =1 as a good principle for time adaptation (*)(**).

For our target set of flows, only *implicit time advancing* is affordable. Small timesteps are CPU costly. Large timesteps may loose the accuracy of the spatial resolution.

Our purpose is to define a space-time global error and optimize it simultaneously in terms of spatial meshes and timestep length.

(*) F. Alauzet, A. Loseille, G. Olivier, Time-accurate multi-scale anisotropic mesh adaptation for unsteady flows in CFD, Journal of Computational Physics 373 (2018) 28-63.

(**) A. Belme, A. Dervieux, F. Alauzet, Time accurate anisotropic goal-oriented mesh adaptation for unsteady flows, Journal of Computational Physics 231 (2012) 6323-6348.

Space-time metric

In space-time domain $\Omega \times [0,T[$, the Transient Fixed Point mesh-adaptation method relies on the notion of *space-time continuous mesh or space-time metric*, $\mathcal{M}_{st}$,

$$\mathcal{M}_{st} = ((t_i)_{i=0,n_{adap}}, \tau, \mathcal{M}) \qquad (t_0 = 0)$$

defined by:
(i) The *splitting* $(t_i)_i$ of $[0,T]$ into n_{adap} subintervals:

$$[0,T] = \bigcup_{i=1}^{n_{adap}} [t_{i-1}, t_i[.$$

(ii) A *continuous timestep length* $\tau : t \in]0,T[\mapsto \tau(t)$.

(iii)A time-dependant *spatial metric* $\mathcal{M}(t) = \mathcal{M}_i$ for $t \in [t_{i-1}, t_i[$, where $\mathcal{M}_i$ is defined as the field $(\mathcal{M}_i(\mathbf{x}), \mathbf{x} \in \Omega)$ with $\mathcal{M}_i(\mathbf{x})$ a positive definite symmetric 3×3 matrix.

Complexity of a space-time metric

The *complexity* $\mathscr{C}_{st}(\mathscr{M}_{st})$, or computational effort, of a space-time metric

$$\mathscr{M}_{st} = ((t_i)_{i=0,n_{adap}}, \tau, \mathscr{M})$$

is the sum of complexities $\mathscr{C}_i$ on each time sub-interval $[t_{i-1}, t_i[$, each $\mathscr{C}_i$ being evaluated as the product of the *spatial complexity*,

$$\mathscr{C}_{space}(\mathscr{M}_i) = \int_\Omega \sqrt{det(\mathscr{M}_i(x))}\mathrm{d}x$$

which is the continuous analog of the number of vertices of spatial discretization, by the *time complexity*, namely the number of timesteps, therefore:

$$\mathscr{C}_{st}(\mathscr{M}_{st}) = \sum_{i=1}^{i=n_{adap}} \mathscr{C}_{space}(\mathscr{M}_i) \int_{t_{i-1}}^{t_i} \tau(t)^{-1}\mathrm{d}t.$$

Error model

Given a metric $\mathscr{M}_{st} = ((t_i)_{i=0,n_{adap}}, \tau, \mathscr{M})$ and a unit mesh $((t_i)_i, \tau, \mathscr{H})$ of it. We can compute on $((t_i)_i, \tau, \mathscr{H})$ a discrete solution $W(\mathbf{x}, t)$ of the Navier-Stokes equations in $\Omega \times [0, T[$, with an *approximation error* $\mathscr{E}$ which we consider as a function of $\mathscr{M}_{st}$.

The *error model* can be based on a goal-oriented analysis, with functional and adjoint. Instead, for simplicity, we consider a L^p feature-based analysis with a *sensor* M (typically the Mach number).

$$\mathscr{E}(\mathscr{M}_{st}) = \mathscr{E}_{time}(\mathscr{M}_{st}) + \mathscr{E}_{space}(\mathscr{M}_{st})$$

$$\mathscr{E}_{time}(\mathscr{M}_{st}) = \int_0^T \int_{-\infty}^{+\infty} [\tau^2 |\frac{\partial^2 M}{\partial t^2}||]^p \mathrm{d}t\mathrm{d}x$$

$$\mathscr{E}_{space}(\mathscr{M}_{st}) = \sum_{i=1}^{n_{adap}} \int_{t_{i-1}}^{t_i} \mathscr{E}^i(t)\mathrm{d}t \quad \text{with}$$

$$\mathscr{E}^i(t) = \int_\Omega \left[\text{trace}\left((\mathscr{M}^i)^{-\frac{1}{2}}(\mathbf{x})\, \mathbf{H}_M(\mathbf{x}, t)\, (\mathscr{M}^i)^{-\frac{1}{2}}(\mathbf{x}) \right)\right]^p \mathrm{d}x$$

and $\mathbf{H}_M = |Hessian(M)|$.

Adaptation optimality

We solve the following optimal problem :

$$\begin{cases} \min\limits_{\mathscr{M}_{st}} \mathscr{E}(\mathscr{M}_{st}), \\ \mathscr{C}_{st}(\mathscr{M}_{st}) = N_{st}. \end{cases}$$

We prescribe:

(a) the time subintervals: n_{adap} and $(t_i)_{i=0,n_{adap}}$ and

(b) an integer N_{st} (prescribed space-time complexity).

There exists

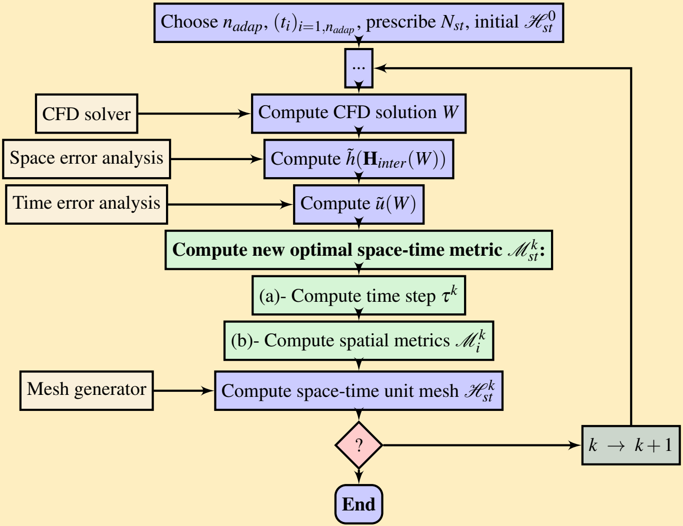
$$\mathscr{M}_{st}^{opt} = \left((t_i)_{i=0,n_{adap}}, \tau^{opt}, (\mathscr{M}_i^{opt})_i \right),$$

where τ^{opt} and $\mathscr{M}_i^{opt}$ can be expressed in terms of the error data:

$$\left|\frac{\partial^2 M}{\partial t^2}\right| \quad \textit{and} \quad \mathbf{H}_M,$$

which minimizes the error $\mathscr{E}(\mathscr{M}_{st})$ under the constraint $\mathscr{C}_{st}(\mathscr{M}_{st}) = N_{st}$.

Space-time Transient Fixed-Point Algorithm



First numerical experiment

2D computation of a flow around a cylinder at Reynolds number 3900, Mach number 0.1, with Spalart-Allmaras turbulence model.

Mesh adaptation options are :
- only one adapted spatial mesh, i.e. $n_{adap} = 1$
- Space-Time complexity N_{st} is prescribed to 2M and 32M.

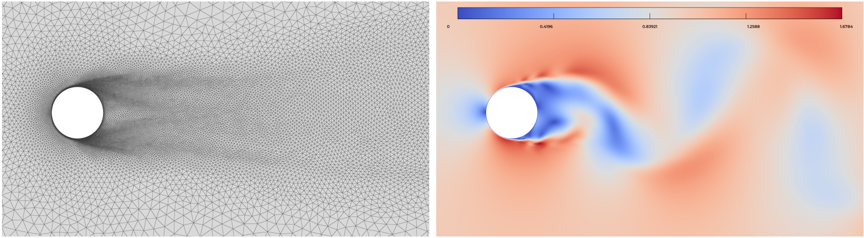


Figure: Flow past a circular cylinder at Re = 3900 : adapted mesh (left) and velocity field (right) in cross-section.

First numerical experiment(2)

N_{st}	k TFP	$\mathscr{C}_{space}(nodes)$	# timesteps	$\mathscr{E}_{space}$	$\mathscr{E}_{time}$	$\mathscr{E}$
–	0	(38K)	16K (CFL100)	–	–	–
$2M$	1	7.6K (10K)	261	$6.4\ 10^{-2}$	$1.2\ 10^{-4}$	$6.4\ 10^{-2}$
$2M$	10	21K (22K)	94	$2.9\ 10^{-2}$	$1.3\ 10^{-2}$	$4.2\ 10^{-2}$
$32M$	1	48K (61K)	658	$1.\ 10^{-2}$	$1.2\ 10^{-4}$	$1.\ 10^{-2}$
$32M$	10	78K (83K)	408	$8.3\ 10^{-3}$	$8.4\ 10^{-4}$	$9.1\ 10^{-3}$

- N_{st} : space-time complexity prescribed
- k : index of the current fixed point iteration
- $\mathscr{C}_{space}$: space complexity
- # timesteps : number of time steps
- $\mathscr{E}_{space}$: theoretical space error.
- $\mathscr{E}_{time}$: theoretical time error.

Second numerical experiment(2)

2D computation of a flow around a cylinder at Reynolds number 1M, Mach number 0.1, with Spalart-Allmaras turbulence model.

Mesh adaptation options are :
- only one adapted spatial mesh,
- Space-Time complexity is prescribed to 40M and 80M.

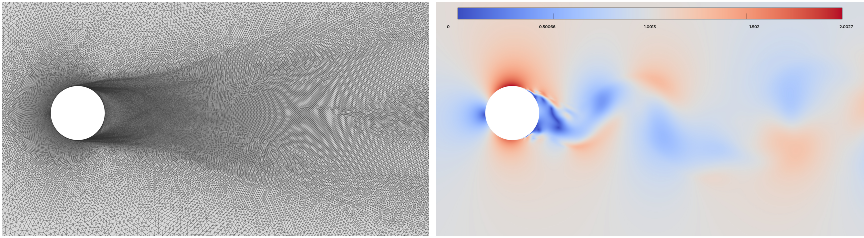


Figure: Flow past a circular cylinder at Re = 1M : adapted mesh (left) and velocity field (right) in cross-section.

Second numerical experiment(2)

N_{st}	k TFP	$\mathscr{C}_{space}(nodes)$	# timesteps	$\mathscr{E}_{space}$	$\mathscr{E}_{time}$	$\mathscr{E}$
–	0	(38K)	6K (FTS 5. 10^{-3})	–	–	–
$40M$	1	48K	829	8.1 10^{-3}	4. 10^{-5}	8.1 10^{-3}
$40M$	5	107K (112K)	374	5.1 10^{-3}	1.3 10^{-3}	6.4 10^{-3}
$80M$	1	76K	1045	5.1 10^{-3}	4. 10^{-5}	5.1 10^{-3}
$80M$	5	150K (156K)	533	3.6 10^{-3}	4.6 10^{-4}	4.1 10^{-3}

- N_{st} : space-time complexity prescribed
- k : index of the current fixed point iteration
- $\mathscr{C}_{space}$: space complexity
- # timesteps : number of time steps
- $\mathscr{E}_{space}$: theoretical space error.
- $\mathscr{E}_{time}$: theoretical time error.

Third numerical experiment

2D computation of a flow around a NACA0021 at Reynolds number 270K and an angle of attack of 60°, Mach number 0.1, with Spalart-Allmaras turbulence model.

Mesh adaptation options are :
- only one adapted spatial mesh,
- Space-Time complexity is prescribed to 2M and 32M.

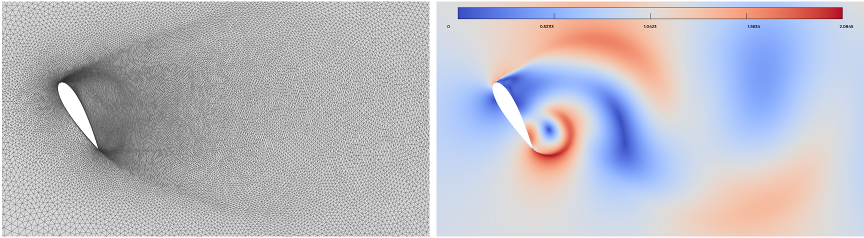


Figure: Flow past a Naca0021 at Re = 270K : adapted mesh (left) and velocity field (right) in cross-section.

Third numerical experiment(2)

N_{st}	k TFP	$\mathscr{C}_{space}(nodes)$	# timesteps	$\mathscr{E}_{space}$	$\mathscr{E}_{time}$
–	0	(18K)	44K (CFL50)	–	–
$2M$	1	1.8K (2.6K)	1132	$1.7\ 10^{-1}$	$2.4\ 10^{-6}$
$2M$	10	16K (18K)	123	$2.6\ 10^{-2}$	$1.3\ 10^{-2}$
$32M$	1	11K (14K)	2852	$2.7\ 10^{-2}$	$2.4\ 10^{-6}$
$32M$	10	57K (61K)	558	$7.5\ 10^{-3}$	$1.8\ 10^{-3}$

- N_{st} : space-time complexity prescribed
- k : index of the current fixed point iteration
- $\mathscr{C}_{space}$: space complexity
- # timesteps : number of time steps
- $\mathscr{E}_{space}$: theoretical space error.
- $\mathscr{E}_{time}$: theoretical time error.

Third numerical experiment(3)

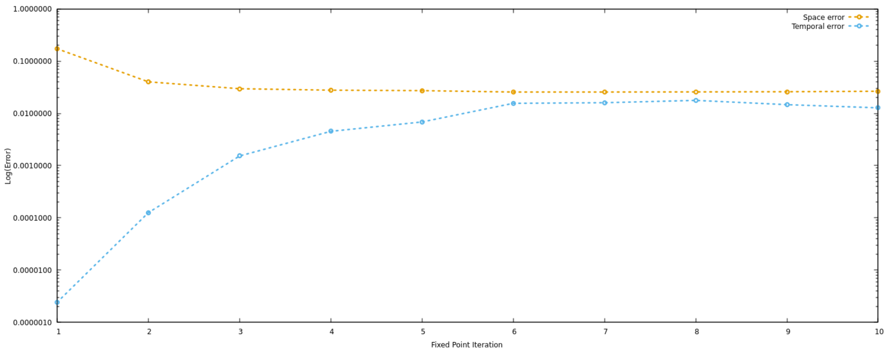


Figure: Evolution of theoretical space error $\mathscr{E}_{space}$ and time error $\mathscr{E}_{time}$ with TFP iterations.

Third numerical experiment(4)

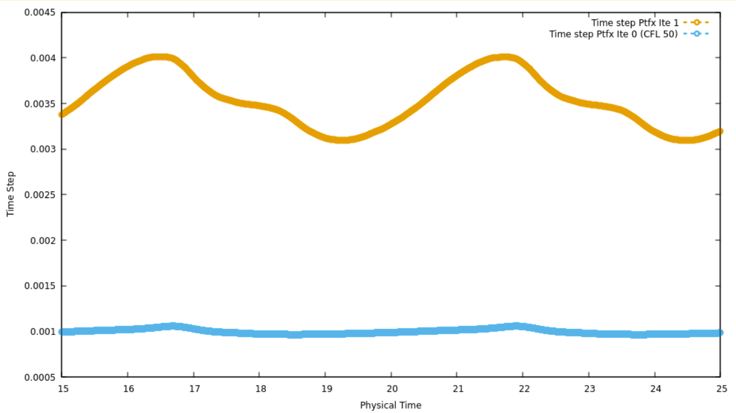


Figure: Blue: Timestep lengths of initial flow at CFL=50, 44K timesteps on 18K vertices, and orange: first timestep lengths proposed by the adaptation algorithm, 1132 timesteps on 1766 vertices.

Concluding remarks

We have proposed an extension of the Transient Fixed Point mesh adaptation algorithm to space-time adaptation for implicit time stepping.

Best time steps are obtained in a few TFP iterations, with CPU improvement.

Future work is the extension to Large Eddy Simulation.

Most details of implementation (except timestep length adaptation) are already available in (*).

(*) F. Alauzet, A. Loseille, G. Olivier, Time-accurate multi-scale anisotropic mesh adaptation for unsteady flows in CFD, Journal of Computational Physics 373 (2018) 28-63.

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Annexe

Space-time metric

In space-time domain $\Omega \times [0,T[$, the Transient Fixed Point mesh-adaptation method relies on the notion of *space-time continuous mesh or space-time metric*, $\mathscr{M}_{st}$,

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$$\mathscr{C}_{spatial}(\mathscr{M}_i) = \int_\Omega \sqrt{\det(\mathscr{M}_i(\mathbf{x}))} \mathrm{d}x.$$

Unit mesh of a space-time metric

A space-time metric $\mathscr{M}_{st} = ((t_i)_{i=0,n_{adap}}, \tau, \mathscr{M})$ allows to define (at least) one *unit space-time mesh* of the metric:

$$\mathscr{H}_{st} \ = \ ((t_i)_{i=0,n_{adap}}, \tau, \mathscr{H})$$

such that:

(i) For $t \in [t_{i-1}, t_i[$, we use $\mathscr{H}_i$, a spatial unit mesh for the metric $\mathscr{M}_i$, i.e. each edge is of length about 1 for metric $\mathscr{M}_i$.

(ii) Time integration is based on time levels $t_{i,k}$ such that:

$$\int_{t_{i,k-1}}^{t_{i,k}} (\tau(t))^{-1} dt = 1.$$

Complexity of a space-time metric

The *complexity* $\mathscr{C}_{st}(\mathscr{M}_{st})$, or computational effort, of a space-time metric

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