Hoel Queffelec

Surface skein algebras, categorification and positivity

joint work with Kevin Walker and Paul Wedrich

Goal:

Definition:

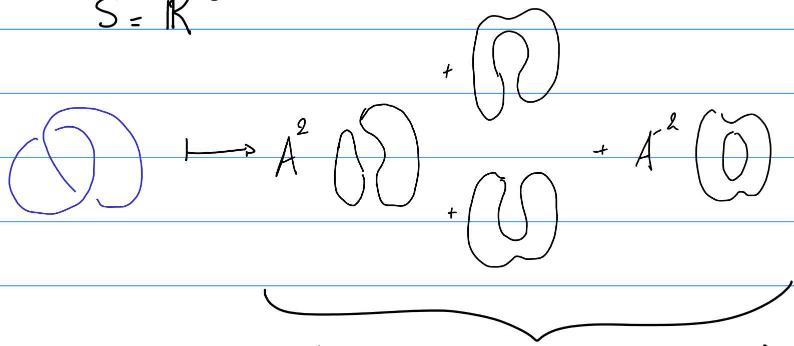
$$Sk(S) := \frac{\mathbb{Z}[A^{\pm 1}]\langle \text{links in } S \times \mathbb{I} \sim \text{diagrams on } S / \text{isotopy} \rangle}{\langle \times = A\,)(+ A^{-1} \asymp \,,\ \bigcirc = -A^{2} - A^{-2} \rangle}$$

||

Why should one care about Sk(S) ?

Example:

$S = \mathbb{R}^2$

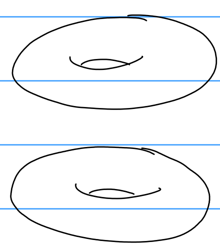


$$\left(A^2(-A^2-A^{-2})^2 + 2(-A^2-A^{-2}) + A^{-2}(-A^2-A^{-2})^2\right)\varnothing$$
$$= \left(A^6 + 2A^4 + \cancel{A^2} - \cancel{2A^2} - \cancel{2}A^{-2} + \cancel{A^2} + 2 + \cancel{A^{-2}}\right)\varnothing$$
$$= \left(A^6 + 2A^4 + 2 - A^{-2}\right)\varnothing$$

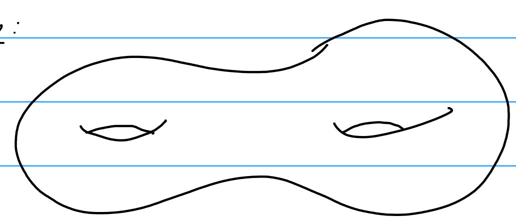
Basis:

Example:

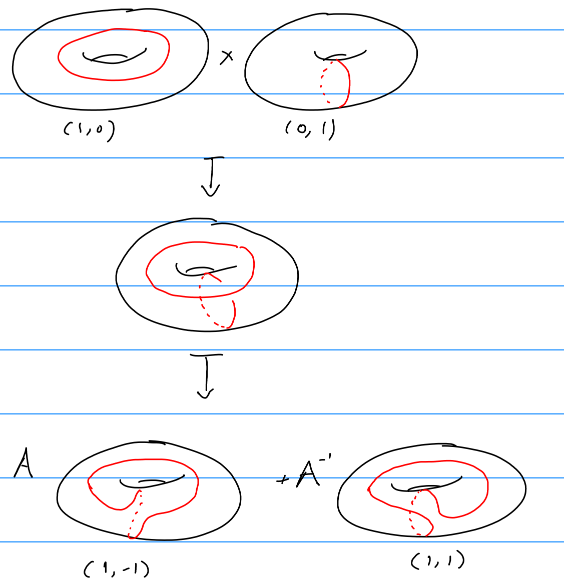
Torus:

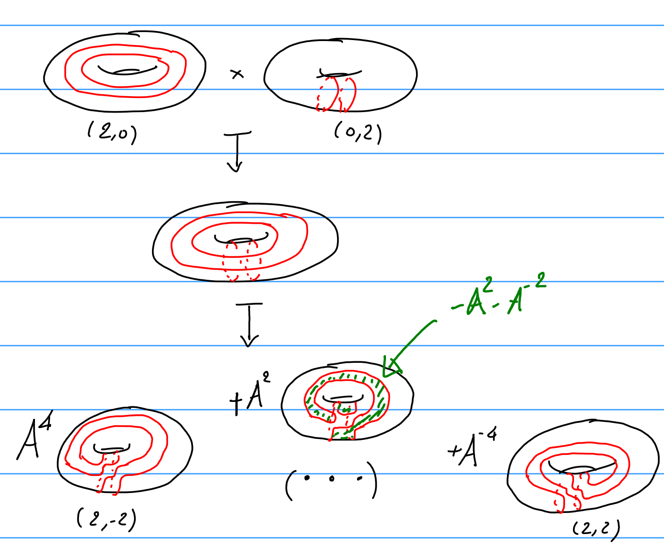


Higher genus:



Algebra structure:





Theorem [Frohman-Gelca 00]

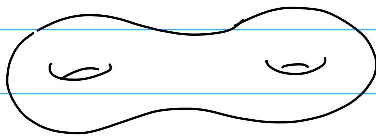
For $S = \mathbb{T}^2$, the Chebyshev basis $\{\emptyset\} \amalg \{T_k(m,n),\ m \wedge n = 1,\ k > 0\}$ is positive, with explicit formulas:

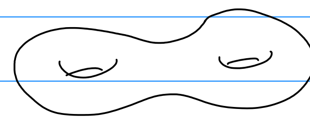
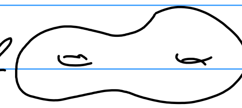
$$(p,q)_T \times (r,s)_T = A^{\begin{vmatrix} p & r \\ q & s \end{vmatrix}} (p-r,\, q-s)_T + A^{-\begin{vmatrix} p & r \\ q & s \end{vmatrix}} (p+r,\, q+s)_T$$

Chebyshev basis

For general S :

$\longleftrightarrow$



 -2 

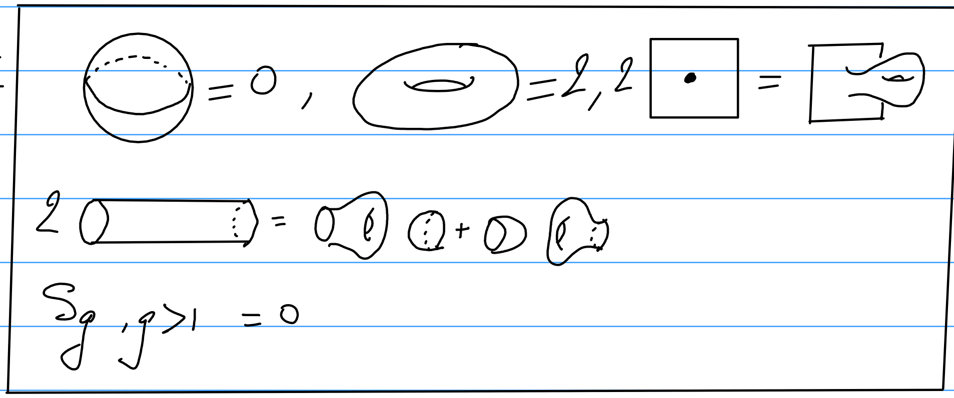
Conjecture: [Fock–Goncharov, Thurston, Lê]

Idea:

Categorification of $Sk(S)$:

- objects:
- morphisms:

Relations:



Grading:

Theorem: [Khovanov, Bar-Natan]: $K_0(BN(S)) \simeq Sk(S)$

$[\bigcirc] = A^2 + A^{-2} \quad \rightsquigarrow \quad [\bigcirc \simeq \emptyset\{1\} \oplus \emptyset\{-1\}]$

Trouble:

Theorem: [QW18]

$\parallel$

Important because:

Defining the grading goes in two steps:

①

② Lemma: $S \neq S^2$.

A connected surface properly embedded in $S \times [0,1]$ is:

-
-
-

↳

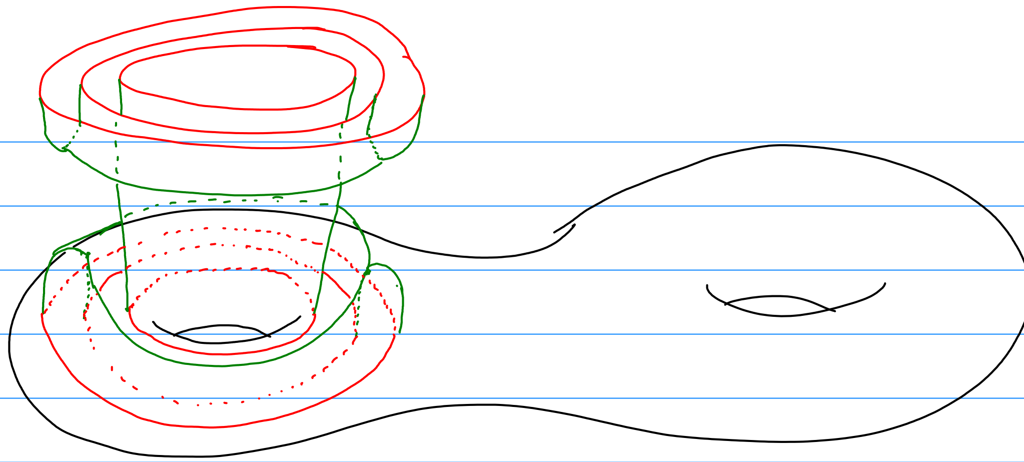
↳

Theorem(ish) [QW18]

Remark:

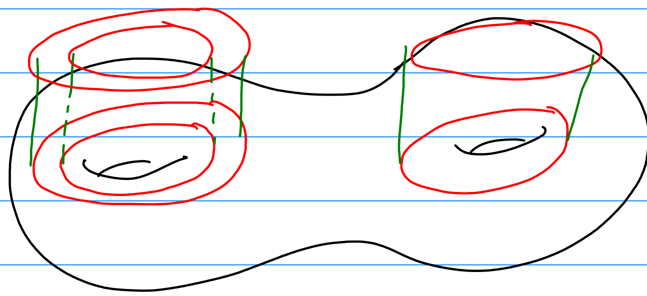
Idea:

Simples:

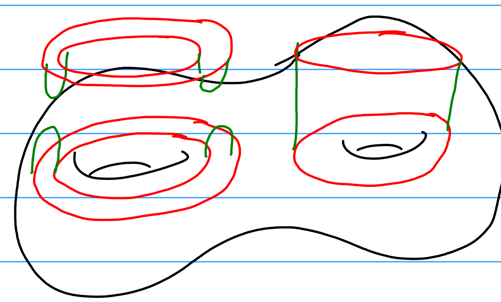


Theorem [QW18]: If $S \neq \mathbb{T}^2$ then $BN(S)^0$ is semi-simple with simples:

Example:



$-\frac{1}{2}$

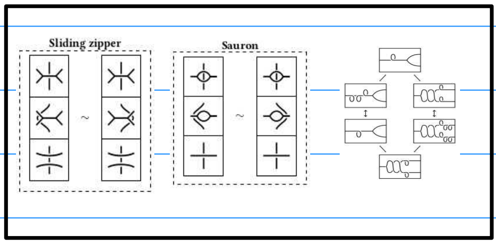


Lemma:

Conjecture:

Theorem [Bousseau 20]: $\{\gamma_T\}$ is positive if $S = \mathbb{T}^2 \setminus \{pt\}$ or $\mathbb{S}^2 \setminus 4$ pts.

① **Theorem**: [QWW, in progress]:
$\|$ full list of MM for framed forms



Theorem: [QWW]

②

③ **Theorem**: [QWW, in progress]
$\|$

$$\boxed{A + B \geq 0 \implies A \geq 0 \text{ and } B \geq 0}$$