Space-time simulations of extreme rainfall : why and how ?

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Abstract

Rainfall-induced urban floods may trigger considerable damage. Free surface flow models incorporating the rainfall forcing are powerful tools for urban flood risk assessment. However, an accurate mapping of the flood risk requires space-time rainfall field resolutions that are not currently available from standard meteorological products (e.g. rainfall radar images). Relying on stochastic rainfall generators may provide promising surrogate rainfall fields. Key aspects that such a generator should reproduce are the alternance of rainy and dry time steps; within a rainy time step, spatial rainfall intermittency; the spatio-temporal dependence structure of rainfall fields and extreme events. The last aspect is especially relevant as far as rainfall-induced urban floods are concerned. However, existing stochastic generators are not designed explicitly to deal with extreme events. We present a review of spatial and spatio-temporal processes for extreme events arising from the extreme value theory framework. The main two classical types of processes are the max-stable and threshold exceedance processes which both assume that the process being modelled is asymptotically dependent. Flexible approaches have been proposed recently allowing also asymptotically independent framework. We discuss the inclusion of processes dedicated to extreme event modelling in a stochastic generator which raises a number of issues. In particular, transitions between regular and extreme events should be modelled, this being especially challenging for the dependence structure. Incorporating stochastic rainfall generators in operational systems flood risk assessment and/or warning systems requires fast running hydraulic simulation engines. Using stochastic generators in combination with upscaled hydraulic models appears as a promising research path.

keywords : Space-time modelling, spatial stochastic generator, extreme rainfall, urban flood risk

1 Why

1.1 Rainfall-induced urban floods

Floods are regarded as the most widespread and globally costly natural disaster. Their human and economic impact is obviously the largest in the most densely populated areas. Since the fraction of the world's population living in urban areas increases steadily, the impact and cost of urban floods can be expected to rise in the future.

In Mediterranean areas, heavy rainfall events occur mostly in autumn owing to temperature contrasts between the moisture coming from the sea and the land surface. Orographic factors may also play a significant role. These rainfall events, that could be either very localized, with high intensities and lasting a few hours, or could be long-lasting events with moderate intensities affecting large areas, might lead to floods.

Not only does urbanization induce an increase in population density (and with it the density of stakes) but it also induces dramatic changes in land use. The fraction of impervious and/or low permeability areas increases. This contributes to reduce the infiltration capacity of soils. Intense to moderate rainfall events that would otherwise infiltrate are retained on the ground and generate large runoff volumes. Roads, parking lots, etc. being significantly smoother than natural grounds, they contribute to speed up the propagation of the runoff signal. Urban drainage networks may also contribute to enhance the dynamics of urban catchments.

Urban flood crisis management gathers a wide variety of stakeholders. These include local authorities, urban planning divisions, rescue and civil protection services, weather forecasting and warning services, insurance and reassurance companies, etc. Not all stakeholders have the same needs. Urban planning divisions and insurance companies are mostly concerned with long-term measures for disaster mitigation and vulnerability reduction. Warning services, local authorities and civil protection play a key role during the crisis, by managing communications, rescue actions, and establishing priorities for stake protection.

Ideally, decision-makers would like to be supplied with real-time knowledge or a forecast of the rainfall event (i.e. how is the rainfall field likely to vary in space and time). The knowledge of the space-time behaviour of the rainfall field would allow rainfall-runoff models to be operated to forecast the consequences of the rainfall event in terms of flood-induced risk and damages on the scale of the conurbation.

An "ideal" urban flood forecasting and warning chain should include meteorological monitoring and forecasting modules, one or several rainfall-runoff and/or free surface flow models, and a decision support system prioritizing and synthesizing information for the crisis management group. The design of flood crisis management systems is beyond the scope of the present chapter. Only rainfall generating and free surface modelling aspects are covered hereafter.

The following subsection presents hydraulic simulations of a rainfall-induced urban flood. These simulations illustrate the need for space-time simulations of extreme rainfall highlighting the effect of the localization and the extension of the rainfall field on the flooding pattern. The second section deals with the question "how to perform space-time simulations of extreme rainfall?". After discussing the spatial stochastic rainfall generators, the emphasis is put on extreme events modelling to understand the main associated issues. The key challenges for the construction of a stochastic rainfall generator geared toward extreme events are then presented. Finally the third section is devoted to outlooks from an operational point of view and a possible framework for an integrated rainfall-induced urban flood crisis management system is proposed.

1.2 Sample hydraulic simulation of a rainfall-induced urban flood

The present section illustrates the sensitivity of the flooding pattern to the localization and extension of the rainfall field. A two-dimensional model of a part of the Ecusson district (Montpellier city, France) is built. Synthetic rainfall fields are used as inputs to the model in the form of a source term in the two-dimensional shallow water equations Guinot & Soares-Frazão (2006).

The rainfall fields are assumed time-independent and follow a radial distribution in the form

$$P_\theta(x,y,t) = P_{\max} \times \exp\left(-\frac{(x-x_0)^2+(y-y_0)^2}{R^2}\right) \forall t \tag{1}$$

where (x, y, t) are the space and time coordinates and $\theta = (x_0, y_0, R, P_{\max})$ with (x_0, y_0) the coordinates of the centre of the field, R a scaling radius and $P_{\max}$ the maximum precipitation. Four such fields are generated with the parameters in Table 1. It is acknowledged that the assumption of a time-independent and static rainfall field undermines the realistic character of the simulations. However, the purpose here is only to illustrate the sensitivity analysis of the computed runoff to the location of the centre of the rainfall field. Figure 1 shows the normalized rainfall fields $\frac{P_\theta}{P_{\max}}(x, y, t)$ for the four simulations. It should be noticed that these four rainfall fields have almost the same average value when averaged on the scale of a 1 km $\times$ 1 km radar pixel centred on the modelled area. Although unusual, intensities of 125 mm/day and 500 mm/day are commensurate with values observed in the South of France during extreme rainfall events (Delrieu *et al.* , 2005; Brunet *et al.* , 2018).

Simulation	$P_{\max}$ (mm/d)	R (m)	x_0 (m)	y_0 (m)
1	125	200	300	500
2	125	200	700	300
3	500	100	300	500
4	500	100	700	300

Table 1: *Rainfall field parameters*

Figure 2 shows the maximum water depth fields obtained by forcing a software package solving the two-dimensional shallow water equations (Guinot & Soares-Frazão (2006)) with the aforementioned rainfall fields. The maximum water depth field $h_{\max}(x, y)$ is computed from the simulated water depth field $h_\theta(x, y, t)$ as

$$h_{\max}(x,y) = \max_{0\le t\le T} h_\theta(x,y,t) \tag{2}$$

where T is the simulated period. In the present simulations, T = 15 minutes. This corresponds to the time needed for the hydraulic fields to reach the asymptotic, steady state under a

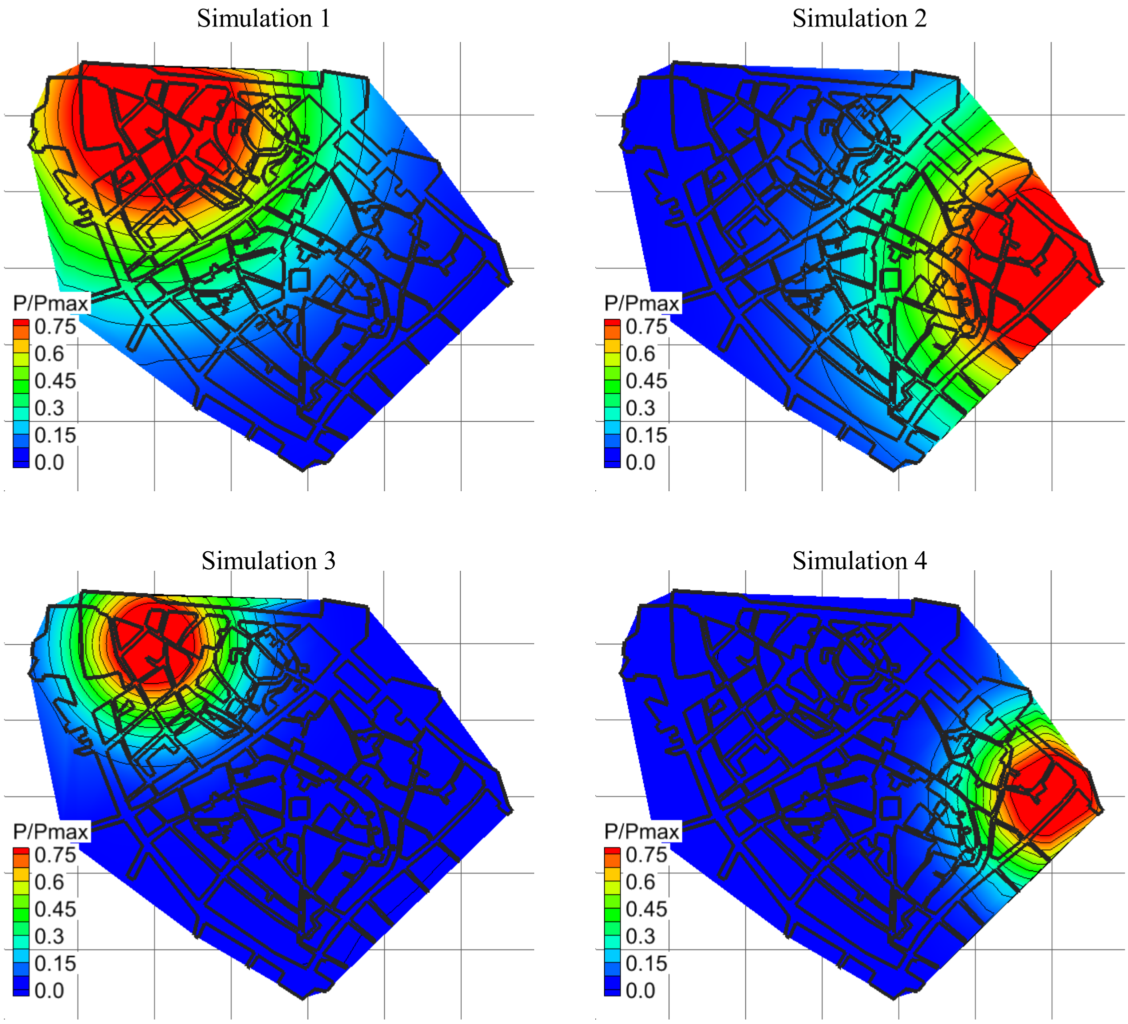

Figure 1: *Normalized rainfall fields* $P_\theta(x, y, t)/P_{\max}$. $x-$ *and* $y-$ *grid spacing: 100 m.*

steady state rainfall forcing. The maximum water depth is selected because it has been identified as danger indicator, in particular for pedestrians, and as a damage indicator for buildings (Blanco-Vogt & Schanze (2014), Merz *et al.* (2010), Wagenaar *et al.* (2016)).

Figure 2 allows the following conclusions to be drawn.

- The maximum rainfall intensity does not exert a significant influence on the range of $h_{\max}$. While $P_{\max}$ varies by a factor 4 between Simulations 1 and 3, it yields very similar $h_{\max}$ maps (see Figure 2, left). The same holds for Simulations 2 and 4 (see Figure 2, right). In contrast, $P_{\max}$ appears to be a controlling factor for the extension of the flooded area.
- Comparing Simulations 1 and 2 (Figure 2, top) with 3 and 4 (Figure 2, bottom) shows that shifting the centre (x_0, y_0) of the rainfall field from (300 m , 500 m) to (700 m , 300 m) also induces significant differences in the mapped $h_{\max}(x, y)$. The contrast is all the more significant as the scaling radius R is smaller.

To conclude, a 1 km $\times$ 1 km rainfall field resolution is too coarse for an accurate mapping of the flood hazard in urban areas. A 100 m × 100 m resolution appears more appropriate. Note that only static rainfall simulations are used in the present example, while rainfall fields are known to exhibit strong space-time dependencies (Cox & Isham, 1988; Kleiber *et al.* , 2012; Baxevani & Lennartsson, 2015) especially at high spatio-temporal resolution, see for instance Benoit *et al.* (2018). Since free surface flows are governed by wave propagation phenomena, it is most likely that the need for highly spatially resolved rainfall fields also comes with a similar need for a high temporal resolution. This latter aspect is not covered in the present chapter for the sake of conciseness.

Therefore, any (deterministic or stochastic) rainfall field generator should be developed and used with the following questions in mind : (i) what is the minimum required spatial and temporal resolution to provide a sound assessment of the free surface flow variables? (ii) how can the temporal and spatial distribution of rainfall over a given area be reproduced?

Stochastic rainfall generators have the potential to characterize several aspects of the space-time dependence structure of rainfall fields. These are described in the next section, with an emphasis on extreme event modelling.

2 How

2.1 Spatial stochastic rainfall generator

Since the outputs of meteorological models are uncertain in essence, an alternative to providing (inevitably biased and inaccurate) deterministic rainfall fields consists in generating ensemble rainfall scenarios, the characteristics of which are controlled in a statistical fashion. These rainfall scenarios can supply boundary conditions for hydraulic models. The latter can simulate water depths, flow velocities and other hydraulic variables, that are in turn used to assess flood risk and to map damages. To generate realistic ensemble rainfall scenarios, including dry sequences, stochastic rainfall generators may be employed (Ailliot *et al.* , 2015).

Spatial generators, that are capable of simulating continuous rainfall fields, are complex probabilistic models that combine several stochastic mechanisms in order to reproduce various features of the rainfall fields. To estimate the parameters that tune the stochastic mechanisms of the generators, statistical inference schemes are designed to draw information from rainfall observation series, whether from rain-gauged stations or from other types of data such as radar data. Thanks to these inference schemes, simulations of the generators are expected to be as close as possible to reality.

One of the foremost features that stochastic rainfall generators seek to reproduce is the alternance of periods with and without rainfall. The alternance of rain and no rain can be thought of as basic **weather types** which are usually modelled with a Markov chain, hidden or not (Ailliot *et al.* , 2015). These weather types can be further refined by decomposing the rainy type into several sub-types such as drizzle, moderate intensities and heavy rainfall. The weather type decomposition is designed to account for the non-stationarity in time and eventually in space if the weather type description includes spatial information (Garavaglia *et al.* , 2010; Leblois,

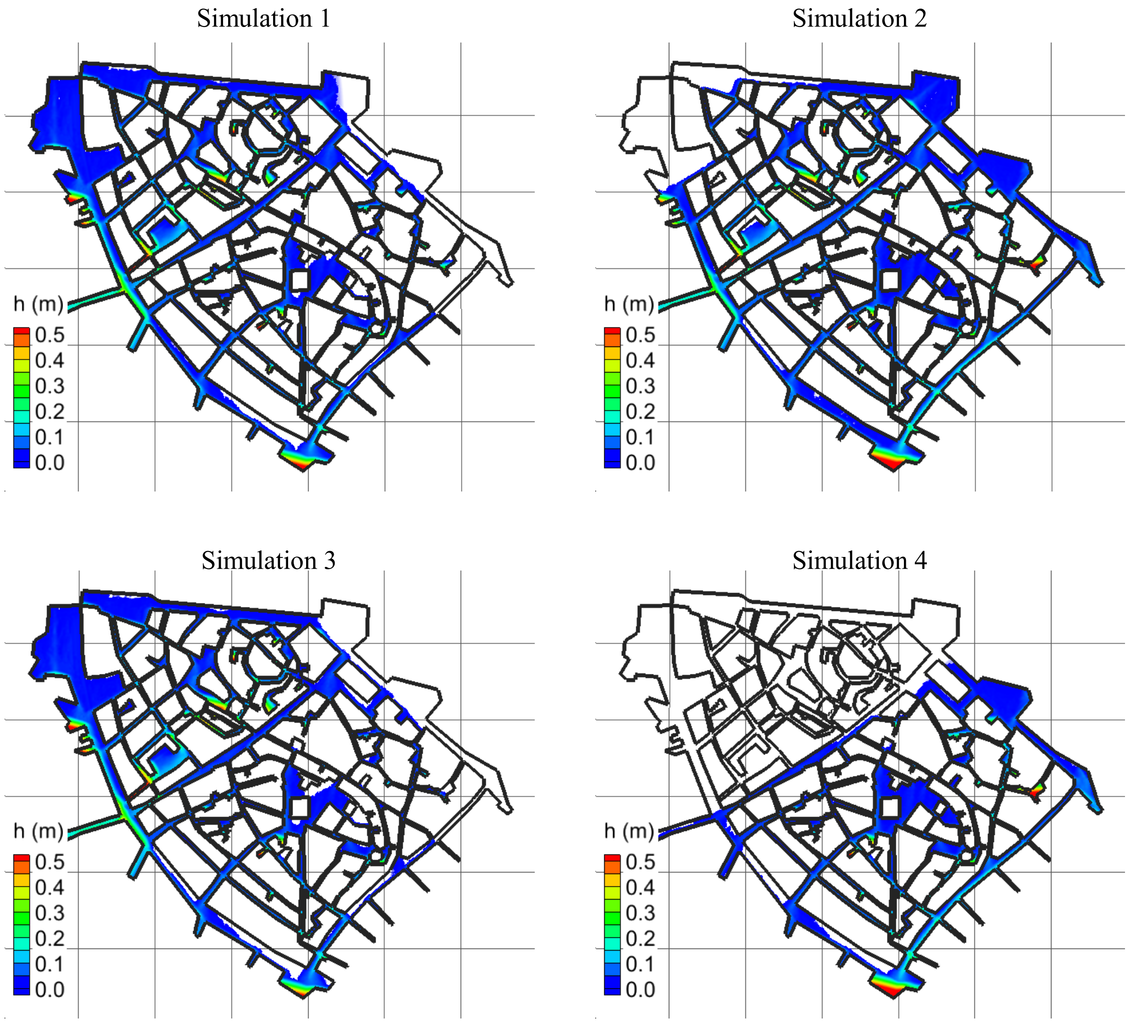

Figure 2: *Maximum simulated water depths $h_{\max}(x, y)$. $x-$ and $y-$ grid spacing: 100 m. The corresponding rainfall fields are shown in Figure 1.*

2012). Further spatial non-stationarity due to orographic effects can be modelled by resorting to landscape variables (Arnaud *et al.* , 2006).

A related feature is the so-called **intermittency** of rainfall, i.e. for a given time step, the alternance of areas with and without rainfall. Intermittency may be modelled by considering a binary random variable that indicates whether it is raining or not (Kleiber *et al.* , 2012; Leblois & Creutin, 2013). Alternatively, the random variable that models positive rainfall may be truncated below zero thereby enforcing, by construction, consistency in the rainfall amounts at the boundaries between dry and wet regions (Allard & Bourotte, 2014; Baxevani & Lennartsson, 2015).

Another core feature is the **spatio-temporal dependence** of the rainfall fields. Within rainy periods (that may be related to different weather types), the generator must simulate positive rainfall values, i.e. *rainfall intensities*, with spatio-temporal patterns similar to the observed ones. Rodriguez-Iturbe *et al.* (1987) and Cox & Isham (1988) propose spatio-temporal rainfall models considering that storm events induce a cluster of rain cells, which are represented as cylinders in space-time. Gaussian processes with spatio-temporal covariance functions can also be employed (Kleiber *et al.* , 2012; Baxevani & Lennartsson, 2015). However, the marginal distributions of rainfall intensities are well-known to be **non-Gaussian**. In particular, they are asymmetrical and, in some areas such as in the South of France, they are heavy tailed (Carreau *et al.* , 2017). This non-gaussianity of the marginals can be handled by a transformation to suitable univariate non-Gaussian distributions. Nevertheless, this increases significantly the complexity of the inference scheme for spatio-temporal gaussian processes (Kleiber *et al.* , 2012; Baxevani & Lennartsson, 2015). In addition, although anisotropy can be accounted for (Baxevani & Lennartsson, 2015), the dependence structure of gaussian processes may be inadequate owing to its symmetry (Carreau & Bouvier, 2016) or to its properties regarding extreme events (in particular, the asymptotic independence property, see § 2.2).

With the exception of Evin *et al.* (2018) in the multivariate case, no spatial stochastic rainfall generator has stochastic mechanisms specially dedicated to account for the spatio-temporal behaviour of **extreme events** (Ailliot *et al.* , 2015). This is a crucial feature in the context of urban flood risk studies. The following § 2.2 presents an overview of existing approaches to model extreme events, in terms of both intensity and spatial and spatio-temporal patterns.

2.2 Modelling extreme events

The key result of Extreme Value Theory (EVT) is due to Fisher & Tippett (1928) and has found many applications in domains such as finance and insurance (Embrechts *et al.* , 1997) along with environmental sciences (Katz, 2002). Interest in extreme value analyses in climate science is relatively recent (Kharin *et al.* , 2007; Goubanova & Li, 2006; Salvadori & Rosso, 2007). The result of Fisher & Tippett (1928) shows that the behaviour of the maximum, i.e. the largest value of a sample, correctly centered and standardized, converges in distribution to the Generalised Extreme Value (GEV) distribution. The GEV distribution encompasses the three extreme-value distributions that share the max-stability property, i.e. the maximum of two independent copies of random variables from a given extreme-value distribution belongs to the same extreme-value distribution.

In practice, modelling is performed on block maxima where each block represents a given time interval. Obviously, this approach is debatable since it may include rather low values for blocks in which no large events occurred and it leaves unexploited information contained in other large values of the sample, e.g. when several large events occurred in the same block. An alternative approach consists in considering the largest values of the sample defined as excesses above a high threshold. The distribution of these strictly positive excesses can then be approximated by a Generalised Pareto Distribution (GPD) (Pickands III, 1975).

In both cases (working on block maxima or excesses), we obtain an approximation of the upper tail of the distribution of the variable of interest. This allows to extrapolate beyond the largest observed values. In particular, probabilities of exceeding high thresholds or high quantiles, i.e. levels that are exceeded with a very low probability, can be estimated thanks to the upper tail approximation by the GEV distribution or by the GPD. Similar approaches allow to obtain a lower tail approximation.

In a spatial context, **max-stable processes** are the straightforward extension of the maxima approach defined previously since they appear as the natural limits for spatial maxima taken site by site (de Haan, 1984). By definition, a max-stable process fulfills the max-stability property which implies, in particular, that its margins follow a GEV distribution. More precisely, a stochastic process Z is max-stable if for each $n \geq 1$ there exist normalizing functions $a_n > 0$ and $b_n \in \mathbb{R}$ such that $\frac{\max_{i=1,\dots,n} Z_i(x) - b_n}{a_n} \stackrel{d}{=} Z$ with $Z_1, \dots, Z_n$ a sequence of independent copies of the stochastic process Z. Max-stable processes spawned a very rich literature with various proposed parametric models (Brown & Resnick, 1977; Smith, 1990; Schlather, 2002; Kabluchko *et al.* , 2009; Davison & Gholamrezaee, 2012; Opitz, 2013) and were widely used as models for spatial extreme events especially for environmental phenomena. A common representation of a max-stable process is the one due to Schlather (2002) in which the process $Z(x)$, a stationary max-stable process on $\mathbb{R}^p$ with unit Fréchet marginal distributions, is written as $\max_j S_j \max\{0, W_j(x)\}$, where $\{S_j\}_{j=1}^{\infty}$ are the points of a Poisson process on $\mathbb{R}^+$ with intensity ds/s^2 and $\{W_j(x)\}_{j=1}^{\infty}$ are independent replicates of a stationary process $W(x)$ on $\mathbb{R}^p$ satisfying $\mathbb{E} \max(0, W_j(o)) = 1$ with o denoting the origin.

As we aim to simulate fields of extreme phenomena, we focus in this chapter on unconditional stochastic simulations. As a first example of climate extreme modelling, Blanchet & Davison (2011) proposed complex models based on max-stable processes designed for heavy snow events. One of their results consists in a risk analysis by computing joint survival probabilities of group-wise annual maxima. Max-stable models can also help to characterize significant heights of extreme waves in the Golf of Lions (Chailan, 2014) and to establish different long-term scenarios of littoral erosion that depend on different inputs like wind.

Max-stable processes in space were extended to the space-time context in Davis *et al.* (2013a). Moreover Davis *et al.* (2013b) proposed statistical inference for such models based on pairwise likelihood. Huser & Davison (2014) obtained consistent estimation thanks to a pairwise censored likelihood to model extreme values of space-time rainfall data.

Max-stable processes being constructed from the pointwise maxima of underlying processes, Dombry *et al.* (2016) presented an algorithm for the exact simulation of a max-stable process at a finite number of locations by focusing on the processes that effectively contribute to the maxima. An extension with a reduced computational cost but restricted to Brown-Resnick processes is introduced in Oesting *et al.* (2018) when the number of locations is large.

The physical interpretation of spatial max-stable processes is not straightforward since they represent maxima of spatial fields. In general, one simulation of a spatial max-stable process do not represent a real event since pointwise maxima are likely to occur at different times. It is even more flagrant in a space-time framework where their use and interpretation for simulation purposes become extremely difficult.

As explained before, in the univariate context, an alternative to maxima models are **threshold exceedance models**. The generalisation of the GPD to spatial processes yields the so-called Pareto processes (Ferreira & de Haan, 2014; Opitz *et al.* , 2015; Dombry & Ribatet, 2015; Thibaud

& Opitz, 2015; de Fondeville & Davison, 2018). Constructively, the Pareto process corresponds to the product of a scale component and a component on the spatial structure called spectral process.

There is no unique definition of a spatial extreme event. Dombry & Ribatet (2015) defined the notion of ℓ-Pareto processes by considering general exceedances introducing a homogeneous cost functional denoted by ℓ. In practice, the choice of ℓ must mainly depend on the nature of the considered phenomenon. Possible examples are functions of the maximum, minimum, or mean. For rainfall, the spatio-temporal mean is a good option combining duration, spatial extent and magnitude of the event. Clear advantages of thresholding techniques are their potential to exploit information from more data and to explicitly model the original event. This last point is really essential for the stochastic spatial generator construction purposes.

So far, Pareto processes were mostly used in a parametric framework, thereby using assumptions on the choice of the underlying dependence structure that may be too strong. In this context, Thibaud & Opitz (2015) are interested in ℓ-Pareto processes and proposed an exact simulation procedure for the limiting processes of threshold exceedances of all asymptotically dependent elliptical processes. It is also possible to relax the parametric assumption for the spectral processes by relying on non-parametric estimates deduced from observed spectral processes.

Building on this non-parametric idea and stemming from original works of Caires *et al.* (2011) and Ferreira & de Haan (2014), Chailan *et al.* (2017) developed a **semi-parametric approach** to generate extreme spatio-temporal fields of waves in the Gulf of Lion (South of France). The first step consists in selecting space-time extreme events called storms among a 52-year hindcast of wave features over the north-western Mediterranean sea. In the second step, the selected storms are uplifted after proper standardisation. As a result, extreme storms of greater intensity than observed ones are generated and are illustrated on a case-study concerning the quantification of the long-shore mass flux of energy in a coastal area. Palacios-Rodríguez *et al.* (2018) extended this semi-parametric approach by setting up a sound space-time framework thanks to links with Pareto processes. A key benefit of the proposed method is the possibility to generate an unlimited number of realisations of extreme storms. Another extension concerns the selection of extreme episodes with a general space-time cost functional ℓ quantifying how extreme episodes are, in other words the extremeness of episodes.

The aforementioned simulation approaches for spatial extremes, in the max-stable framework, rely on the hypothesis of asymptotic dependence. This entails that dependence is assumed to remain constant regardless of the extreme level under consideration. These approaches are not suitable when the dependence strength decreases at high levels and may vanish ultimately. This behaviour, called **asymptotic independence (AI)**, is very difficult to detect in practice. However, analyses of hourly precipitation in the South of France (Bacro *et al.* , 2019) suggest AI behaviour (see also Davison *et al.* (2013); Thibaud *et al.* (2013); Le *et al.* (2018)). Models allowing for AI behaviour requires the development of specific approaches. Stationary Gaussian processes are examples of AI processes because, except in case of perfect correlation, bivariate Gaussian variables are AI (Sibuya, 1960). AI processes can also be obtained by inverting max-stable processes (Wadsworth & Tawn, 2012) or as pointwise maxima of samples from a ratio of

Gaussian processes with common correlation function (Padoan, 2013).

Models with flexible dependencies such as max-mixture models (Wadsworth & Tawn, 2012; Bacro *et al.* , 2016) for maxima and other processes for threshold exceedances such as Gaussian scales mixture processes or related works (Opitz, 2016; Huser *et al.* , 2017; Huser & Wadsworth, 2018) constitute inspiring and promising works. Following an idea of Wadsworth & Tawn (2012), Bacro *et al.* (2016) exploited a max-mixture approach to propose a general spatial model which is capable to deal with extremal dependence at small distances, possible independence at large distances and AI at intermediate ones.

These models, allowing AI behaviour commonly assume temporal independence for inference purposes. However, developing flexible space-time modelling for extremes is crucial to characterize the temporal dynamics and the persistence of extreme events spanning several time steps. Bacro *et al.* (2019) proposed a two-stage model for spatio-temporal exceedances that remains physically interpretable in an AI context. Following Bortot & Gaetan (2014), they use the representation of the GPD as a Gamma mixture of an exponential distribution to formulate a hierarchical model integrating space-time dependence thanks to a latent space-time Gamma process (Wolpert & Ickstadt, 1998). This Gamma process relies on an elliptical cylinder allowing a nice physical interpretation in terms of storms. Statistical inference of model parameters is performed thanks to a pairwise log-likelihood for the observed censored excesses. The interest of this model was exemplified on a real dataset of rainfall in the South of France and it was validated by computing empirical estimates of various multivariate conditional probabilities involving spatio-temporal aspects. This hierarchical model is related to the temporal trawl processes (Barndorff-Nielsen *et al.* , 2014; Noven *et al.* , 2015) of which Opitz (2017) proposed spatial extensions.

2.3 Stochastic rainfall generator geared toward extreme events

As argued in § 2.1, to be useful for urban flood risk studies, a spatial stochastic rainfall generator should be capable of mimicking the spatio-temporal patterns of extreme events. Therefore, the stochastic mechanisms associated with extreme events should draw from the modelling techniques described in § 2.2. We identify the following three key challenges.

As extreme events modelling is not adapted for regular events, a natural strategy would be to define a special weather type dedicated to extreme events. To our knowledge, in existing approaches, although extreme weather type may be identified *a posteriori*, no particular technique has been proposed for their identification (Leblois, 2012). Both intensity and spatio-temporal pattern information should enter in the definition of this extremal weather type. To this end, non-parametric descriptors of the extremal dependence structure, such as the madogram, could be useful (Cooley *et al.* , 2006; Vannitsem & Naveau, 2007; Erhardt & Smith, 2012).

Within the weather type dedicated to extreme events, regular and extreme events are likely to be present in different areas and in different time periods. Transitions between regular and extreme events must be modelled both in the univariate marginal distributions and in the spatio-temporal dependence structure. Therefore, a second issue to consider is the need to rely on mixed univariate marginal distributions that can characterize both regular and extreme events. To this end, Carreau & Bengio (2009) stitched together a Gaussian distribution for the lower part and a GPD for the upper tail. This hybrid Pareto distribution was used in a mixture model. It was shown to be able to adapt to various complex heavy tailed distributions. One drawback of this approach is the presence of a potentially non-negligible lower tail which may be inadequate to model phenomena such as precipitation. A recent alternative to this hybrid was proposed in Naveau *et al.* (2016) who take benefit from EVT for both large and low values (excluding zeros). Their statistical model ensures a smooth transition between the upper and the lower tails with a reasonable number of parameters.

The last and probably most challenging issue concerns the need to design spatio-temporal processes with transitions in the dependence structure between extreme and ordinary events. As far as we know, there is no proposition along these lines. Nevertheless, one possibility to simulate spatial fields that contain both regular and extreme events is to rely on the approach in Thibaud *et al.* (2013) in which a single dependence structure, inherited from either a max-stable or an inverted max-stable process, is employed. A related problem is the non-stationarity of the dependence structure in space and time where few proposals exist. For instance, Fox & Dunson (2015) developed dynamic latent factor models in the Gaussian framework. In the max-stable framework, Huser & Genton (2016) integrated covariates in the dependence structure. In both cases, the dependence structure is non-stationary but is not able to change of distribution family, as would be required to make a transition between ordinary and extreme rainfall events.

3 Outlook

As far as an operational use is concerned, using the fields from a stochastic rainfall generator as inputs for a free surface flow model such as that presented in Section 1 raises a number of issues. In each of the 15 minute shallow water simulations presented in Section 1, 2 CPU seconds are needed to simulate 1 second using a standard PC. This is because the urban geometry is complex. Refined computational grids are needed to capture hydraulic singularities accurately. The typical computational cell width in a two-dimensional urban shallow water model is 1 m. This precludes refined shallow water models to be used on the scale of the district (let alone the entire conurbation) for real-time purposes. Besides, a stochastic rainfall generator implies that many realizations of the rainfall field are to be used as inputs for as many shallow water simulations, so as to obtain a statistical description of danger/damage over the area of concern. This increases the computational effort even more.

An alternative consists in upscaling the free surface flow model. Fast-running free surface flow models can be obtained by averaging the shallow water equations over large domains (the size of e.g. a house block). The properties of the buildings and other singularities that influence the flow are described in a statistical fashion. A key parameter that emerges from averaging is the porosity of the urban medium, that is, the plan view fraction of the urban area available for the storage and transport of water. A wide variety of porosity models (Guinot & Soares-Frazão (2006); Sanders *et al.* (2008); Guinot (2012); Özgen *et al.* (2016); Guinot *et al.* (2017, 2018);

Viero (2019)) have been proposed and the field is in rapid development. Porosity models have been reported to run two to three orders of magnitude faster that their classical shallow water counterparts (Sanders *et al.* (2008); Guinot *et al.* (2018)). Such computational rapidity is compatible with the stochastic simulation of multiple rainfall scenarios.

A possible framework for an integrated rainfall-induced urban flood crisis management system could be the following.

1. Generate stochastic rainfall fields on the scale of the conurbation, with the temporal and spatial resolution required by the free surface flow model (i.e. 100 m, see Section 1).

2. Use the multiple realizations of the rainfall fields to run fast, porosity-based shallow water simulations. This allows the urban areas with the higher risk to be identified on a coarse scale.

3. Select those areas where the flood risk has been identified as the higher, and where a detailed mapping of the risk on the metric scale is deemed necessary. For these, run refined simulations covering the (limited) spatial extension where the detailed mapping is needed. To do so, the initial and boundary conditions for the refined flow model must be interpolated from those of the porosity model. The same goes with the rainfall field.

Although simple in its principle, the above sequence is not straightforward to implement.

Concerning point 1, with a few exceptions (Benoit *et al.* , 2018), observation series are not available over a sufficiently dense network of sites and over long enough observation periods at the desired resolution. In these cases, downscaling techniques can be employed by making use of auxiliary information such as rainfall radar data (Delrieu *et al.* , 2014). Few downscaling techniques were proposed in the extreme value framework. Very recently, Engelke et al (2019) developed a method based on a theoretical link between the extremal distribution of the aggregated data and the corresponding underlying process.

Concerning Step 3, very little is known about the constraints attached to initial and boundary condition scale transfer from the porosity model to the local, refined shallow water model. This specific issue is currently under study in the Inria Lemon research team.

As far as urban floods are concerned, the generation of rainfall scenarios is not the only possible application field of the extreme value theory. In coastal areas, urban flooding may also result from storm surges. Applying the EVT to wave forcing so as to obtain probabilistic assessments of the marine submersion risk is also a path for research.

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