Technical Analysis Compared to Mathematical Models under Misspecification

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Outline

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Motivation

Technical Analysis

- avoids model specification and calibration problems

Mathematical Models

- theoretically better
- liable to miscalibration

Framework

Bond	$dS^0_t = S^0_t r dt,$
Stock	$dS_t = \mu(t) S_t dt + \sigma S_t dB_t,$

- B standard Brownian motion,
- $\mu(t) \in \{\mu_1, \mu_2\}$ independent of B,
- $\pi_t \in \{0, 1\}$ proportion of the wealth invested in the stock
- terminal time T

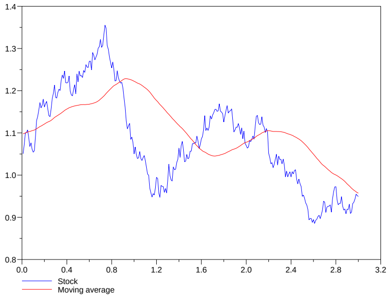
Technical Analyst Strategy

Technical analyst: moving average strategy

$$M_t^\delta = \frac{1}{\delta}\int_{t-\delta}^{t} S_u du$$

If $S_t > M_t^\delta$ buy

If $S_t < M_t^\delta$ sell



$\mu_1 = -0.2$, $\mu_2 = 0.2$, $\sigma = 0.15$, $\delta = 0.8$.

Previous work

BLANCHET, DIOP, GIBSON, KAMINSKI, TALAY, TANRÉ (2005)

One change of drift:

- $\mu(t) = \mu_1$ if $t < \tau$
- $\mu(t) = \mu_2$ if $t \geq \tau$

with $\mathbb{P}(\tau > t) = e^{-\lambda t}$

Optimal strategy: detect τ.

Outline

Our Model

- Several changes of drift
  (ξ_{2n+1}) iid Exp(λ_1), (ξ_{2n}) iid Exp(λ_2)
  $\tau_0 = 0$, $\tau_n = \xi_1 + \cdots + \xi_n$

$$\mu(t) = \begin{cases} \mu_1 & \text{if } \tau_{2n} \leq t < \tau_{2n+1} \\ \mu_2 & \text{if } \tau_{2n+1} \leq t < \tau_{2n+2} \end{cases}$$

- Transaction costs
  - g_{01} buying cost
  - g_{10} selling cost

Admissible Strategies

$\pi_t \in \{0, 1\}$ proportion of the wealth invested in the stock

$$\mathcal{F}_t^S = \sigma(S_u, u \le t) \neq \mathcal{F}_t^B = \sigma(B_u, u \le t)$$

We want π_t measurable w.r.t. $\mathcal{F}_t^S$

$\Longrightarrow$ change of framework

Filtering Theory

Optional projection: $F_t = \mathbb{P}(\mu(t) = \mu_1 \mid \mathcal{F}_t^S)$

$$\overline{B}_t = \frac{1}{\sigma}\Big(\log\frac{S_t}{S_0} - \int_0^t (\mu_1 F_s + \mu_2(1-F_s)) - \frac{\sigma^2}{2})ds\Big)$$

Proposition (Martinez, Rubenthaler, Tanré 2005)

- *$\overline{B}$ is a $(\mathcal{F}^S)$ Brownian Motion*
- $\mathcal{F}^S = \mathcal{F}^{\overline{B}}$

$$\frac{dS_t}{S_t} = \big(\mu_1 F_t + \mu_2(1-F_t)\big)dt + \sigma d\overline{B}_t$$

Theorem (Kurtz, Ocone 1988)

New Framework

W_t^π : Wealth process with strategy π

$$
\begin{aligned}
\frac{dW_t^\pi}{W_{t-}^\pi} &= \pi_t \frac{dS_t}{S_t} + (1-\pi_t)\frac{dS_t^0}{S_t^0} \\
&\quad - g_{01}\delta(\Delta\pi_t = 1) - g_{10}\delta(\Delta\pi_t = -1) \\
&= \big(\pi_t(\mu_1 F_t + \mu_2(1-F_t)) + (1-\pi_t)r\big)dt + \pi_t \sigma d\bar{B}_t \\
&\quad - g_{01}\delta(\Delta\pi_t = 1) - g_{10}\delta(\Delta\pi_t = -1)
\end{aligned}
$$

$$
dF_t = \big(-\lambda_1 F_t + \lambda_2(1-F_t)\big)dt + \frac{\mu_1-\mu_2}{\sigma}F_t(1-F_t)d\bar{B}_t,
$$

Outline

Value Functions

Utility: $U(x) = x^{\alpha}$, $\alpha \in]0,1[$

Value functions:

$$\begin{aligned} V^0(t,x,f) &= \sup_{\pi} \mathbb{E}[U(W_T^{\pi}) \mid \pi_{t^-} = 0, W_{t^-}^{\pi} = x, F_t = f] \\ V^1(t,x,f) &= \sup_{\pi} \mathbb{E}[U(W_T^{\pi}) \mid \pi_{t^-} = 1, W_{t^-}^{\pi} = x, F_t = f] \end{aligned}$$

Properties of V

Proposition (Continuity)

For all $i \in \{0; 1\}$, $0 \le t \le \hat{t} \le T$, $x, \hat{x} > 0$, $0 \le f, \hat{f} \le 1$:

$$\left|V^i(\hat{t}, \hat{x}, \hat{f}) - V^i(t, x, f)\right| \le C(1 + x^{\alpha-1} + \hat{x}^{\alpha-1})\big(|\hat{x} - x| + x(|\hat{f} - f| + |\hat{t} - t|^{1/2})\big)$$

Theorem (Dynamic Programming Principle)

For all $0 \le s \le t \le T$ *and* x, f, i:

$$V^i(s, x, f) = \sup_{\pi} \mathbb{E}[V^{\pi_{t-}}(t, W_t^{s,x,f,\pi}, F_t^{s,f})]$$

Hamilton Jacobi Bellman Equations

$\mathcal{V}_\alpha$: set of continuous functions φ on $[0;T]\times[0;+\infty[\times[0;1]$ satisfying $\varphi(t,0,f)=0$ and

$$\sup_{[0;T]\times]0;+\infty[^2\times[0;1]^2}\frac{|\varphi(t,x,f)-\varphi(t,\hat{x},\hat{f})|}{(1+x^{\alpha-1}+\hat{x}^{\alpha-1})(|x-\hat{x}|+x|f-\hat{f}|)}<\infty.$$

Theorem

(V^0,V^1) is the unique viscosity solution of

$$\begin{cases}\min\left\{-\frac{\partial\varphi^0}{\partial t}-\mathcal{L}^0\varphi^0;\ \varphi^0(t,x,f)-\varphi^1(t,x(1-g_{01}),f)\right\}=0\\ \min\left\{-\frac{\partial\varphi^1}{\partial t}-\mathcal{L}^1\varphi^1;\ \varphi^1(t,x,f)-\varphi^0(t,x(1-g_{10}),f)\right\}=0\end{cases}$$

in $\mathcal{V}_\alpha\times\mathcal{V}_\alpha$ such that $V^0(T,x,f)=V^1(T,x,f)=U(x)=x^\alpha$

Discretization Scheme

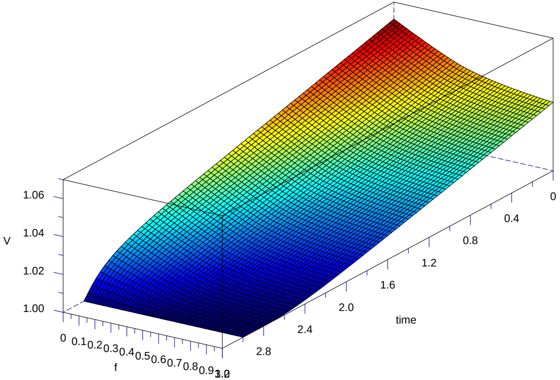
Dependence on x:

$$\begin{aligned} V^i(t,x,f) &= \sup_\pi \mathbb{E}[U(W_T^\pi) \mid \pi_{t-} = 0, W_{t-}^\pi = x, F_t = f] \\ &= x^\alpha V^i(t,1,f) \end{aligned}$$

- First step: compute $\overline{V^0}(t,\cdot)$ and $\overline{V^1}(t,\cdot)$ that solve HJB equation from $\hat{V}^0(t+dt,\cdot)$ and $\hat{V}^1(t+dt,\cdot)$
- Second step: comparison
  - if $\overline{V^0}(t,f) \geq (1-g_{01})^\alpha \overline{V^1}(t,f)$, take $\hat{V}^0(t,f) = \overline{V^0}(t,f)$ otherwise take $\hat{V}^0(t,f) = (1-g_{01})^\alpha \overline{V^1}(t,f)$
  - if $\overline{V^1}(t,f) \geq (1-g_{10})^\alpha \overline{V^0}(t,f)$, take $\hat{V}^1(t,f) = \overline{V^1}(t,f)$ otherwise take $\hat{V}^1(t,f) = (1-g_{10})^\alpha \overline{V^0}(t,f)$

Value Function V^0

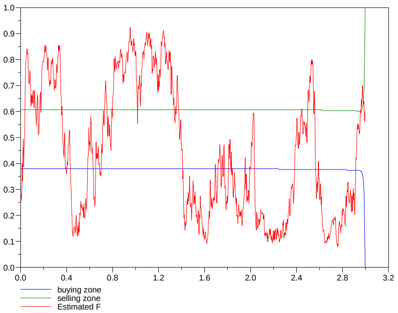
Parameters: $T = 3$, $\mu_2 = -\mu_1 = 0.2$, $\lambda_1 = \lambda_2 = 2$, $\sigma = 0.15$, $g_{01} = g_{10} = 0.001$



Outline

An efficient strategy

- Compute $\hat{V}^0$ and $\hat{V}^1$
- Estimate $\hat{F}_t$ from the stock
- Compare $\hat{V}^0(t,\hat{F}_t)$ and $\hat{V}^1(t,\hat{F}_t)$:
  - buy if $\hat{V}^0(t,\hat{F}_t) = (1-g_{01})^\alpha \hat{V}^1(t,\hat{F}_t)$
  - sell if $\hat{V}^1(t,\hat{F}_t) = (1-g_{10})^\alpha \hat{V}^0(t,\hat{F}_t)$



$\mu_1 = -0.2$, $\mu_2 = 0.2$, $\sigma = 0.15$, $\lambda_1 = 2$, $\lambda_2 = 2$, $T = 3$

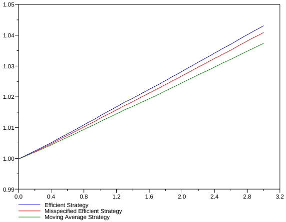
Efficient Strategy vs Value Function

100000 Monte Carlo simulations of the efficient strategy

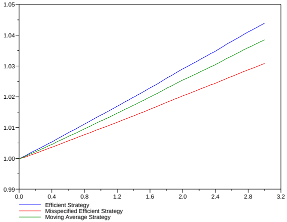
F_0	0	0.1	0.2	0.3	0.4	0.5
Computed V^0	1.061	1.057	1.053	1.049	1.045	1.043
Efficient strategy	1.061	1.057	1.052	1.049	1.045	1.043

F_0	0.6	0.7	0.8	0.9	1
Computed V^0	1.041	1.039	1.038	1.037	1.036
Efficient strategy	1.040	1.039	1.038	1.037	1.036

Misspecified Strategy vs Moving Average



Misspecified parameters:
$\mu_1 = -1.8$, $\mu_2 = 1.8$, $\sigma = 0.15$,
$\lambda_1 = 4$, $\lambda_2 = 4$



Misspecified parameters:
$\mu_1 = -1.8$, $\mu_2 = 1.8$, $\sigma = 0.25$,
$\lambda_1 = 4$, $\lambda_2 = 4$

Real parameters: $\mu_1 = -0.2$, $\mu_2 = 0.2$, $\sigma = 0.15$, $\lambda_1 = 2$, $\lambda_2 = 2$, $T = 3$, $\delta = 0.8$, 100000 Monte Carlo simulations