Artificial viscosity in WENO-EBR

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Consider Euler equations. Schemes with complete Riemann solvers suffer from shock-wave instabilities which lead to additional oscillations (that are not damped by limiters) or carbuncle phenomenon. To overcome this, additional viscosity is needed. Two main ways to induce it:

- artificial viscosity via additional term in governing equations;
- local switching to incomplete Riemann solvers.

Exotic way:

- Balsara’s multidimensional Riemann solvers.
Rodionov’s approach

Artificial viscosity as local increase of the viscosity and heat conductivity coefficients *.

\[ \mu_{AV} = \begin{cases} C_{AV} \rho h^2 \sqrt{(\nabla \cdot u)^2 - (C_{th} a/h)^2}, & -(\nabla \cdot u) > C_{th} a/h; \\ 0, & \text{otherwise.} \end{cases} \]

Recommended values: \( C_{AV} = 0.5, \ C_{th} = 0.05. \)

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Shortcoming: monotonicity

P1-Galerkin approximation of Laplacian is a negative definite operator. On meshes of acute-angled triangles, it is monotone. If the are obtuse-angled triangles, the method loses monotonicity.

- In theory, this may lead to pits in front of a shock.
- Numerical results show that it’s not a big problem.
In most of the computations we provide, we use a simplified viscosity approximation. This method is quicker. Simplified viscosity is unusable for the artificial viscosity.

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Rodionov’s function can be used as a sensor for switching to LF solver.

- If $-(\nabla \cdot u) > C_{th}a/h$, use LF solver.
- Otherwise use Roe solver.

This is enough to cure the carbuncle phenomenon, but the results are worse than the ones obtained with the artificial viscosity. Continuous switching between Roe and LF solvers does not improve the results.
Rodionov’s approach – new

Laplace operators at the right-hand side:

\[
\frac{\partial (\rho u_j)}{\partial t} + \ldots = \nabla \cdot (\mu \nabla u_j);
\]

\[
\frac{\partial E}{\partial t} + \ldots = \nabla \cdot (\mu \nabla h),
\]

where \(E\) is the full energy and \(h\) is the specific entalpy.

Variant: use full specific entalpy in the last formula.
Non-linear finite volume method\textsuperscript{‡} can be used to approximate Laplace operator. It’s designed for cell-centered schemes but applicable as well to vertex-centered schemes. For the approximation of the term $\nabla \cdot (\mu \nabla f)$, it’s simpler than the P1-Galerkin method.