

Artificial viscosity in WENO-EBR

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Consider Euler equations. Schemes with complete Riemann solvers suffer from shock-wave instabilities which lead to additional oscillations (that are not damped by limiters) or carbuncle phenomenon. To overcome this, additional viscosity is needed. Two main ways to induce it:

- artificial viscosity via additional term in governing equations;
- local switching to incomplete Riemann solvers.

Exotic way:

- Balsara's multidimensional Riemann solvers.

Artificial viscosity as local increase of the viscosity and heat conductivity coefficients *.

$$\mu_{AV} = \begin{cases} C_{AV} \rho h^2 \sqrt{(\nabla \cdot \mathbf{u})^2 - (C_{th} a/h)^2}, & -(\nabla \cdot \mathbf{u}) > C_{th} a/h; \\ 0, & \textit{otherwise.} \end{cases}$$

Recommended values: $C_{AV} = 0.5$, $C_{th} = 0.05$.

*Rodionov A. V. Artificial viscosity to cure the shock instability in high-order Godunov-type schemes. *Computers and Fluids*. 190 (2019) 77–97

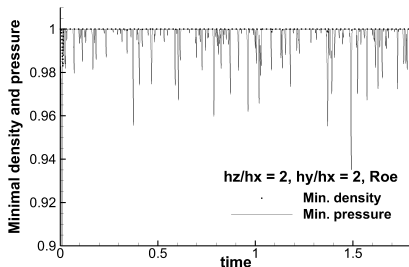
P1-Galerkin approximation of Laplacian is a negative definite operator. On meshes of acute-angled triangles, it is monotone. If there are obtuse-angled triangles, the method loses monotonicity.

- In theory, this may lead to pits in front of a shock.
- Numerical results show that it's not a big problem.

Simplified viscosity?!

In most of the computations we provide, we use a simplified viscosity approximation[†]. This method is quicker.

Simplified viscosity is unusable for the artificial viscosity.



[†]A.V. Gorobets, P.A. Bakhvalov, A.P. Duben, P.V. Rodionov. Acceleration of NOISEtte Code for Scale-resolving Supercomputer Simulations of Turbulent Flows // Lobachevskii Journal of Mathematics, 2020, Vol. 41, No. 8, pp. 1463–1474

Rodionov's function can be used as a sensor for switching to LF solver.

- If $-(\nabla \cdot \mathbf{u}) > C_{th}a/h$, use LF solver.
- Otherwise use Roe solver.

This is enough to cure the carbuncle phenomenon, but the results are worse than the ones obtained with the artificial viscosity. Continuous switching between Roe and LF solvers does not improve the results.

Laplace operators at the right-hand side:

$$\frac{\partial(\rho u_j)}{\partial t} + \dots = \nabla \cdot (\mu \nabla u_j);$$

$$\frac{\partial E}{\partial t} + \dots = \nabla \cdot (\mu \nabla h),$$

where E is the full energy and h is the specific entalpy.
Variant: use *full* specific entalpy in the last formula.

Non-linear finite volume method[‡] can be used to approximate Laplace operator. It's designed for cell-centered schemes but applicable as well to vertex-centered schemes. For the approximation of the term $\nabla \cdot (\mu \nabla f)$, it's simpler than the P1-Galerkin method.

[‡]Lipnikov K., Svyatskiy D., Vassilevski Yu. Minimal stencil finite volume scheme with the discrete maximum principle. Russ. J. Numer. Anal. Math. Modelling, Vol. 27, No. 4, pp. 369–385 (2012)