WALL MODELING FOR IMMERSED BOUNDARY METHOD

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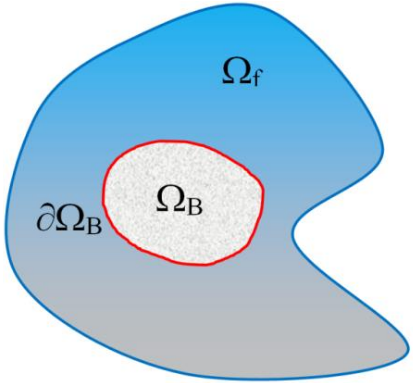
BRINKMAN PENALIZATION METHOD

$$\begin{cases} \dfrac{\partial \mathbf{Q}_\eta}{\partial t} + \nabla \cdot \mathbf{F}(\mathbf{Q}_\eta) + \dfrac{1}{\eta}\chi \begin{pmatrix} 0 \\ \rho_\eta(\mathbf{u}_\eta - \mathbf{u}_B) \\ \rho_\eta \mathbf{u}_\eta(\mathbf{u}_\eta - \mathbf{u}_B) + E_\eta - \rho\varepsilon_B \vee 0 \end{pmatrix} = \dfrac{1}{\mathrm{Re}} \nabla \cdot \mathbf{F}_v(\mathbf{Q}_\eta), \quad \mathbf{x} \in \Omega_f, t > 0 \\ \mathbf{Q}_\eta(\mathbf{x}, 0) = \mathbf{Q}_0(\mathbf{x}), \quad \mathbf{x} \in \Omega_f \end{cases}$$

$\eta << 1$ - penalty parameter

$\chi(\mathbf{x}) = \begin{cases} 1, & \mathbf{x} \in \bar{\Omega}_b \\ 0, & \mathbf{x} \in \Omega_f \end{cases}$ - mask function

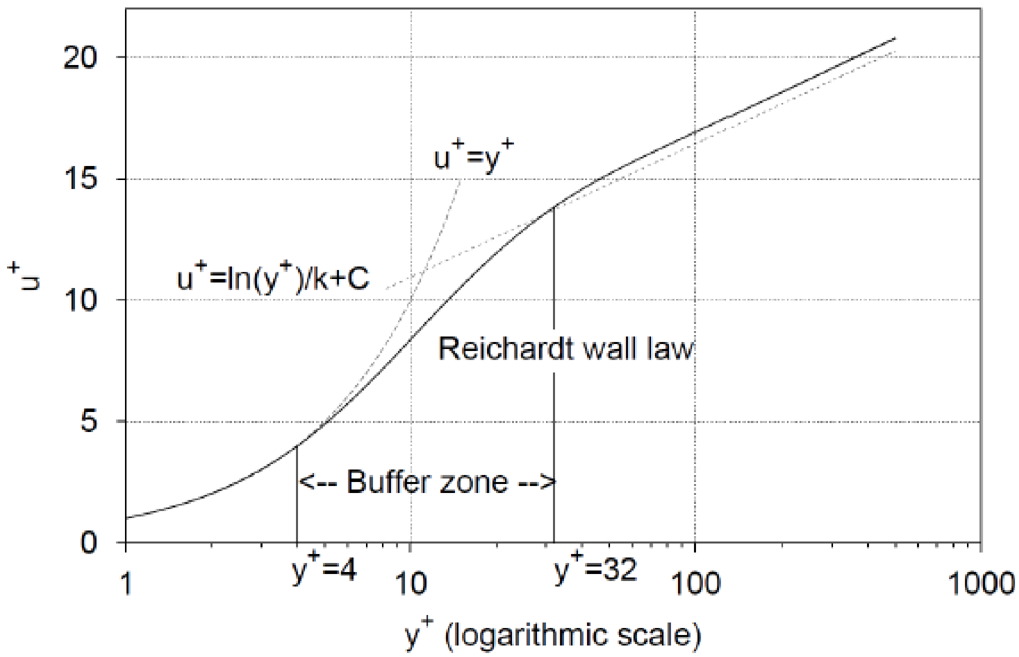


Boiron O., et al. (Comp.&Fluids, 2009)
Abgrall R., et al. (JCP, 2014)

WALL FUNCTION FOR HIGH REYNOLDS NUMBER FLOW (1)

Wall functions: motivated by the universal nature of the flat plate boundary layer

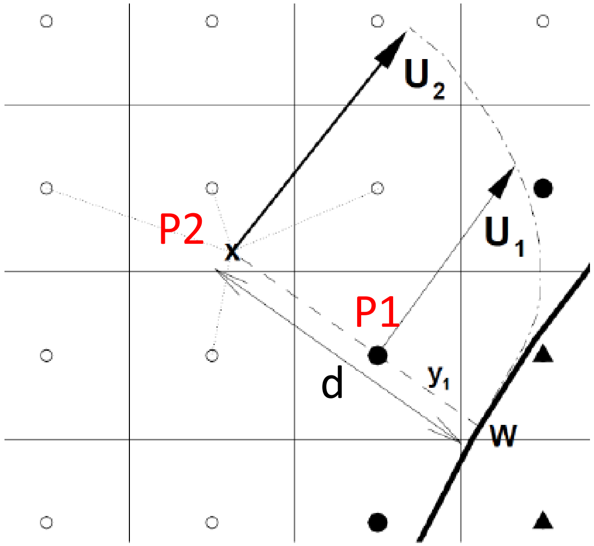
Reichardt's wall law



$$u^+ = f_w(y^+) = \frac{1}{k}\log(1+ky^+) + 7.8\left(1 - e^{\frac{y^+}{11}} - \frac{y^+}{11}e^{-0.33y^+}\right)$$

WALL FUNCTION FOR HIGH REYNOLDS NUMBER FLOW (2)

Implementation of the wall law using Brinkman penalization



● - interface point

∘ - fluid point

▲ - body point

The wall function adjusts the velocity field after penalization step.

- Consider interface point P1.
- Point P2 is found, along the normal-to-the-wall direction, at distance *d*, for instance, equal to *twice the largest distance from the wall to the interface points*.
- Velocity at P2 (U_2) is found by interpolation on the surrounding points.
- Using U_2 and wall law, the friction velocity is calculated.
- Using the friction velocity and wall law, the tangential velocity at P1 is calculated.

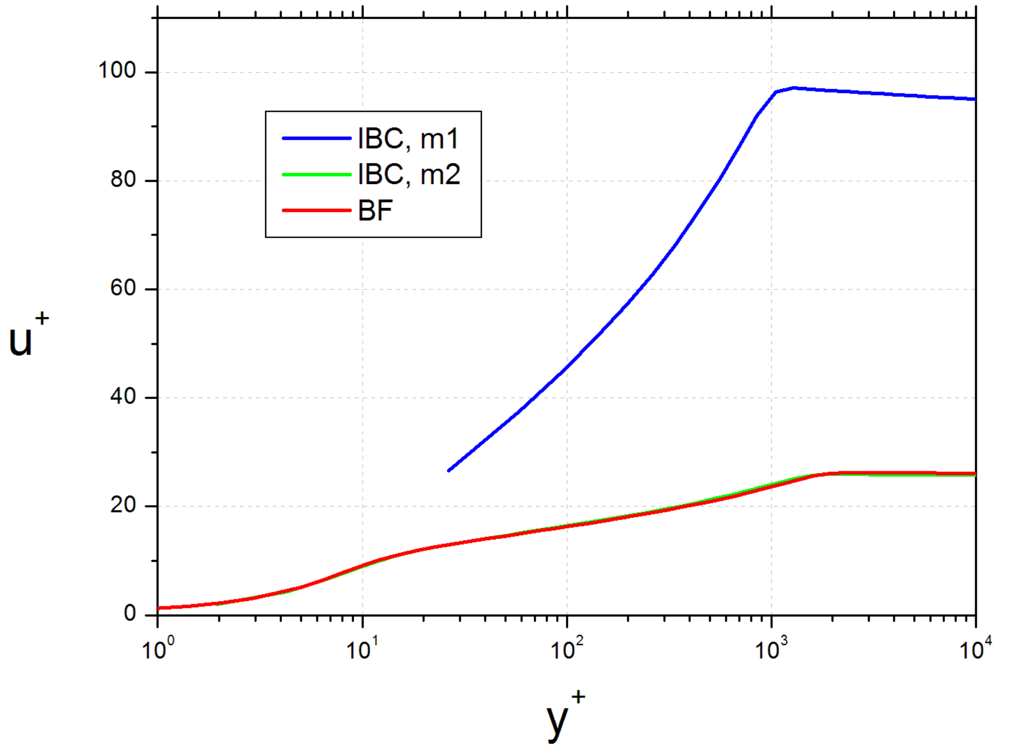
M.D.Tullio. PhD thesis. Politecnico di Bari. Italy. 2006.

FLAT PLATE (1)

$$Re = 1e + 06$$

$$M = 0.2$$

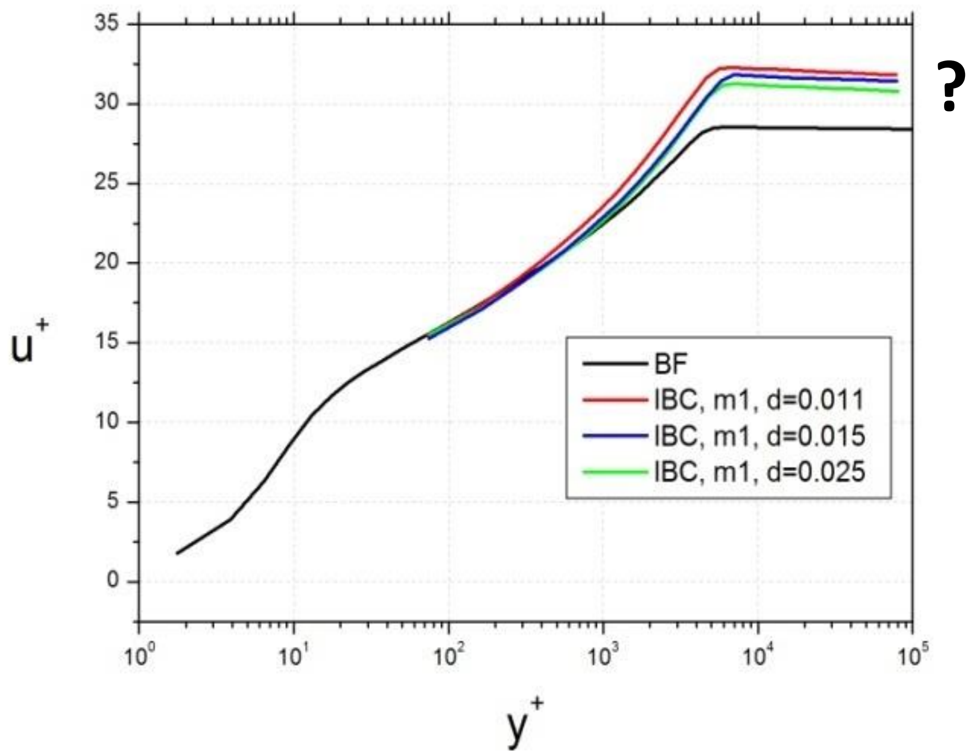
No-slip boundary condition



IBC, m1 – immersed boundary method using coarse mesh **m1** with near-wall dy = 1e-02
IBC, m2 - immersed boundary method using fine mesh **m2** with near-wall dy = 1e-05
BF – body-fitted approach using mesh with near-wall dy = 1e-05

FLAT PLATE (2)

No-slip boundary condition vs Wall-function (WF)



IBC, m1 – immersed boundary method with **WF** using coarse mesh m1 with near-wall dy = 1e-02

BF – body-fitted approach with **No-Slip** using mesh with near-wall dy = 1e-05

d – distance from the wall to interpolation point P2

Work in progress

- Investigate the robustness and efficiency issues in more detail.

- Review and implementation alternative wall modelling approaches applicable with penalty immersed boundary method.

- To use the Characteristic-Based Volume Penalization method instead of Brinkman Penalization? The mathematical ground can be provided.