High-order scheme for rotating machines
Norma Meeting

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Inria
Research topics

- Study of numerical high-order scheme
- Rotating machine with mesh adaptation
Finite volume formulation

Advection equation:

\[
\frac{\partial u}{\partial t}(x, y, t) + \nabla \cdot f(u(x, y, t)) = 0
\]

We can integrate over \( C_i \) and use the Green formula:

\[
\frac{d}{dt} \int_{C_i} u(x, y, t) \, dx \, dt + \int_{\partial C_i} f(u(x, y, t)) \cdot \mathbf{n} \, ds = 0
\]

Figure – Cell construction: from triangular mesh (a), from tetrahedral mesh (b)
Main Property

In order to evaluate flux, we construct a quadratic polynomial $P^n_i$ on every cells $C_i$. It’s necessary that the mean of $P^n_i$ and $u^n$ are equal on cell $i$. This condition writes $\bar{P}^i,n = \bar{u}^i,n$, with:

\[
\begin{align*}
\bar{P}^i,n &= \frac{1}{\text{aire}(C_i)} \int_{C_i} P^n_i(x, y) dxdy \\
\bar{u}^i,n &= \frac{1}{\text{aire}(C_i)} \int_{C_i} u^n(x, y) dxdy
\end{align*}
\]

Before evaluating each coefficient of the polynomial function, we need to introduce the molecular partition of our mesh.
Least square approximation

For a given cell $i$ we note $M$ the molecule such that $C_i \subset M$. Then the polynomial approximation $P_i^n$ is defined as:

$$\text{argmin} \sum_{C_k \neq i \subset M} \left( \bar{P}^{k,n}_i - \bar{u}^{k,n} \right)^2$$
We implement our algorithm by the following steps:

1. Mesh Data
2. Molecular Partition
3. Polynomial Approximation
4. Flux evaluation
5. Time loop
The initial CENO scheme requires a molecule construction for every cell: the number of molecules is equal to the number of cells. The new version will be more efficient.

At the end of the study we shall consider the $h, p$ mesh adaptation. This method varies not only the local mesh size $h$, but also the degree $p$ of polynomials reconstruction.
We focus on a mesh method that can be used for rotating machines study.

**Figure** – Suitable mesh construction for rotating machines. On the left: top view of the mesh geometry, center area represents the spinning part. On the right: mesh inner surfaces.
The main difficulty is to evaluate the solution at the border between both areas.

Figure – Mesh configuration.

As the mesh can be non-conforming, we use an interpolation technique to evaluate values from one mesh to another.
Example of calculation

Figure – Solution and mesh for a mixer. On the left the density. Right, the fluid velocity.
Currently done.

1. Specification of new CENO.
2. Niceflow training with rotating machine.

Next.

1. Ceno 3D coding.
2. New geometry.