Predictive maintenance for the heated hold-up tank

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Outline

1. Maintenance problem
   - The heated hold-up tank
   - Optimization problem

2. Numerical solution
   - Mathematical solution
   - Numerical scheme

3. Numerical results

4. Conclusion
The heated hold-up tank

Test case

- continuous variables
  - liquid level $h$
  - temperature $\theta$
- discrete variables
  - state of the 3 units and controller

in closed loop interaction
Dynamics: units

Possible states for each unit: ON, OFF, Stuck ON, Stuck OFF

Transitions for unit $i$

Jump intensity depending on the temperature

$$\lambda^i = a(\theta) \ell^i$$
The heated hold-up tank

Dynamics: liquid level

Liquid level depends on the units positions

\[ \frac{dh}{dt} = nG \]

\( n = \) number of inlet pumps ON or Stuck ON – number of outlet valves ON or Stuck ON

Control laws if controller ON

- if \( h \geq 8m \) turn pumps OFF and valve ON if unstuck
- if \( h \leq 6m \) turn pumps ON and valve OFF if unstuck
Dynamics: temperature and controller

Temperature depends on the liquid level and units positions

\[
\frac{d\theta}{dt} = \frac{mG(\theta_{in} - \theta) + K}{h}
\]

\(m\): number of inlet pumps ON or Stuck ON

Controller succeeds with probability \(p\) at each solicitation. Once failed stays failed.
The heated hold-up tank

Top events

Starting point
Unit 1 ON, Unit 2 OFF, Unit 3 ON, Controller ON,
\( h = 7 \text{m} \), \( \theta = 30.9261^\circ C \) equilibrium point

Top events: systems stops
- dry out \( h < 4 \text{m} \)
- overflow \( h > 10 \text{m} \)
- hot temperature \( \theta > 100^\circ C \)
The heated hold-up tank

Examples of trajectories
Problem

The heated hold-up tank

Examples of trajectories

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Examples of trajectories
Maintenance optimization

Find the best time to stop the process

- before reaching the top events
- letting the system evolve in the operational states as long as possible
Optimal stopping problem

\((X_t)\) stochastic process, \(T\) optimization horizon

\[ V = \sup_{\tau \leq T} \mathbb{E}[g(X_\tau, \tau)] \]

Find

- the optimal performance \(V\)
- the optimal stopping time \(\tau^*\) such that \(\mathbb{E}[g(X_{\tau^*}, \tau^*)] = V\)

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Optimal stopping for the tank

Horizon \( T = 1000h \)

gain function
\[
g(h, \theta, t) = f(h, \theta)t^\alpha
\]

\[
f(h, \theta) =
\begin{align*}
1 & \text{ if } 6 \leq h \leq 8 \text{ and } \theta \leq 50 \\
0 & \text{ if top events}
\end{align*}
\]
Mathematical solution

Modeling

### Piecewise deterministic Markov process

\[ X_t = (m_t, x_t) \]

- \( m_t \) discrete **mode**: state of the units and controller
- \( x_t = (h_t, \theta_t) \) euclidean variable

### Underlying Markov chain

\[ (S_n, Z_n) \]

\( S_n \) time between jumps \( n - 1 \) and \( n \)
\( Z_n \) value of the process after jump \( n \)

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### Problem

Mathematical solution

### Solution

Iterative theoretical resolution

#### Dynamic programming

- $v_N = g$
- $v_n = L(v_{n+1}, g)$ for $n \leq N - 1$

$$v_0 = \sup_{\tau \leq T} \mathbb{E}[g(X_\tau)] = V$$

$$L(v_{n+1}, g)(Z_n) = \sup_{u \leq t^*(Z_n)} \left\{ \mathbb{E}[v_{n+1}(Z_{n+1}) \mathbb{1}_{S_{n+1} < u} + g(\phi(Z_n, u)) \mathbb{1}_{S_{n+1} \geq u} | Z_n] \right\}$$

$$\quad \vee \mathbb{E}[v_{n+1}(Z_{n+1}) | Z_n]$$

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Quantization

**Strategy**

Discretize the Markov chain \((S_n, Z_n)\) using quantization

Standard gaussian random variable \(\mathcal{N}(0, I_2)\):

![Standard Gaussian Random Variable](image)
Quantization

Strategy

Discretize the Markov chain \((S_n, Z_n)\) using quantization.

Standard gaussian random variable \(\mathcal{N}(0, I_2)\) :
Quantization

Strategy

Discretize the Markov chain \((S_n, Z_n)\) using quantization

Standard gaussian random variable \(\mathcal{N}(0, I_2)\) :
Main difficulties

Simulation
- closed-loop interactions
- high cardinality of the mode
- rare events
Optimal stopping time

\[ h = 7m, \ \theta = 30.9261^\circ C, \ \text{ON, OFF, ON} \]
Optimal stopping time

\[ h = 7m, \theta = 30.9261^\circ C, \text{ON, OFF, Stuck OFF} \]
Optimal stopping time

\[ h = 8m, \; \theta = 30.9261^\circ C, \; \text{OFF, OFF, Stuck OFF} \]
Optimal stopping time

\[ h = 8m, \theta = 78.25^\circ C, \text{OFF, Stuck ON, Stuck OFF} \]
Optimal stopping time

\[ h = 8.86m, \ \theta = 73.66^\circ C, \ \text{gain} = 10.20 \]
Optimal stopping time

\[ h = 7m, \theta = 30.9261^\circ C, \text{ON, OFF, ON} \]
Optimal stopping time

\[ h = 7m, \theta = 30.9261^\circ C, \text{ Stuck OFF, OFF, ON} \]
Optimal stopping time

\[ h = 6m, \theta = 33.38^\circ C, \text{Stuck OFF, ON, OFF} \]
Optimal stopping time

\[ h = 8m, \theta = 32.77^\circ C, \text{ Stuck OFF, OFF, ON} \]
Optimal stopping time

\[ h = 6m, \, \theta = 37.35^\circ C, \text{ Stuck OFF, ON, OFF} \]
Optimal stopping time

\[ h = 7.66 \text{m}, \ \theta = 35.96^\circ \text{C}, \ \text{Stuck OFF, Stuck ON, OFF} \]
Optimal stopping time

$h = 8m, \theta = 35.74^\circ C$, Stuck OFF, Stuck ON, ON
Optimal stopping time

\[ h = 8m, \; \theta = 30.9261^\circ C, \; \text{Stuck OFF, Stuck ON, Stuck OFF} \]
Optimal stopping time

\[ h = 8.75\, m, \ \theta = 30.9261^\circ C, \ \text{gain} = 99.07 \]
## Mean optimal performance

<table>
<thead>
<tr>
<th>Disc. points</th>
<th>Value function</th>
<th>MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>334.34</td>
<td>305.55</td>
</tr>
<tr>
<td>300</td>
<td>333.04</td>
<td>319.45</td>
</tr>
<tr>
<td>400</td>
<td>332.95</td>
<td>322.20</td>
</tr>
<tr>
<td>800</td>
<td>330.43</td>
<td>323.63</td>
</tr>
<tr>
<td>1000</td>
<td>330.87</td>
<td>324.04</td>
</tr>
</tbody>
</table>
Distribution at optimality

Distribution of the computed stopping time

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Distribution at optimality

Distribution of the computed stopping time (zoom)
Validity of the results

- No analytic solution to compare
- Theoretical proof of convergence of the algorithm

<table>
<thead>
<tr>
<th></th>
<th>without maintenance</th>
<th>with maintenance</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean performance</td>
<td>211.80</td>
<td>330.87</td>
</tr>
<tr>
<td>gain=0</td>
<td>80.33%</td>
<td>0.02%</td>
</tr>
<tr>
<td>$6 \leq h \leq 8$</td>
<td>28.25%</td>
<td>90.02%</td>
</tr>
<tr>
<td>$\theta \leq 50^\circ C$</td>
<td>80.33%</td>
<td>95.09%</td>
</tr>
</tbody>
</table>
Conclusion and perspectives

- powerful numerical method
- stopping time adapted to each trajectory
- rigorous mathematical context

- impulse control: maintenance with partial repair
Conclusion and perspectives

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- rigorous mathematical context
- impulse control: maintenance with partial repair