Predictive maintenance for the heated hold-up tank

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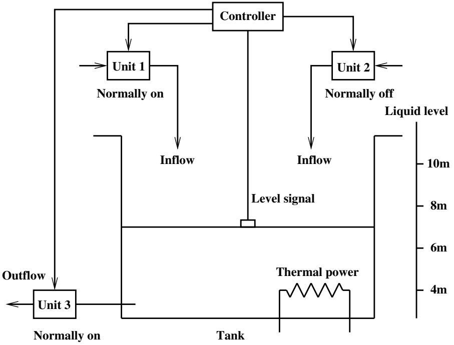
Outline

The heated hold-up tank



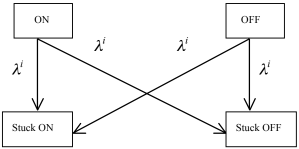
Test case

- continuous variables
  liquid level h,
  temperature θ
- discrete variables
  state of the 3 units
  and controller

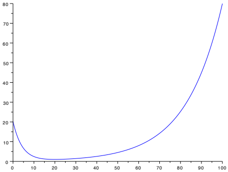
in closed loop interaction

Dynamics: units

Possible states for each unit: ON, OFF, Stuck ON, Stuck OFF



Transitions for unit i



$\theta \longmapsto a(\theta)$

Jump intensity depending on the temperature

$$\lambda^i = a(\theta)\ell^i$$

Dynamics: liquid level

Liquid level depends on the units positions

$$\frac{dh}{dt} = nG$$

$n =$ number of inlet pumps ON or Stuck ON $-$ number of outlet valves ON or Stuck ON

Control laws if controller ON

- if $h \geq 8m$ turn pumps OFF and valve ON if unstuck
- if $h \leq 6m$ turn pumps ON and valve OFF if unstuck

Dynamics: temperature and controller

Temperature depends on the liquid level and units positions

$$\frac{d\theta}{dt} = \frac{mG(\theta_{in} - \theta) + K}{h}$$

m: number of inlet pumps ON or Stuck ON

Controller succeeds with probability p at each solicitation. Once failed stays failed.

Top events

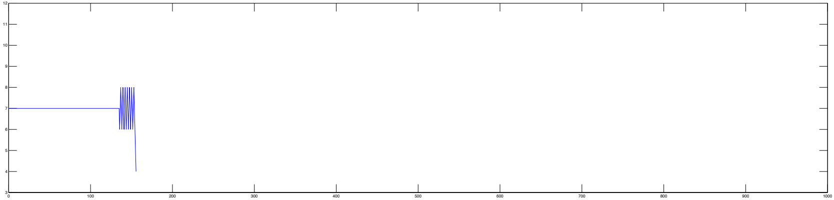
Starting point

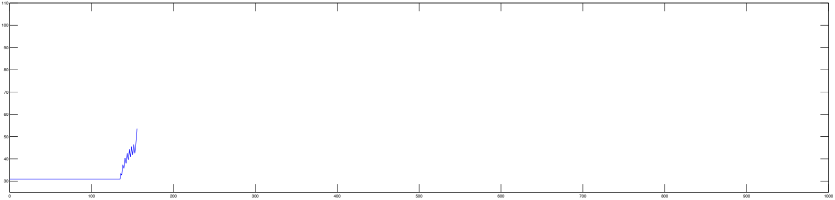
Unit 1 ON, Unit 2 OFF, Unit 3 ON, Controller ON,
$h = 7m$, $\theta = 30.9261^\circ C$ equilibrium point

Top events: systems stops

- dry out $h < 4m$
- overflow $h > 10m$
- hot temperature $\theta > 100^\circ C$

Examples of trajectories





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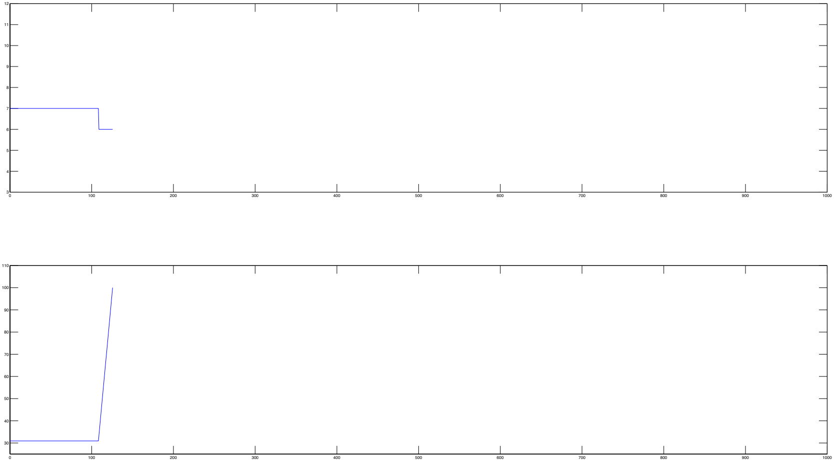

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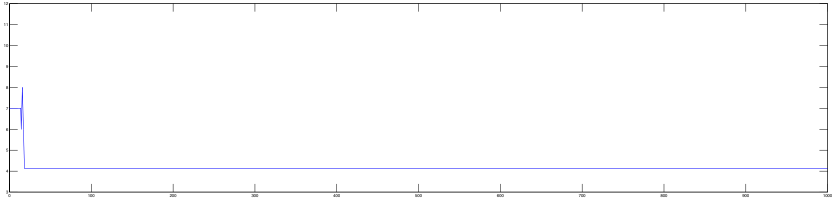
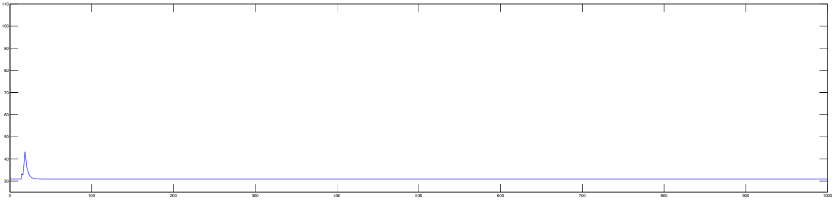




Maintenance

Maintenance optimization

Find the best time to stop the process

- before reaching the top events
- letting the system evolve in the operational states as long as possible

Mathematical formulation

Optimal stopping problem

(X_t) stochastic process, T optimization horizon

$$V = \sup_{\tau \leq T} \mathbb{E}[g(X_\tau, \tau)]$$

Find

- the optimal performance V
- the optimal stopping time τ^* such that $\mathbb{E}[g(X_{\tau^*}, \tau^*)] = V$

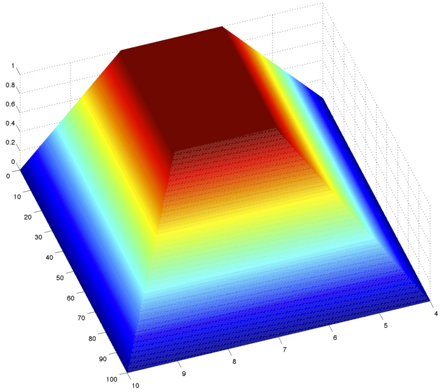
Optimal stopping for the tank

Horizon $T = 1000h$

gain function
$g(h, \theta, t) = f(h, \theta)t^{\alpha}$

$f(h, \theta) =$

- 1 if $6 \leq h \leq 8$ and $\theta \leq 50$
- 0 if top events



Modeling

Piecewise deterministic Markov process

$$X_t = (m_t, x_t)$$

- m_t discrete mode: state of the units and controller
- $x_t = (h_t, \theta_t)$ euclidean variable

Underlying Markov chain

$$(S_n, Z_n)$$

S_n time between jumps $n-1$ and n
Z_n value of the process after jump n

Iterative theoretical resolution

Dynamic programming

- $v_N = g$
- $v_n = L(v_{n+1}, g)$ for $n \leq N-1$

$$v_0 = \sup_{\tau \leq T} \mathbb{E}[g(X_\tau)] = V$$

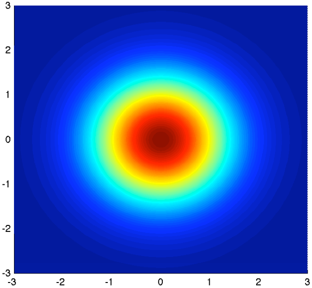
$$L(v_{n+1}, g)(Z_n)$$
$$= \sup_{u \leq t^*(Z_n)} \left\{ \mathbb{E}\left[v_{n+1}(Z_{n+1}) \mathbb{1}_{\{S_{n+1} < u\}} + g\big(\phi(Z_n, u)\big) \mathbb{1}_{\{S_{n+1} \geq u\}} \;\middle|\; Z_n \right] \right\}$$
$$\vee\ \mathbb{E}\left[v_{n+1}(Z_{n+1}) \;\middle|\; Z_n \right]$$

Quantization

Strategy

Discretize the Markov chain (S_n, Z_n) using quantization

Standard gaussian random variable $\mathcal{N}(0, I_2)$:



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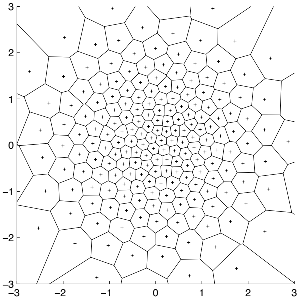

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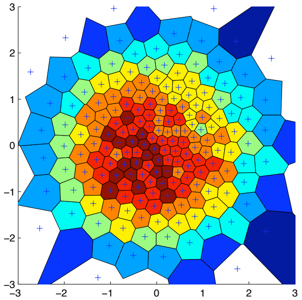
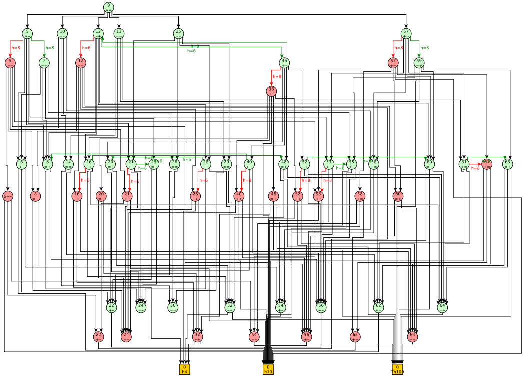


Main difficulties

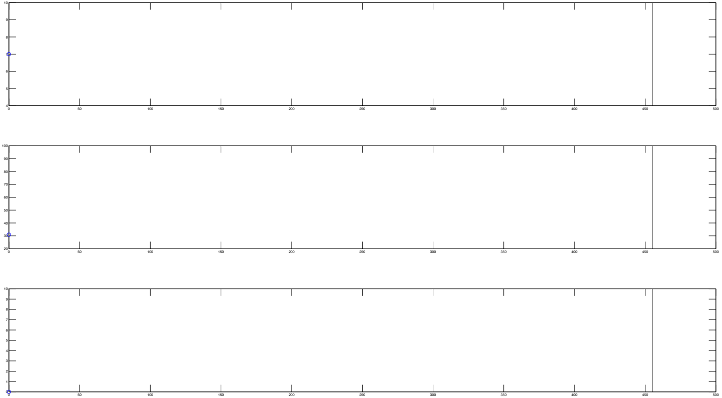
Simulation

- closed-loop interactions
- high cardinality of the mode
- rare events



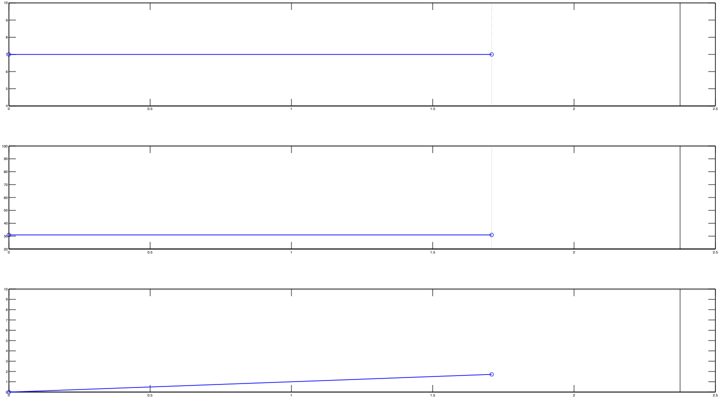
Optimal stopping time

$h = 7m$, $\theta = 30.9261^{\circ}C$, ON, OFF, ON



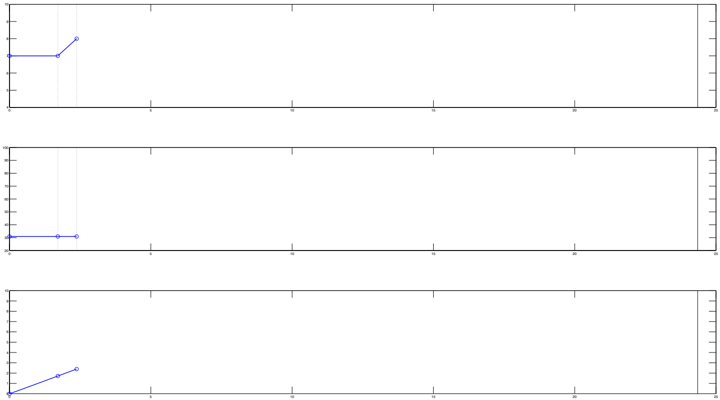
Optimal stopping time

$h = 7m$, $\theta = 30.9261°C$, ON, OFF, Stuck OFF



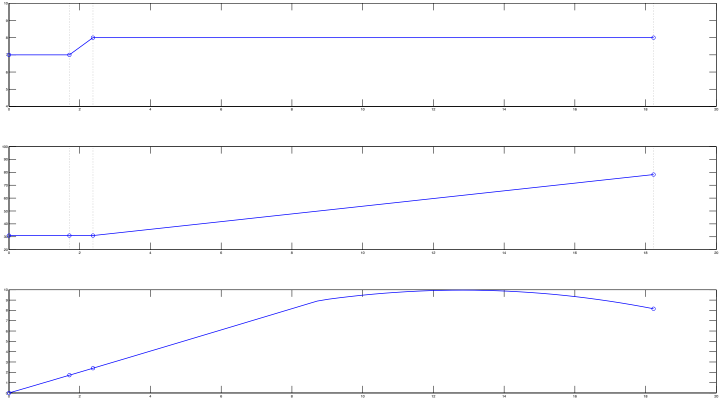
Optimal stopping time

$h = 8m$, $\theta = 30.9261°C$, OFF, OFF, Stuck OFF



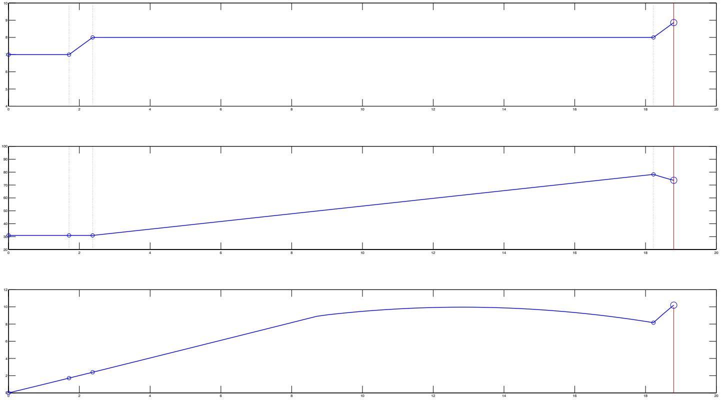
Optimal stopping time

$h = 8m$, $\theta = 78.25^\circ C$, OFF, Stuck ON, Stuck OFF



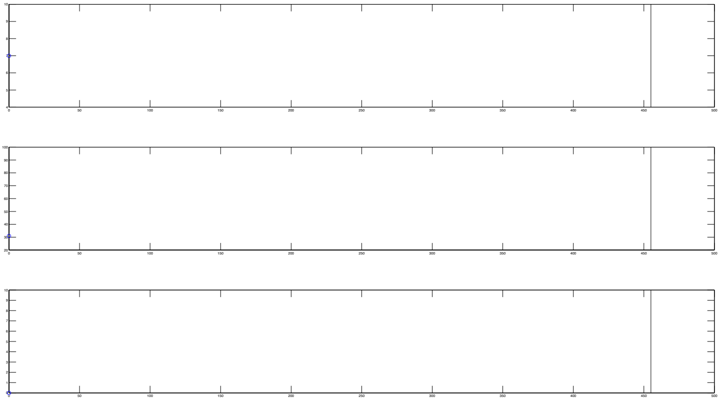
Optimal stopping time

$h = 8.86m$, $\theta = 73.66^\circ C$, gain$= 10.20$



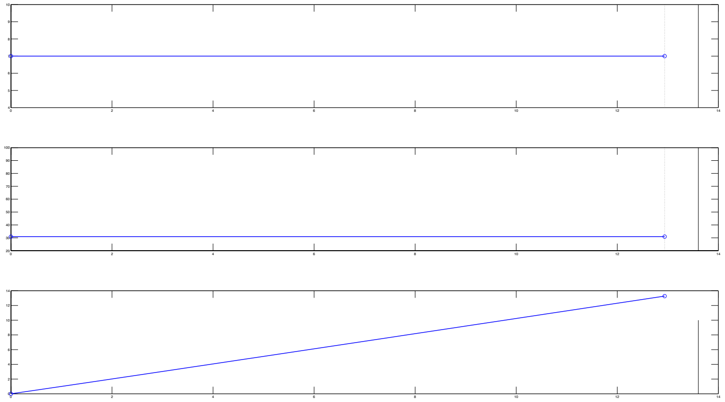
Optimal stopping time

$h = 7m$, $\theta = 30.9261°C$, ON, OFF, ON



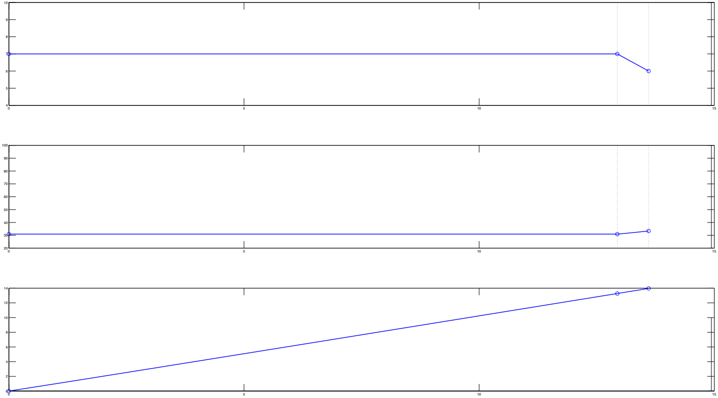
Optimal stopping time

$h = 7m$, $\theta = 30.9261^\circ C$, Stuck OFF, OFF, ON



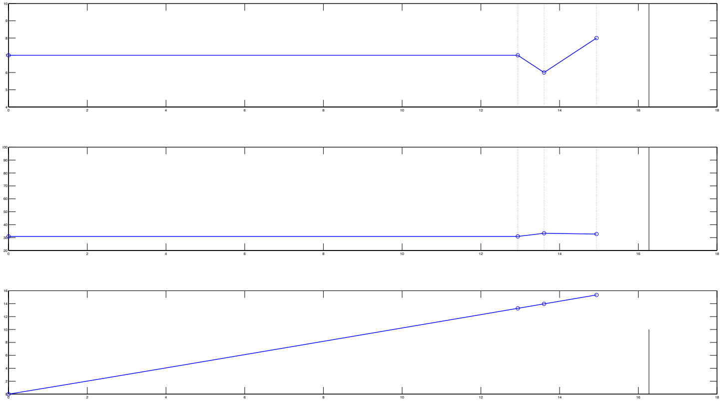
Optimal stopping time

$h = 6m$, $\theta = 33.38^\circ C$, Stuck OFF, ON, OFF



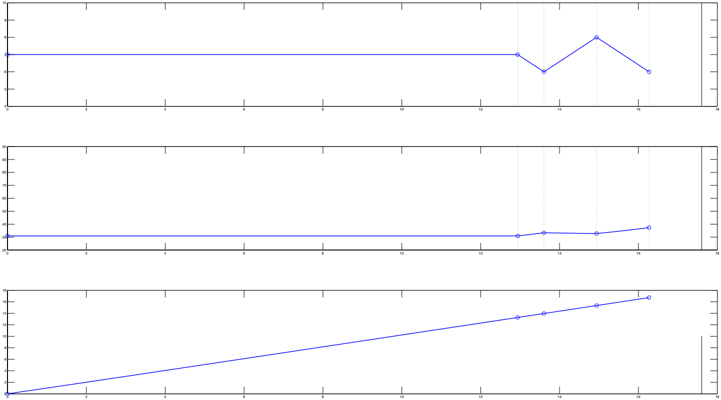
Optimal stopping time

$h = 8m$, $\theta = 32.77^\circ C$, Stuck OFF, OFF, ON



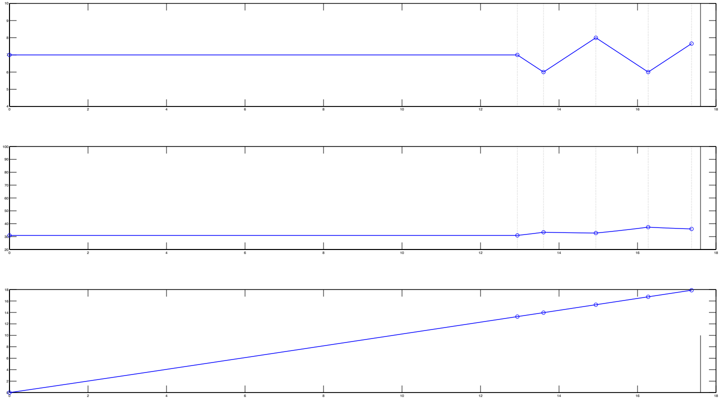
Optimal stopping time

$h = 6m$, $\theta = 37.35^\circ C$, Stuck OFF, ON, OFF



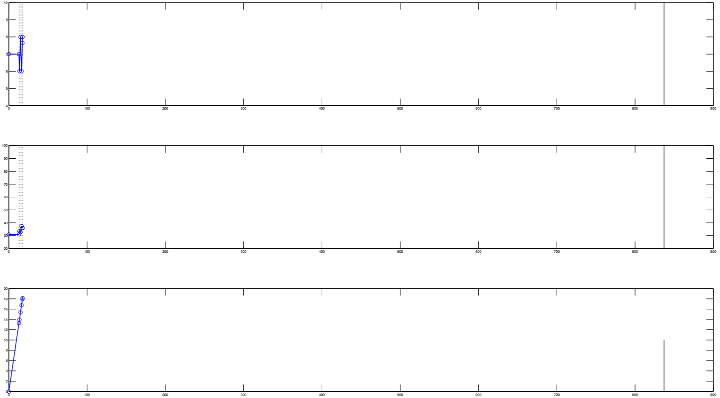
Optimal stopping time

$h = 7.66m$, $\theta = 35.96^\circ C$, Stuck OFF, Stuck ON, OFF



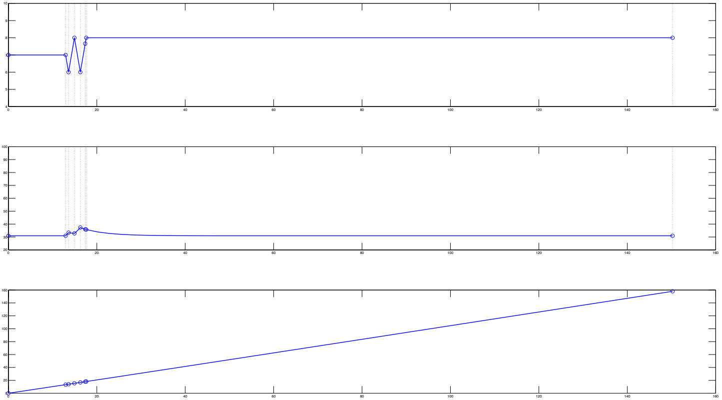
Optimal stopping time

$h = 8m$, $\theta = 35.74^\circ C$, Stuck OFF, Stuck ON, ON



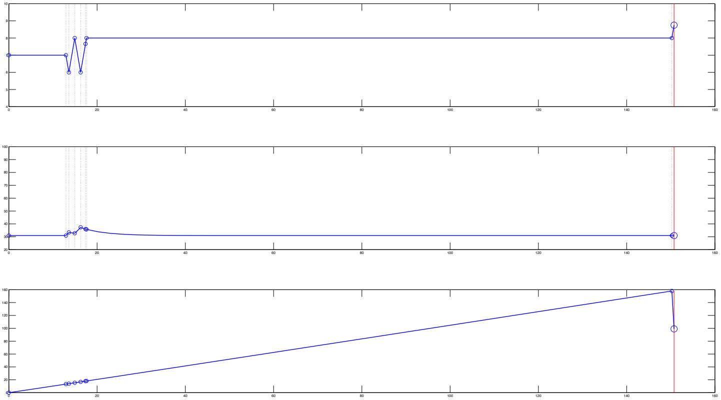
Optimal stopping time

$h = 8m$, $\theta = 30.9261°C$, Stuck OFF, Stuck ON, Stuck OFF



Optimal stopping time

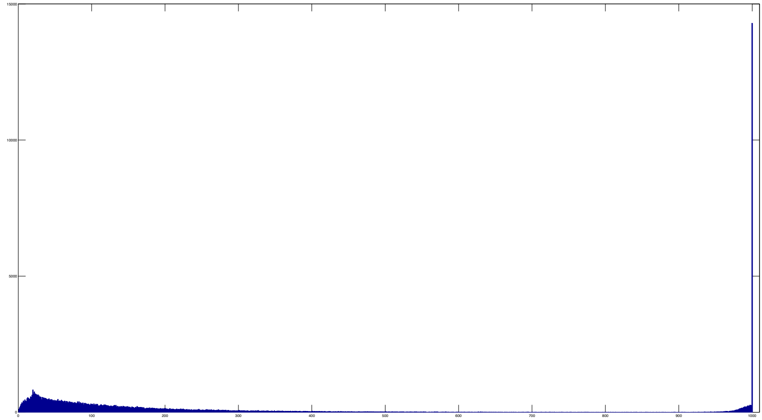
$h = 8.75m$, $\theta = 30.9261°C$, gain$= 99.07$



Mean optimal performance

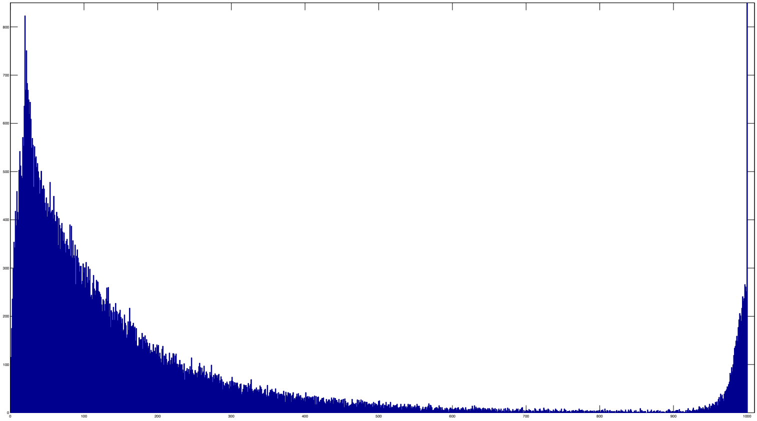
Disc. points	Value function	MC
200	334.34	305.55
300	333.04	319.45
400	332.95	322.20
800	330.43	323.63
1000	330.87	324.04

Distribution at optimality



Distribution of the computed stopping time

Distribution at optimality



Distribution of the computed stopping time (zoom)

Validity of the results

- No analytic solution to compare
- Theoretical proof of convergence of the algorithm

	without maintenance	with maintenance
mean performance	211.80	330.87
gain=0	80.33%	0.02%
$6 \leq h \leq 8$	28.25%	90.02%
$\theta \leq 50^\circ C$	80.33%	95.09%

Conclusion and perspectives

- powerful numerical method
- stopping time adapted to each trajectory
- rigorous mathematical context

- impulse control: maintenance with partial repair

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