Technical Analysis Compared to Mathematical Models under Misspecification

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Outline

1. Introduction
2. Our Model
3. Optimal Portfolio Allocation
   - Mathematical Results
   - Numerical Results
4. Comparisons of Strategies
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Benoîte de Saporta Technical Analysis vs Mathematical Models
Technical Analysis
- avoids model specification and calibration problems

Mathematical Models
- theoretically better
- liable to miscalibration
Framework

Bond \quad dS^0_t = S^0_t r dt,

Stock \quad dS_t = \mu(t) S_t dt + \sigma S_t dB_t,

- \( B \) standard Brownian motion,
- \( \mu(t) \in \{\mu_1, \mu_2\} \) independent of \( B \),
- \( \pi_t \in \{0, 1\} \) proportion of the wealth invested in the stock
- terminal time \( T \)
Technical Analyst Strategy

Technical analyst: moving average strategy

\[ M_t^\delta = \frac{1}{\delta} \int_{t-\delta}^{t} S_u du \]

If \( S_t > M_t^\delta \) buy

If \( S_t < M_t^\delta \) sell

\[ \mu_1 = -0.2, \ \mu_2 = 0.2, \ \sigma = 0.15, \ \delta = 0.8. \]
Previous work

Blanchet, Diop, Gibson, Kaminski, Talay, Tanré (2005)

One change of drift:
- $\mu(t) = \mu_1$ if $t < \tau$
- $\mu(t) = \mu_2$ if $t \geq \tau$

with $\mathbb{P}(\tau > t) = e^{-\lambda t}$

Optimal strategy: detect $\tau$. 
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Technical Analysis vs Mathematical Models
Several changes of drift

\((\xi_{2n+1}) \, \text{iid Exp}(\lambda_1), \, (\xi_{2n}) \, \text{iid Exp}(\lambda_2)\)

\(\tau_0 = 0, \ \tau_n = \xi_1 + \cdots + \xi_n\)

\[\mu(t) = \begin{cases} 
\mu_1 & \text{if } \tau_{2n} \leq t < \tau_{2n+1} \\
\mu_2 & \text{if } \tau_{2n+1} \leq t < \tau_{2n+2}
\end{cases}\]

Transaction costs
- \(g_{01}\) buying cost
- \(g_{10}\) selling cost
Admissible Strategies

\( \pi_t \in \{0, 1\} \) proportion of the wealth invested in the stock

\[ \mathcal{F}^S_t = \sigma(S_u, u \leq t) \neq \mathcal{F}^B_t = \sigma(B_u, u \leq t) \]

We want \( \pi_t \) measurable w.r.t. \( \mathcal{F}^S_t \)

\( \implies \) change of framework
Filtering Theory

Optional projection: $F_t = \mathbb{P}(\mu(t) = \mu_1 \mid \mathcal{F}^S_t)$

$$
\overline{B}_t = \frac{1}{\sigma} \left( \log \frac{S_t}{S_0} - \int_0^t \left( \mu_1 F_s + \mu_2 (1 - F_s) - \frac{\sigma^2}{2} \right) ds \right)
$$

Proposition (Martinez, Rubenthaler, Tanré 2005)

- $\overline{B}$ is a $(\mathcal{F}^S)$ Brownian Motion
- $\mathcal{F}^S = \mathcal{F}^{\overline{B}}$

$$
\frac{dS_t}{S_t} = \left( \mu_1 F_t + \mu_2 (1 - F_t) \right) dt + \sigma d\overline{B}_t
$$

Theorem (Kurtz, Ocone 1988)
New Framework

$W_t^\pi$ : Wealth process with strategy $\pi$

\[
\frac{dW_t^\pi}{W_t^\pi} = \pi_t \frac{dS_t}{S_t} + (1 - \pi_t) \frac{dS_0^0}{S_0^0}
- g_{01} \delta(\Delta \pi_t = 1) - g_{10} \delta(\Delta \pi_t = -1)
= \left(\pi_t (\mu_1 F_t + \mu_2 (1 - F_t)) + (1 - \pi_t) r\right) dt + \pi_t \sigma d\tilde{B}_t
- g_{01} \delta(\Delta \pi_t = 1) - g_{10} \delta(\Delta \pi_t = -1)
\]

\[dF_t = \left(-\lambda_1 F_t + \lambda_2 (1 - F_t)\right) dt + \frac{\mu_1 - \mu_2}{\sigma} F_t (1 - F_t) d\tilde{B}_t,\]
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Utility: $U(x) = x^\alpha$, $\alpha \in ]0, 1[$

Value functions:

\[
V^0(t, x, f) = \sup_{\pi} \mathbb{E}[U(W^\pi_T) | \pi_{t^-} = 0, W^\pi_{t^-} = x, F_t = f]
\]

\[
V^1(t, x, f) = \sup_{\pi} \mathbb{E}[U(W^\pi_T) | \pi_{t^-} = 1, W^\pi_{t^-} = x, F_t = f]
\]
Properties of $V$

**Proposition (Continuity)**

For all $i \in \{0; 1\}$, $0 \leq t \leq \hat{t} \leq T$, $x, \hat{x} > 0$, $0 \leq f, \hat{f} \leq 1$:

$$|V^i(\hat{t}, \hat{x}, \hat{f}) - V^i(t, x, f)| \leq C(1 + x^{\alpha - 1} + \hat{x}^{\alpha - 1})(|\hat{x} - x| + x(|\hat{f} - f| + |\hat{t} - t|^{1/2}))$$

**Theorem (Dynamic Programming Principle)**

For all $0 \leq s \leq t \leq T$ and $x, f, i$:

$$V^i(s, x, f) = \sup_{\pi} \mathbb{E}[V^{\pi_{t-}}(t, W^{s,x,f,\pi}_t, F^{s,f}_t)]$$
Hamilton Jacobi Bellman Equations

\( \mathcal{V}_\alpha \) : set of continuous functions \( \varphi \) on \( [0; T] \times [0; +\infty[ \times [0; 1] \) satisfying \( \varphi(t, 0, f) = 0 \) and

\[
\sup_{[0; T] \times [0; +\infty[ \times [0; 1]^2} \frac{|\varphi(t, x, f) - \varphi(t, \hat{x}, \hat{f})|}{(1 + x^{\alpha-1} + \hat{x}^{\alpha-1})(|x - \hat{x}| + x|f - \hat{f}|)} < \infty.
\]

Theorem

\((V^0, V^1)\) is the unique viscosity solution of

\[
\begin{align*}
\min \left\{ -\frac{\partial \varphi^0}{\partial t} - \mathcal{L}^0 \varphi^0; \quad \varphi^0(t, x, f) - \varphi^1(t, x(1 - g_{01}), f) \right\} &= 0 \\
\min \left\{ -\frac{\partial \varphi^1}{\partial t} - \mathcal{L}^1 \varphi^1; \quad \varphi^1(t, x, f) - \varphi^0(t, x(1 - g_{10}), f) \right\} &= 0
\end{align*}
\]

in \( \mathcal{V}_\alpha \times \mathcal{V}_\alpha \) such that \( V^0(T, x, f) = V^1(T, x, f) = U(x) = x^\alpha \)
Discretization Scheme

Dependence on $x$:

$$V^i(t, x, f) = \sup_{\pi} \mathbb{E}[U(W^\pi_T) \mid \pi_{t^-} = 0, W^\pi_{t^-} = x, F_t = f]$$

$$= x^\alpha V^i(t, 1, f)$$

- **First step**: compute $\hat{V}^0(t, \cdot)$ and $\hat{V}^1(t, \cdot)$ that solve HJB equation from $\hat{V}^0(t + dt, \cdot)$ and $\hat{V}^1(t + dt, \cdot)$

- **Second step**: comparison
  - if $\hat{V}^0(t, f) \geq (1 - g_{01})^\alpha \hat{V}^1(t, f)$, take $\hat{V}^0(t, f) = \overline{V}^0(t, f)$
  - otherwise take $\hat{V}^0(t, f) = (1 - g_{01})^\alpha \hat{V}^1(t, f)$
  - if $\hat{V}^1(t, f) \geq (1 - g_{10})^\alpha \hat{V}^0(t, f)$, take $\hat{V}^1(t, f) = \overline{V}^1(t, f)$
  - otherwise take $\hat{V}^1(t, f) = (1 - g_{10})^\alpha \hat{V}^0(t, f)$
Value Function $V^0$

Parameters: $T = 3$, $\mu_2 = -\mu_1 = 0.2$, $\lambda_1 = \lambda_2 = 2$, $\sigma = 0.15$, $g_{01} = g_{10} = 0.001$
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Technical Analysis vs Mathematical Models
An efficient strategy

- Compute $\hat{V}^0$ and $\hat{V}^1$
- Estimate $\hat{F}_t$ from the stock
- Compare $\hat{V}^0(t, \hat{F}_t)$ and $\hat{V}^1(t, \hat{F}_t)$:
  - buy if $\hat{V}^0(t, \hat{F}_t) = (1 - g_{01})^\alpha \hat{V}^1(t, \hat{F}_t)$
  - sell if $\hat{V}^1(t, \hat{F}_t) = (1 - g_{10})^\alpha \hat{V}^0(t, \hat{F}_t)$

\[ \mu_1 = -0.2, \mu_2 = 0.2, \]
\[ \sigma = 0.15, \lambda_1 = 2, \]
\[ \lambda_2 = 2, T = 3 \]
100000 Monte Carlo simulations of the efficient strategy

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<th>0.1</th>
<th>0.2</th>
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<tr>
<td>Computed $V^0$</td>
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<table>
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<td>1.038</td>
<td>1.037</td>
<td>1.036</td>
</tr>
</tbody>
</table>
Misspecified Strategy vs Moving Average

Misspecified parameters:

\[ \mu_1 = -1.8, \quad \mu_2 = 1.8, \quad \sigma = 0.15, \]
\[ \lambda_1 = 4, \quad \lambda_2 = 4 \]

Real parameters:

\[ \mu_1 = -0.2, \quad \mu_2 = 0.2, \quad \sigma = 0.15, \quad \lambda_1 = 2, \]
\[ \lambda_2 = 2, \quad T = 3, \quad \delta = 0.8, \quad 100000 \text{ Monte Carlo simulations} \]