Fiche 03 — Formulaire de trigonométrie

$$\cos^2(x)+\sin^2(x)=1$$

Parité

$$\cos(-x)=\cos(x)$$
$$\sin(-x)=-\sin(x)$$
$$\tan(-x)=-\tan(x)$$

Périodicité

$$\cos(x+2\pi)=\cos(x)$$
$$\sin(x+2\pi)=\sin(x)$$
$$\tan(x+\pi)=\tan(x)$$

$x \leftarrow \pi + x$

$$\cos(\pi+x)=-\cos(x)$$
$$\sin(\pi+x)=-\sin(x)$$
$$\tan(\pi+x)=\tan(x)$$

$x \leftarrow \pi - x$

$$\cos(\pi-x)=-\cos(x)$$
$$\sin(\pi-x)=\sin(x)$$
$$\tan(\pi-x)=-\tan(x)$$

$x \leftarrow \frac{\pi}{2} - x$

$$\cos\left(\frac{\pi}{2}-x\right)=\sin(x)$$
$$\sin\left(\frac{\pi}{2}-x\right)=\cos(x)$$
$$\tan\left(\frac{\pi}{2}-x\right)=\frac{1}{\tan(x)}=\cot\text{an}(x)$$

$x \leftarrow \frac{\pi}{2} + x$

$$\cos\left(\frac{\pi}{2}+x\right)=-\sin(x)$$
$$\sin\left(\frac{\pi}{2}+x\right)=\cos(x)$$
$$\tan\left(\frac{\pi}{2}+x\right)=-\text{co}\tan(x)$$

$x \leftarrow a + b$

$$\cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$$
$$\sin(a+b)=\sin(a)\cos(b)+\cos(a)\sin(b)$$
$$\tan(a+b)=\frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}$$

$x \leftarrow a - b$

$$\cos(a-b)=\cos(a)\cos(b)+\sin(a)\sin(b)$$
$$\sin(a-b)=\sin(a)\cos(b)-\cos(a)\sin(b)$$
$$\tan(a-b)=\frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}$$

produit → somme

$$\cos(a)\cos(b)=\frac{1}{2}\left[\cos(a+b)+\cos(a-b)\right]$$
$$\sin(a)\sin(b)=\frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right]$$
$$\sin(a)\cos(b)=\frac{1}{2}\left[\sin(a+b)+\sin(a-b)\right]$$

somme → produit

$$\cos(p)+\cos(q)=2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$
$$\cos(p)-\cos(q)=-2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$
$$\sin(p)+\sin(q)=2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$
$$\sin(p)-\sin(q)=2\sin\left(\frac{p-q}{2}\right)\cos\left(\frac{p+q}{2}\right)$$

$a \leftrightarrow 2a$

$$\cos(2a)=\cos^2(a)-\sin^2(a)$$
$$=2\cos^2(a)-1=1-2\sin^2(a)$$
$$\sin(2a)=2\sin(a)\cos(a)$$
$$\tan(2a)=\frac{2\tan(a)}{1-\tan^2 a}$$

$$\cos^2 a=\frac{1+\cos(2a)}{2} \qquad \sin^2(a)=\frac{1-\cos(2a)}{2}$$

$$1+\cos(a)=2\cos^2\frac{a}{2} \qquad 1-\cos(a)=2\sin^2\frac{a}{2}$$

Formules d'Euler

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2} \qquad \sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

Formule de Moivre

$$(\cos(\theta) + i\sin(\theta))^n = \cos(n\theta) + i\sin(n\theta)$$

Dérivées

$$\sin' = \cos$$

$$\cos' = -\sin$$

$$\tan' = \frac{1}{\cos^2} = 1 + \tan^2$$

$$\cot\text{an}' = -\frac{1}{\sin^2} = -1 - \cot\text{an}^2$$

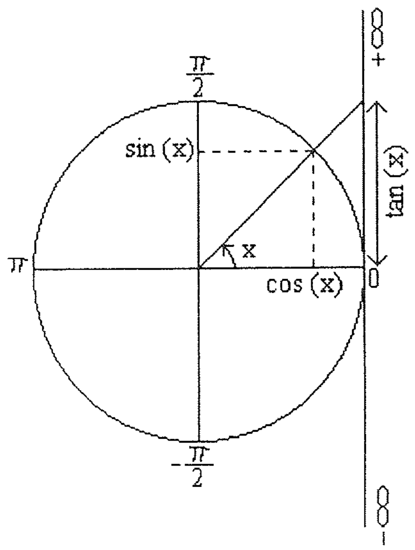
Changement d'inconnue

Soit $\theta \in]-\pi;\pi[$: on pose $t = \tan\left(\frac{\theta}{2}\right)$ et alors :

$\sin(\theta) = \frac{2t}{1+t^2}$	$\cos(\theta) = \frac{1-t^2}{1+t^2}$	$\tan(\theta) = \frac{2t}{1-t^2}$

Angles remarquables

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	



Fiche 04 — Les fonctions trigonométriques réciproques

<table>
<tr><th>La fonction Arcsin</th><th>La fonction Arccos</th><th>La fonction Arctan</th></tr>
<tr>
<td>Arcsin : $[-1,1] \to \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$
• $\forall x \in [-1,1]$, $\forall \theta \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$
$\theta = \text{Arcsin}\, x \Leftrightarrow x = \sin\theta$
• $\forall x \in [-1,1]$:
$x = \sin(\theta) \Leftrightarrow \begin{cases} \theta = \text{Arcsin}(x) + 2k\pi \\ ou \\ \theta = \pi - \text{Arcsin}(x) + 2k\pi \end{cases}$
• Dérivée : $\forall x \in \left]-1,1\right[$ $\text{Arcsin}'(x) = \frac{1}{\sqrt{1-x^2}}$</td>
<td>Arccos : $[-1,1] \to [0,\pi]$
• $\forall x \in [-1,1]$, $\forall \theta \in [0,\pi]$
$\theta = \text{Arccos}\, x \Leftrightarrow x = \cos\theta$
• $\forall x \in [-1,1]$:
$x = \cos(\theta) \Leftrightarrow \begin{cases} \theta = \text{Arccos}(x) + 2k\pi \\ ou \\ \theta = -\text{Arccos}(x) + 2k\pi \end{cases}$
• Dérivée : $\forall x \in \left]-1,1\right[$ $\text{Arccos}'(x) = \frac{-1}{\sqrt{1-x^2}}$</td>
<td>Arctan : $\mathbb{R} \to \left]-\frac{\pi}{2};\frac{\pi}{2}\right[$
• $\forall x \in \mathbb{R}$, $\forall \theta \in \left]-\frac{\pi}{2};\frac{\pi}{2}\right[$
$\theta = \text{Arctan}\, x \Leftrightarrow x = \tan\theta$
• $\forall x \in \mathbb{R}$:
$x = \tan(\theta) \Leftrightarrow \theta = \text{Arctan}(x) + k\pi$
• Dérivée : : $\forall x \in \mathbb{R}$ $\text{Arctan}'(x) = \frac{1}{1+x^2}$</td>
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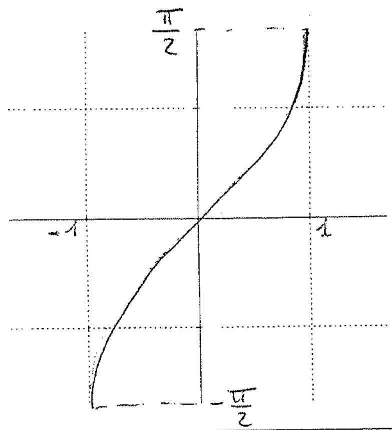
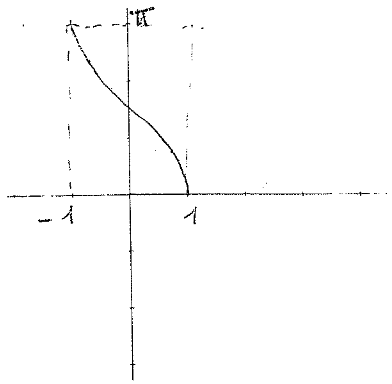
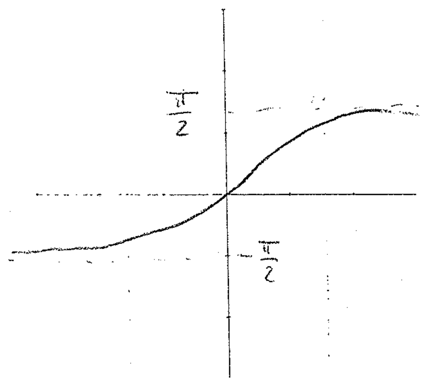
<td></td>
<td></td>
<td></td>
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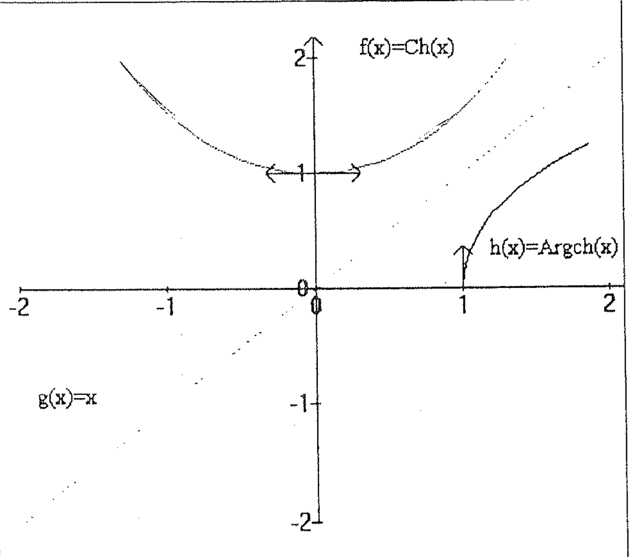
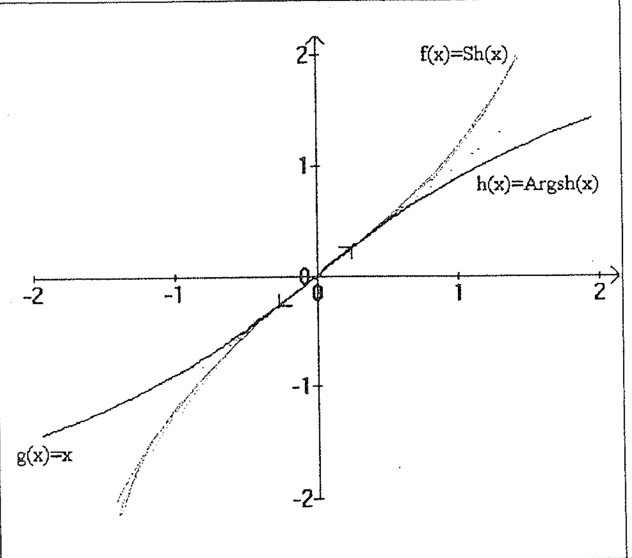
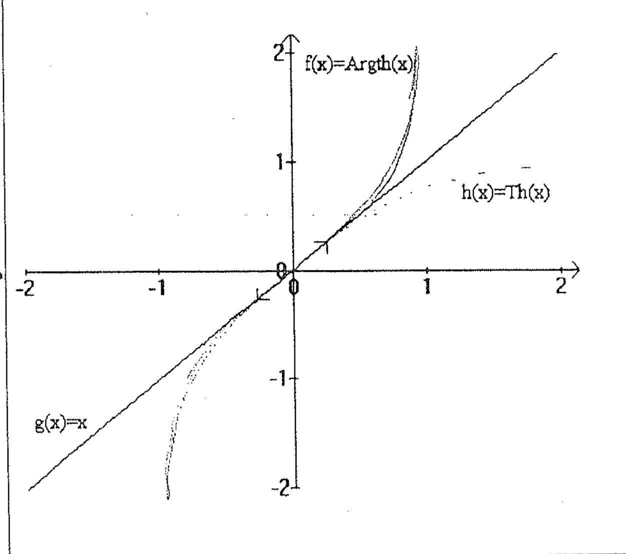
Fiche 08 — Trigonométrie hyperbolique

ch et Argch	sh et Argsh	th et Argth
		
$ch = \frac{e^x + e^{-x}}{2}$	$sh = \frac{e^x - e^{-x}}{2}$	$th = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
$ch' = sh$	$sh' = ch$	$th' = \frac{1}{ch^2} = 1 - th^2$
$Argch = \ln\left(x + \sqrt{x^2 - 1}\right)$	$Argsh = \ln\left(x + \sqrt{x^2 + 1}\right)$	$Argth = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$
$Argch' = \frac{1}{\sqrt{x^2 - 1}}$	$Argsh' = \frac{1}{\sqrt{1 + x^2}}$	$Argth' = \frac{1}{1 - x^2} = Argcoth'$