High-order scheme for rotating machines

Norma Meeting

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Research topics

- Study of numerical high-order scheme
- Rotating machine with mesh adaptation

Finite volume formulation

Advection equation :

$$\frac{\partial u}{\partial t}(x, y, t) + \nabla \cdot \mathbf{f}(u(x, y, t)) = 0$$

We can integrate over C_i and use the Green formula :

$$\frac{d}{dt} \int_{C_i} u(x, y, t) dxdt + \int_{\partial C_i} \mathbf{f}(u(x, y, t)) \cdot \mathbf{n} ds = 0$$

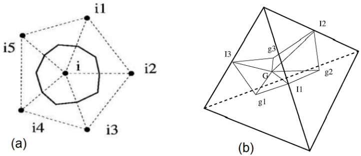


Figure – Cell construction : from triangular mesh (a), from tetrahedral mesh (b)

Polynomial approximation

Main Property

In order to evaluate flux, we construct a quadratic polynomial P_i^n on every cells C_i.
It's necessary that the mean of P_i^n and u^n are equal on cell i.
This condition writes $\bar{P}_i^{i,n} = \bar{u}^{i,n}$, with :

$$\begin{cases} \bar{P}_i^{i,n} = \dfrac{1}{aire\,(C_i)} \displaystyle\int_{C_i} P_i^n(x,y)dxdy \\ \bar{u}^{i,n} = \dfrac{1}{aire\,(C_i)} \displaystyle\int_{C_i} u^n(x,y)dxdy \end{cases}$$

Before evaluating each coefficient of the polynomial function, we need to introduce the molecular partition of our mesh.

Moleculecular Partition

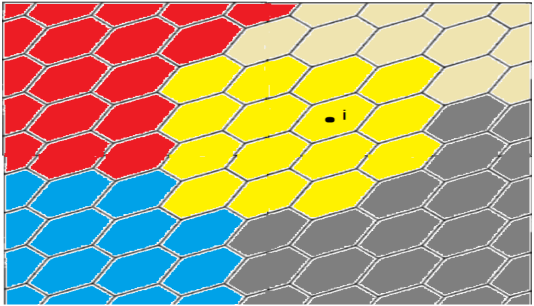


Figure – Molecular partition : each color area represent a molecule

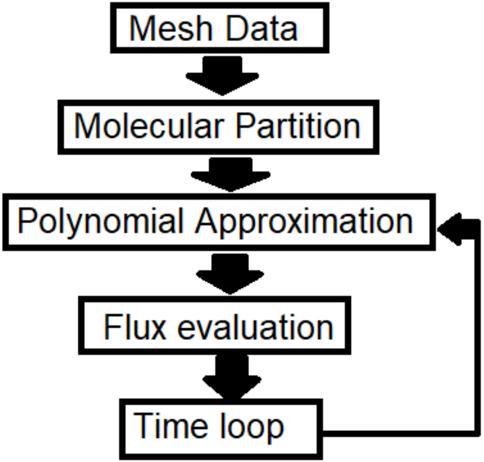
Least square approximation

For a given cell i we note M the molecule such that $C_i \subset M$.
Then the polynomial aproximation P_i^n is defined as :

$$argmin \sum_{C_{k \neq i} \subset M} \left(\bar{P}_i^{k,n} - \bar{u}^{k,n} \right)^2$$

Algorithm structure

We implement our algorithm by the following steps



The initial CENO scheme requires a molecule construction for every cells : the number of molecules is equal to the number of cells. The new version will be more efficient.

At the end of the study we shall consider the h, p mesh adaptation. This method varies not only the local mesh size h, but also the degree p of polynomials reconstruction.

Chimera

We focus on a mesh method that can be used for rotating machines study.

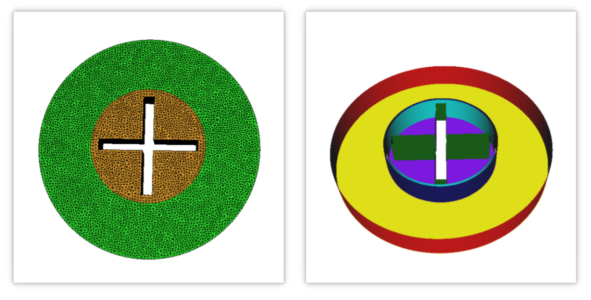


Figure – Suitable mesh construction for rotating machines. On the left : top view of the mesh geometrie, center area represents the spinning part. On the right : mesh inner surfaces.

The main difficulty is to evaluate the solution at the border between both areas.

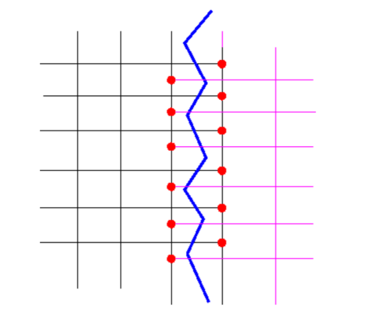


Figure – Mesh configuration.

As the mesh can be non-conforming, we use an interpolation technique to evaluate values from one mesh to another.

Example of calculation

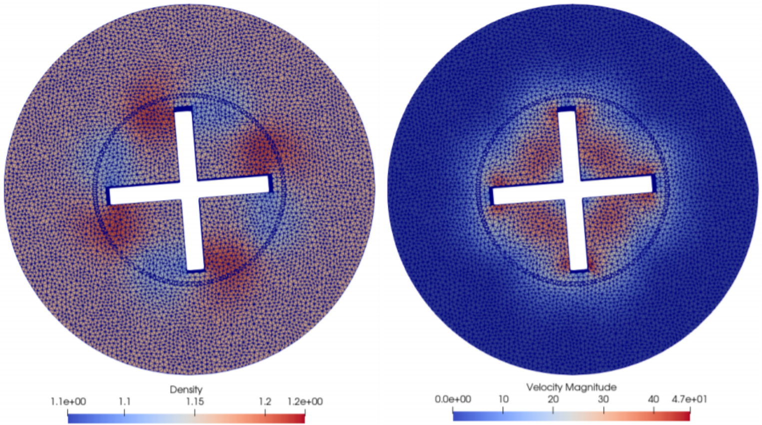


Figure – Solution and mesh for a mixer. On the left the density. Right, the fluid velocity.

Conclusion

- Currently done.

1. Specification of new CENO.
2. Niceflow training with rotating machine.

- Next.

1. Ceno 3D coding.
2. New geometry.